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CALCULATION OF LINEAR FRACTIONAL FUZZY TRANSPORTATION PROBLEM USING SIMPLEX METHOD

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ABSTRACT

In this research article, we implement a methodology for solving fuzzy transportation problems involving linear fractional fuzzy numbers. The main aim of this paper is to find optimum values of the fuzzy transportation problems by simplex method with the help of a triangular fuzzy number (TFN) as the costs of objective function. The outcome of this method is explained with a numerical example.

Keywords: Transportation Problem, Linear Fractional Fuzzy Programming Problem, Linear Fractional Fuzzy, Simplex Method, Triangular Fuzzy Number

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1. INTRODUCTION

Operation research is widely used for developing a new method in case of real-world problem. "This transportation problem was first introduced by Hitchcock in 1941 Hitchcock (1941). The objective of this concept is to find an optimal solution of the transportation problem in case of economics and Mathematics". This problem was first introduced by the French mathematician Gaspard Monge.

For solvtation of transportation problem, all the parameters like supply, demands and unit transportation cost are represented in a crisp value. These values can also be represented in case of fuzzy numbers. If the cost associated in transportation problem are fuzzy, then the optimal solution of the problem will be fuzzy, then this type of problem is termed as fuzzy transportation problem Charnes and Cooper (1973).

Fractional transportation problem be a unique class of mathematical technique, in which all the constraint variables are form of linear and in the objective function is upgraded into two linear functions Chandra (1968). In 1960, Hungarian mathematician B. Metros first exposed the linear fractional problem. In actual existence problem, this idea is commonly utilized in stock income real cost-preferred cost, and income cost Pandian and Jayalakshmi (2013), Bitran and Novaes (1972) and additionally it enables in finance & business etc. In this paper, we proposed a technique for finding an optimal solution of transportation problem using simplex method. The linear fractional problem i.e.,

$$\text{Max } z = \frac{(cx+\alpha)}{(dx+\beta)}$$

Where β 0. In section 2, we discuss some basic definitions on fuzzy set theory with properties and theorems. In section 4, we anticipated an algorithm about the LFFPP. In next section, we discuss a simple numerical example on LFFPP by simplex method. In last section, we provided the concluding remarks

2. PRELIMINARIES

This section contains basic preliminary definitions and its some properties, which will be used in a sequel.

2.1. DEFINITIONS

1) **Fuzzy set**: Let X be a general set and x be a member of X, then fuzzy set on X is denoted by a membership value μ_ () (x), which identifies the function maps from every element to the interval Bitran and Novaes (1972) and it can be defined as

$$\overline{A} = \{(x, \mu_{\overline{A}}(x)), x \in X\}$$

$$\text{Where } \mu_{\overline{A}}(x): x \to [0,1]$$

2) **α − cut**: Let be a fuzzy set on X and α be a real number between Bitran and Novaes (1972), then cut of $\overline{A}$ is determined by

$$\alpha_{\overline{A}} = \{x \in X: \mu_{\overline{A}}(x) \geq \alpha\}$$

3) **Strong α − cut**: Let $\overline{A}$ be a fuzzy set on X and α be a real number between [0, 1], then strong α − cut of fuzzy set $\overline{A}$ is defined as

$$\alpha_{\overline{A}} + = \{x \in X: \mu_{\overline{A}}(x) > \alpha\}$$

4) **Support of a fuzzy set**: Consider X be a crisp set and $\overline{A}$ be a fuzzy set associate on X. Then support of $\overline{A}$ is described by

$$\boldsymbol{Supremum}\,(\overline{A}) = \{\boldsymbol{x} \in \boldsymbol{X}: \mu_{\overline{A}}(\boldsymbol{x}) > 0\}$$

5) **Normal Fuzzy set**: Let X be a universal set $\overline{A}$ be a fuzzy section X. Then normal of fuzzy set is termed as

$$Sup_{x \in X}\, \mu_{\overline{A}}(x) = 1$$

6) **Convex of fuzzy set**: consider X be a crisp set. A fuzzy subset $\overline{A}$ on X is convex if and only if

$$\mu_{\overline{A}}(\boldsymbol{\lambda x} + (\mathbf{1} - \boldsymbol{\lambda})\,\boldsymbol{y}) \geq (\mu_{\overline{A}}(\boldsymbol{x}) \wedge \mu_{\overline{A}}(\boldsymbol{y})), \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{X} \text{ and } \boldsymbol{\lambda} \in [\mathbf{0}, \mathbf{1}]$$

7) **Fuzzy Number**: A fuzzy set $\overline{A}$ on X is a fuzzy number if and only if it is normal and convex in X

8) **Triangular fuzzy number**: The triangular fuzzy number of fuzzy sets $\overline{A}$ is a triplet product [*l, m, n*] and its value of it corresponding the membership function is defined by

$$\mu_{\overline{A}}(X) = \begin{cases} 0\ , & x \leq l \text{ or } x \geq n \\ \frac{(x-l)}{(m-l)}\ , & l \leq x \leq m \\ \frac{(n-x)}{(n-m)}\ , & m \leq x \leq n \end{cases}$$

Where l and n are representing the lower and the upper boundaries respectively, with membership degree is 0 and m is the centre with membership degree is 1

9) **Arithmetic Operations**: Let $\overline{A}$ = (l_1, m_1, n1) and = (l_2, m_2, n_2) be two triangular fuzzy numbers, then

- **Addition**

$$\overline{A} + \overline{B} = (l_1 + l_2, m_1 + m_2$$

- **Substraction**

$$\overline{A} - \overline{B} = (l_1 - l_2) + (m_1 - m_2) + (n_1 - n_2)$$

- **Multiplication**

$$\overline{A} \times \overline{B} = l_1 l_2, m_1 m_2, n_1 n_2$$

- **Scalar Multiplication**

$$k(l_1, m_1, n_1) = (kl_1, km_1, kn_1) \quad \text{if } k > 0$$

$$k(l_1, m_1, n_1) = (kn_1, km_1, kl_1) \quad \text{if } k < 0$$

- **Division**

$$\frac{\overline{A}}{\overline{B}} = \frac{l_1}{l_2}, \frac{m_1}{m_2}, \frac{n_1}{n_2}$$

10) **Ranking Function**: The ranking function "R" is a mapping from each fuzzy number into the set of real line and it is defined by

$$R : F(R) \rightarrow R,$$

Where the *F(R)* consists of triangular fuzzy numbers.

If $\overline{A} = (l, m, n)$ be the triangular fuzzy number, then the corresponding ranking function of $\overline{A}$ is given by

$$R\left(\overline{A}\right) = \frac{l+4m+n}{6}$$

11) **Efficient point**: A point x0 is called to be an efficient point if there does not exist other feasible point x other than x0,

i.e

$$R\left(\overline{A}\right) = \frac{l + 4m + n}{6}$$

$$where\ i = 1, 2.$$

Theorem 1. Narayanamoorthy and Kalyani (2015) Suppose X_t is the set of efficient points for P_t, t = 1, 2, then X_t is a subset of the set of all efficient points X of P

Theorem 2. Hitchcock (1941) Suppose $\overline{X} = \{x_i \in P_i\}$ be an optimal solution of P_1 and = {x_i ∈P} be an another optimal solution of P_2,then $x = \frac{\overline{X}}{\overline{X}}$ is an optimal solution of P,

Where all elements of x_1, x_2. are in P_1 and P_2.

1) Linear Fractional Fuzzy Programming Problem (LFFPP)

In this section, we expand the linear fractional problem into linear fractional fuzzy programming problem. The LFFPP can be considered as:

$$\max\ f(x) = \frac{p(x)}{q(x)}$$

Subject to

$$x \in S \subset R^n, \qquad \textbf{Equation 1}$$

Where *p(x)* and *q(x)* are continuous linear functions and $q(x) > 0$, x now the set S is designated by

$$\bar{S} = \{x \mid \bar{A}x = b, x > 0\},$$

Where S is a bounded polyhedron and $\bar{A}$ is a fuzzy matrix of order. Now n×n the above equation may be represented as

$$maxp(x)$$

$$subject\ to\ x \in S \subset R^n,$$

$$maxp(x)$$

$$subject\ to\ x \in S \subset R^n. \qquad \textbf{Equation 2}$$

Then by using above theorem-2, we have the set of efficient points for p_t, t = 1, 2 is a subset of *p*.

3. ALGORITHM

This section deals with a methodology for solving a linear fractional fuzzy transpotation problem (LFFTP). The LFFTP can be considered as

$$p: minZ = \frac{\sum_{i=1}^{n} c_i x_i}{\sum_{i=1}^{n} d_i x_i}$$

$$\sum_{i=1}^{n} c_i x_i = l_j$$

$$\sum_{i=1}^{n} d_i x_i = m_j$$

$$xi \gg 0, \qquad \textbf{Equation 3}$$

Where the objective function is of fractional type. This problem can be decomposed into two linear fuzzy problem such as

$$p{:}\, minZ^N = \sum_{i=1}^{n} c_i {x_i}^N$$

Subject to

$$\sum_{i=1}^{n} c_i {x_i}^N = l_j$$

$$\sum_{i=1}^{n} d_i {x_i}^N = m_j$$

$$x_i^N \geq 0, \qquad \textbf{Equation 4}$$

$$P_2{:}\, minZ^D = \sum_{i=1}^{n} d_i {x_i}^D$$

Subject to

$$\sum_{i=1}^{n} c_i {x_i}^D = l_j$$

$$\sum_{i=1}^{n} d_i {x_i}^D = m_j$$

$$x_i^D \geq 0 \qquad \textbf{Equation 5}$$

Then by simplex method, we have to find the solution of two linear fuzzy problems. Thus, the solutions min z^N and min z^D are determined distinctly. Then by the above theorem, we have to obtain the optimal solution of *P*

4. NUMERICAL EXAMPLE

Here, we demonstrate a numerical example on linear fractional transportation problems under fuzzy values which are expressed by a triangular fuzzy number. Consider a LFFTP as:

$$P : min\ z = ((8, -2, -5)x_1 + (-10, 2, 6)x_2)$$

$$3x_1 + 2x_2 \leq 6,$$

$$4x_1 + 5x_2 \leq 10$$

$$x_1, x_2 \geq 0,$$ **Equation 6**

Where objective function is expressed in case of triangular fuzzy number and crisp values for case of supply and demands.

In computationally, this problem can be written as

$$P_1 : min\ z^N = ((8, -2, -5)x_1 + (-10, 2, 6)x_2$$

Subject to

$$3x_1 + 2x_2 \leq 6,$$

$$4x_1 + 5x_2 \leq 10$$

and $x_1, x_2 \geq 0,$

Subject to

$$P_2 : min\ zD = ((6, -4, -4)x_1 + (-4, 4, 4)x_2)$$

$$3x_1 + 2x_2 \leq 6,$$

$$4x_1 + 5x_2 \leq 10$$

$$x_1, x_2 \geq 0,$$ **Equation 7**

The standard form of the linear problem P1 can be expressed as

$$P : \max z = \left((8, -2, -5)x_1 + (-10, 2, 6)x_2\right)$$

Subject to

$$3x_1 + 2x_2 + s_1 = 6$$

$$4x_1 + 5x_2 + s_2 = 10$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Where s_1 and s_2 are basic slack variables.

Now to solve

LFTP $'P_1'$ with the help of simplex method. The initial simplex table is given as

Table 1

		cj	(-8,2,5)	(10, -2, -6)	(0,0,0)	(0,0,0)3	Min
CB	Y_B	X_B	X_1	X_2	S_1	S_2	
(0,0,0)	S_1	6	3	2	1	0	2
(0,0,0)	S_2	10	4	5	0	1	2.5
	Z_j	0	0	0	0	0	
	Z_j-C_j		(-5, -2, -8)	(6,2, -10)	0	0	
	R		-0.84	0.67	0	0	

In this above table, we have one zj cj value is non-positive. So, the feasible optimal solution has been not reached. The non -basic variable s1 will go away the basis cell and the basic variable x1 come into the basis cell.

Table 2

		cj	(-8,2,5)	(10, -2,6)	(0,0,0)	(0,0,0)3
C_B	Y_B	X_B	X_1	X_2	S_1	S_2
(-8,2,5)	X_1	2	1	2/3	1/3	0
0	S_2	2	0	7/3	-4/3	1
	Z_j	(-16,4,10)	(-8,2,5)	(-16/3,4/3,10/3)	(-8/3,2/3,5/3	(0,0,0)
	Z_j-C_j		0	(-46/3,10/3,28/3)	(-8/3,2/3,5/3)	0
	R		0	1.23	0.28	0

Now using the procedure of simplex method, we get the following table.

This table shows that all zj - cj ≥ 0 so optimal solution to the LFTP is obtained

Thus, the solution is

$$x_1 = 2,\ x_2 = 0,\ max\ z^N = (-16, 4, 10)$$

i.e.

$$x_1 = 2,\ x_2 = 0,\ min\ z^N = (-10, -4, 16). \qquad \textbf{Equation 8}$$

Similarly consider the standard form of the linear problem P2 can be expressed as

$$P_2:\ \ max\ \ z^D = ((6, -4, -4)\, x_1 + (-4, 4, 4)\, x_2)$$

Subject to

$$3x_1 + 2x_2 + s_1 = 6$$

$$4x_1 + 5x_2 + s_2 = 10$$

$$x_1,\ x_2, s_1, s_2 \geq 0, \qquad \textbf{Equation 9}$$

Where s_1 and s_2 are slack variables. Now to solve the LFTP "p_2" and simplex method. The primary datas are given below.

Table 3

		C_j	(-6,4,4)	(4, -4, -4)	(0,0,0)	(0,0,0)3	Min
C_B	Y_B	X_B	X_1	X_2	S_1	S_2	
(0,0,0)	S_1	6	3	2	1	0	2
(0,0,0)	S_2	10	4	5	0	1	2.5
	Z_j	0	0	0	0	0	
	Z_j-C_j		(-4, -4,6)	(4,4, -4)	0	0	
	R		-2.34	2.67	0	0	

From the above table, we get one $z_j - c_j \geq 0$ value is negative. So, the optimal feasible solution is not satisfied. Thus, the non-basic variable s1 will leave the basis and the basic variable x1 enter the basis.

Now by using the procedure of simplex method, we have the following table.

Table 4

		cj	(-6,4,4)	(4, -4, -4)	(0,0,0)	(0,0,0)3
C_B	Y_B	X_B	X_1	X^2	S_1	S_2
(-6,4,4)	X_1	2	1	2/3	1/3	0
0	S_2	2	0	7/3	-4/3	1
	Z_j	(-12,8,8)	(-6,4,4)	(-4,8/3,8/3)	(-2,4/3,4/3)	(0,0,0)
	Z_j-C_j		0	(-8,20/3,20/3)	(-2,4/3,4/3)	0
	R		0	4.23	0.78	0

This table shows that all. $z_j - c_j \geq 0$, it implies that the optimality condition is satisfied so,

$$x_1 = 2, x_2 = 0 \ max\ z^D = (-12, 8, 8)$$

i.e.

$$x_1 = 2, x_2 = 0, min\ z^D = (-8, -8, 12) \qquad \textbf{Equation 10}$$

So, the required optimum value is

$$\min z = \frac{minz^n}{minz^D} = \frac{(-10,-4,16)}{(-8,-8,12)} \qquad \textbf{Equation 11}$$

5. CONCLUSION

In this research article, we anticipated a methodology for finding a solution of linear fractional fuzzy transportation problem, where objective functions are expressed by tri-angular fuzzy number. This proposed method i.e., simplex method is one of the exclusive techniques for calculating the optimal solution of any transportation problem. This additionally be prolonged into fractional quadratic problems.

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