ESTABLISHING THE EQUATIONS THAT GENERATE THE VIBRATION MOVEMENT OF THE MAIN SPINDLE IN TRANSVERSAL DIRECTION AT CNC LATHE

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ABSTRACT

The paper presents a mathematical model for determining the movement equations that generate the transversal vibration of the main spindle at CNC lathe. For this purpose, we take into account the general equations which generate the vibration of the spindle, obtained by means of Hamilton's Variational Principle as well as the impulse derivative and kinetic moment axioms. The solutions to the movement equations are arranged under the form of a system of first order partial derivatives equations, for which we determine the integration constants. This original way of establishing these equations allows further analysis of the main spindle vibrations in transversal direction and finding ways to decrease the vibration amplitude.

Keywords: Transversal Vibration; CNC Lathe; Main Spindle; Movement Equations; Hamilton's Variational Principle.

1. INTRODUCTION

Large scale usage of CNC machines is a significant step forward for increasing the processing capability, dimensional precision and surface quality [1]. It fills the need for producing highly complex products and achieving cost productive automation but at the same time requires research in the particular domain [2]. Given the specific structure, the high build quality and the use of special or adequate tools, the CNC machine tools must ensure a corresponding dynamic stability [3]. For this reason, the dynamic analysis of the elastic structure and mainly of the main shaft of the machine tool provides for creating equipment with far much superior processing characteristics [4].

2. ESTABLISHING THE MOVEMENT EQUATIONS

In order to determine the movement equations, a series of simplifying assumptions is required in order to establish the mathematical model for analyzing the vibrations of the main spindle of the CNC machine tool (e.g. CNC lathe).

- There are no additional distributed superficial forces or couples on the external surface of the shaft (the functioning is in natural environment only);
- There are no supplementary links that could leads to occurrence of shocks;
- The initial shaft state is considered tension-free (no remnant tensions, only elastic domain stresses might occur);
- A plane section perpendicular on the shaft axis remains plane but not necessarily perpendicular. Although the considered section can move under deformation, it will be

considered plane such that taking into account the previous assumption the approximation of the deformations and tensions will be as close as possible to the real case.

The mathematical model used to generate the movement equations is based on Hamilton's variational principle [5].

We consider the movement equations that generate the vibration movement of the main spindle at CNC lathe [6]. For simplicity, the transversal equations in X_1OX_2 and X_1OX_3 planes as being decoupled, non-interfering with each other.

$$\begin{aligned}
&\rho A \ddot{v}_1 - EAv_{1,11} = 0\\
&\rho A \ddot{v}_3 - \rho I \ddot{v}_{3,11} + 2\rho I \Omega \dot{v}_{2,11} + \rho I \Omega^2 v_{3,11} + EIv_{3,1111} = 0\\
&-\rho A \ddot{v}_2 + \rho I \ddot{v}_{2,11} + 2\rho I \Omega \dot{v}_{3,11} - \rho I \Omega^2 v_{2,11} - EIv_{2,1111} = 0
\end{aligned} \quad (1)$$

Where: ρ – specific mass

A – section area

I – inertial moment in transversal section of spindle

Ω – angular rotation speed of the spindle

We further study only transversal vibrations in one plane, the study of others being similar. The movement equation in transversal direction becomes:

$$\rho A \ddot{v}_3 - \rho I \ddot{v}_{3,11} + \rho I \Omega^2 v_{3,11} + EIv_{3,1111} = 0 \quad (2)$$

Applying the Laplace transform with respect to time, under null initial conditions, it becomes:

$$s^2 \rho A \bar{v}_3 - s^2 \rho I \bar{v}_{3,11} + \rho I \Omega^2 \bar{v}_{3,11} - EI\bar{v}_{3,1111} = 0 \quad (3)$$

The equation (3) can be written under the form of a system of first order differential equations:

$$\{\bar{v}\}_{,1} = [A]\{\bar{v}\} \quad (4)$$

Where: $\{\bar{v}\} = \{\bar{v}_3; \bar{v}_{3,1}; \bar{v}_{3,11}; \bar{v}_{3,1111}\}^t$ (5)

and
$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{s^2 \rho A}{EI} & 0 & \dfrac{\rho}{E}(s^2 - \Omega^2) & 0 \end{bmatrix} \quad (6)$$

Thus, the solution of the system (4) becomes:

$$\{\bar{v}\} = e^{[A]x_1}\{v_0\} \tag{7}$$

Where $\{v_0\} = \{a_1, a_2, a_3, a_1\}$ represents the integration constants vector (8)

Next we compute

$$\det[p[I] - [A]x_1] = 0 \tag{9}$$

which is a bi-square equation with unknown variable p, with the solutions:

$$p_{1,2} = \pm i \cdot A(s)x_1 \tag{10}$$

$$p_{3,4} = \pm i \cdot B(s)x_1 \tag{11}$$

where:

$$A(s) = \frac{i}{\sqrt{2}}\sqrt{\frac{\rho}{E}(\Omega^2 - s^2) + \sqrt{\frac{\rho^2}{E^2}(\Omega^2 - s^2)^2 - 4s^2\frac{\rho A}{EI}}} \tag{12}$$

$$B(s) = \frac{i}{\sqrt{2}}\sqrt{\frac{\rho}{E}(\Omega^2 - s^2) - \sqrt{\frac{\rho^2}{E^2}(\Omega^2 - s^2)^2 - 4s^2\frac{\rho A}{EI}}} \tag{13}$$

We construct the polynomial [2]:

$$R(X) = CX^3 + DX^2 + EX + F \tag{14}$$

in which the constants C, D, E, F are given by:

$$\begin{cases} R(p_1) = e^{p_1} \\ R(p_2) = e^{p_2} \\ R(p_3) = e^{p_3} \\ R(p_4) = e^{p_4} \end{cases} \tag{15}$$

Replacing (12), (13) into (15) we obtain:

$$F = \frac{A^2\cos(Bx_1) - B^2\cos(Ax_1)}{A^2 - B^2}$$

$$D = \frac{\cos(Bx_1) - \cos(Ax_1)}{(A^2 - B^2)x_1^2}$$
$$E = \frac{A^3 \sin(Bx_1) - B^3 \sin(Ax_1)}{AB(A^2 - B^2)x_1}$$
$$C = \frac{A\sin(Bx_1) - B\sin(Ax_1)}{AB(A^2 - B^2)x_1^3} \tag{16}$$

The matrix $e^{[A]x_1}$ has the form:

$$e^{[A]x_1} = C[A]^3 x_1^3 + D[A]^2 x_1^2 + E[A]x_1 + F[I] \tag{17}$$

Replacing C, D, E, F and A into (17) by identification we obtain the components of the matrix $e^{[A]x_1}$, namely e11, e12, e13, …, e44.

The solution of the system on interval i is:

$$\begin{aligned}
\bar{v}_3^i &= e_{11}a_1^i + e_{12}a_2^i + e_{13}a_3^i + e_{14}a_4^i \\
\bar{v}_{3,1}^i &= e_{21}a_1^i + e_{22}a_2^i + e_{23}a_3^i + e_{24}a_4^i \\
\bar{v}_{3,11}^i &= e_{31}a_1^i + e_{32}a_2^i + e_{33}a_3^i + e_{34}a_4^i \\
\bar{v}_{3,111}^i &= e_{41}a_1^i + e_{42}a_2^i + e_{43}a_3^i + e_{44}a_4^i
\end{aligned} \tag{18}$$

In order to find the integration constants $a_1^i, a_2^i, a_3^i, a_4^i, i \in \{0,1,2\}$, we impose the limit conditions:

$$\begin{aligned}
&x_1 = 0 \\
&EI\bar{v}_{3,111}^{(0)}(0) + \rho I\Omega^2 \bar{v}_{3,1}^{(0)}(0) = 0 \\
&EA\bar{v}_{3,11}^{(0)}(0) = 0
\end{aligned} \tag{19}$$

$$\begin{aligned}
&x_1 = l_1 \\
&EI(\bar{v}_{3,111}^{(1)}(l_1) - \bar{v}_{3,111}^{(0)}(l_1)) + \rho I\Omega^2 (\bar{v}_{3,1}^{(1)}(l_1) - \bar{v}_{3,1}^{(0)}(l_1)) = -k_{33}\bar{v}_3^{(0)}(l_1) \\
&EI(\bar{v}_{3,11}^{(1)}(l_1) - \bar{v}_{3,11}^{(0)}(l_1)) = -k_{54}\bar{v}_{3,1}(l_1) \\
&\bar{v}_3^{(0)}(l_1) = \bar{v}_3^{(1)}(l_1) \\
&\bar{v}_{3,1}^{(0)}(l_1) = \bar{v}_{3,1}^{(1)}(l_1)
\end{aligned} \tag{20}$$

$$\begin{aligned}
&x_1 = l_2 \\
&EI(\bar{v}_{3,111}^{(2)}(l_2) - \bar{v}_{3,111}^{(1)}(l_2)) + \rho I\Omega^2 (\bar{v}_{3,1}^{(2)}(l_2) - \bar{v}_{3,1}^{(1)}(l_2)) = -k_{33}\bar{v}_3(l_2)
\end{aligned}$$

$$EI(\bar{v}^{(2)}_{3,11}(l_2)-\bar{v}^{(1)}_{3,11}(l_2))=-k_{54}\bar{v}^{(1)}_{3,1}(l_2)$$
$$\bar{v}^{(1)}_{3}(l_2)=\bar{v}^{(2)}_{3}(l_2)$$
$$\bar{v}^{(1)}_{3,1}(l_2)=\bar{v}^{(2)}_{3,1}(l_2) \quad (21)$$

$$x_1=l$$
$$EI\bar{v}^{(2)}_{3,111}(l)+\rho\Omega^2\bar{v}^{(2)}_{3,1}(l)+m_p\Omega^2\bar{v}^{(2)}_{3}(l)=-\bar{R}^e_3$$
$$EI\bar{v}^{(2)}_{3,11}(l)-J_0\Omega^2\bar{v}^{(2)}_{3,1}(l)=-\bar{M}^e_{R_2} \quad (22)$$

Where: J0 is the inertial moment of the tool with respect to the symmetry axis (Fig. 1);
$R^e_3, M^e_{R_2}$ are the bending force and moment due to grinding;
k33, k54 are the bearing rigidities, usually having constant values.

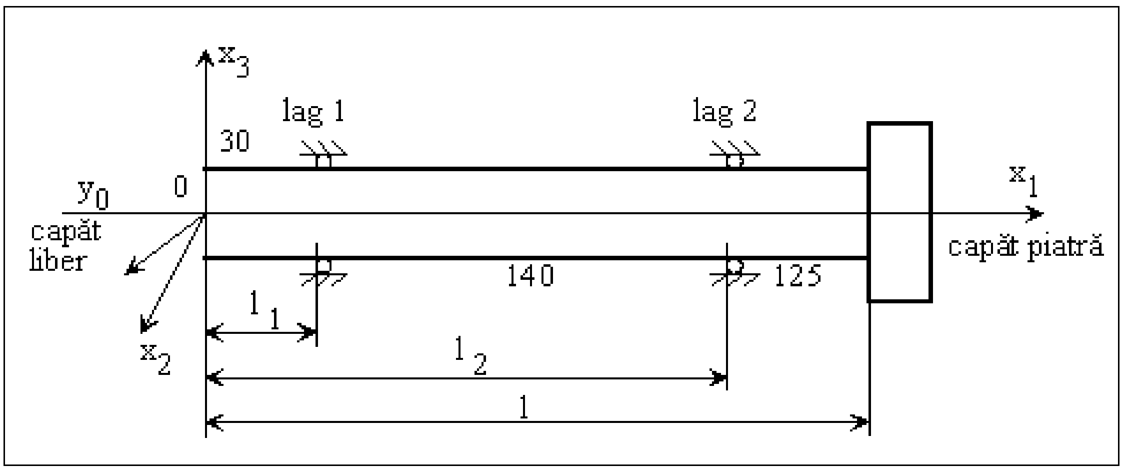

Figure 1:

The system formed of equations (19), (20), (21) and (22) is linear and can be solved by means of Kramer's formulas.

By solving the system, the field of transversal displacements on each interval $v^{(0)}_3, v^{(1)}_3, v^{(2)}_3$ can be obtained from (18) by applying the inverse Laplace transform.

3. CONCLUSIONS

1) Establishing the movement equations that generate the transversal vibrations of the main spindle of the CNC lathe are the basis for analyzing the spindle stability [7].
2) The mathematical model used is based on Hamilton's variation principle;
3) The calculus is simplified by particularization of the transversal equations with respect to the axis system x_1Ox_2 și x_1Ox_3;
4) The results obtained are a system of 1st degree differential equations;
5) The main contact points on the bearings and between the tool and workpiece are shown in Fig. 1 in case of CNC milling operation.

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