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A STUDY OF OPTION PRICING MODELS WITH DISTINCT INTEREST RATE

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ABSTRACT

This paper analyses the effect of different interest rates on two Option Pricing Models, Black-Scholes', and Heston. Here, the parameter interest rate is focused, and a comparison is done amongst the two models. An error estimator, UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is calculated for pricing various European call options. The real market data is collected from NSE (National Stock Exchange). Moneyness (percentage difference of stock price and strike price) and Time-To-Maturity are used as the base for comparison. All the mathematical calculation is done in MATAB software. We observe that Black-Scholes' model is preferred for lower interest rates than Heston options pricing model and vice-versa. This study is helpful in derivatives market.

Keywords: European Call Option, Black-Scholes' Model, Heston Model, Moneyness, Time-To-Maturity, Interest Rate

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1. INTRODUCTION

Financial markets are one of the most demanding and considered to be quick source of income nowadays. It is the place where various securities are traded like – stock market, bond market, derivatives market etc. The derivative market is the market for financial instruments like – futures contracts or options. The value of financial contract depends on an underlying asset. Prices for derivatives derive from fluctuations in the underlying asset. Options are a type of derivative product that allows investors to contemplate against the volatility of an underlying stock. Options are of two types, Call and Put option. Call option - the option holder purchases an asset at a specified price on or before a specified time. Put option is vice versa of call option. In this, option holders sell an asset at a specified price on or before a specified time. It is again divided among two styles European and American. European options are traded in both NSE and BSE in Indian stock market. They can be exercised only at the time of expiration. Black-Scholes' model is a well-known basic option pricing model for determining the theoretical premium value for a call or a put option with six parameters such as, volatility, underlying stock price, type of option, strike price, time, and risk-free interest rate.

Since 1960's, large number of mathematicians are working on the valuation of options. In 1973, Fischer Black and Myron Scholes Black F. and Scholes M. (1973). The Pricing of Options and Corporate Liabilities, Journal of Political Economy,81(3), 637-644. gave the theoretical closed form solution for pricing

European call options. With several assumptions, this is more convenient and easiest model for calculation of theoretical premium values. In 1993, Heston Heston S.L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options,The Review of Financial Studies, 6(2),327-343. came up with a stochastic volatility model which assumed that the asset variance v_t, follows a mean reverting Cox-Ingersoll-Ross process. Stochastic volatility is favourable with the practical market and also it removes excess kurtosis and asymmetry of Black-Scholes' model. Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26..

Interest rate is one of the important parameters in market fluctuations. It is the amount a lender charges a borrower and is a percentage of the principal- the amount loaned. The risk-free rate is used in both mathematical models Black-Scholes' and Heston option pricing model. It is the zero risk investments, theoretical rate of return. However, a truly risk-free rate does not exist because even the safest investments carry a very small risk.

An error estimator UMBRAE (Unscaled Mean Bounded Relative Absolute Error) Chao C, Jamie T. and Jonathan M. (2017). A new accuracy measure based on bounded relative error for time series forecasting, Tianjin University, China, 12(3), 6-7. is used in the model. It helps in elimination of symmetric and bounded error that occurs in forecasting. Naive method is used as the benchmark for forecasting. The performance of Heston Model and Black-Scholes' Model is observed for three different health companies. The comparison is done for the parameters like time-to-maturity and moneyness (The percentage difference of stock price and strike price) for European call option. Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

In this paper, initially we discuss the basic terminologies, Black-Scholes' model, Heston Model and then Methodology. At last, the result is discussed regarding comparison of the two models for real market Indian stock data.

1.1. LITERATURE REVIEW

It was in early 1960's study of financial market started, many mathematicians like Ayres, H. F. (1963). Risk aversion in the warrant markets., Samuelson, P. A. (1965). Rational theory of warrent prices. Indust. manag. Rev., 6, 13-31., Boness, A. J. (1964). Elements of a theory of stock-option value. Journal of Political Economy, 72(2), 163-175., Baumol, W. J., Malkiel, B. G., & Quandt, R. E. (1966). The valuation of convertible securities. The Quarterly Journal of Economics, 80(1), 48-59. etc.. worked on this concept. In Black F. and Scholes M. (1973). The Pricing of Options and Corporate Liabilities, Journal of Political Economy,81(3), 637-644. developed the formula which gives theoretical premium value for European call Options. This also works with corporate bonds and warrants. With large number of assumptions this formula is widely used all over stock market in India. Sisodia, N. and Gor, R.

(2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

Many other mathematicians worked on the modified Black-Scholes models. Recently, Singh, A. Gor, R. (2020a). Relevancy of pricing European put option based on Gumbel distribution in actual market, Alochana Chakra Journal, 9(6), 4339-4342. worked on the model where underlying at maturity, stock returns follow the Gumbel distribution Singh, A. Gor, R. (2020a). Relevancy of pricing European put option based on Gumbel distribution in actual market, Alochana Chakra Journal, 9(6), 4339-4342.. Also, Singh, A. Gor, R. (2020b). Relevancy of pricing European put option based on truncated Gumbel distribution in actual market, IOSR Journal of Mathematics, 16(5), 12-15. compared the basic B-S model to a new model in which stock returns follow truncated Gumbel distribution Singh, A. Gor, R. (2020b). Relevancy of pricing European put option based on truncated Gumbel distribution in actual market, IOSR Journal of Mathematics, 16(5), 12-15.. Chauhan, A, and Gor, R. (2020b), A Comparative Study of Modified Black-Scholes option pricing formula for selected Indian call options, IOSR Journal of Mathematics, 16(5), 16-22. worked upon the modified truncated Black-Scholes model and compared the result Chauhan, A, and Gor, R. (2020b), A Comparative Study of Modified Black-Scholes option pricing formula for selected Indian call options, IOSR Journal of Mathematics, 16(5), 16-22.. Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42. worked on the relevancy of option pricing model. They have used various error estimators for relevancy check Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42.. orked on the comparative study of oprion pricing models. They have compared the B-S Model and Heston model for various option of moneyness Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42.. worked on the effect of implied volatility on two option pricing models and compared the result Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42.. worked on the historical volatility and its effect on option pricing model and compared the result of error estimation Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26..

It has been seen that modified B-S model gives better output than the original B-S model. In continuation to this, a stochastic volatility model was developed by Heston in 1993. He proposed a stochastic volatility model which gives a closed form solution for calculating theoretical premium value of European Call options. It assumes underlying stock price and volatility as stochastic quantity. The stochastic volatility models help in removing excess of skewness and kurtosis and asset variance follow a mean reverting CIR process. Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

In the paper of Shinde A.S. and Takale K.C. (2012). Study of Black-Scholes Model and its Applications, International Conference on Modelling, (38),270-279., basic terminologies of option pricing model are explained in a simplified manner. Zhang, J.E. Shu, J. (2003). Pricing S&P 500 index options with Heston's model, IEEE International Conference on Computational Intelligence for Financial Engineering., Yuang Y. (2013). Valuing a European Options with the Heston Model, Rochester Institute of Technology,25-28. , and Crisostomo R. (2014), An Analysis of the Heston Stochastic Volatility Model : Implementation and Calibration using Matlab, 58, 6-14. derived the Heston characteristic function and graphically compared the B-S model and Heston option pricing formula on the basis of different parameters of moneyness and Time-To-Maturity.

In the paper of Santra A. and Chakrabarti, B. (2017). Comparison of Black-Scholes and Heston Models for Pricing Index Options, Indian Institute of Management, 796,2-6., Matlab software is introduced for all kind of mathematical calculation. An accuracy measure UMBRAE is used as provided in the paper of Chao C, Jamie T. and Jonathan M. (2017). A new accuracy measure based on bounded relative error for time series forecasting, Tianjin University, China, 12(3), 6-7.. The two models are compared for real market data with this error estimator.

1.2. BASIC CONCEPTS AND THE MODELS Hull J. C. (2009). Options, Futures and other Derivatives, Pearson Publication, Toronto., Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

1. **Option:** An option is defined as the right, but not the obligation, to buy (call option) or sell (put option) a specific asset by paying a strike price on or before a specific date.
   - *Call option:* An option which allows its holder the right to buy the underlying asset at a strike price at some particular time in the future.
   - *Put option:* An option which allows its holder the right to sell the underlying asset at a strike price at some particular time in the future.
2. **Stochastic Process**: Any variable whose value changes over time in an uncertain way is said to follow a stochastic process.
3. **Strike Price:** The predetermined price of an underlying asset is called strike price.
4. **Stochastic Volatility:** Volatility is a measure for variation of price of a stock over time. Stochastic in this sense refers to successive values of a random variable that are not independent.
5. **Expiration Date/ Time-to-maturity:** The date on which an option right expires and becomes worthless if not exercised. In European options, an option cannot be exercised until the expiration date.
6. **Moneyness:** It is the relative position of the current price of an underlying asset with respect to the strike price of a derivative, most commonly a call/put option.
7. **Geometric Brownian Motion:** A continuous time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion.
8. **Black-Scholes Inputs:**

   Black-Scholes model uses following six parameters in option pricing model.
   - Underlying stock price
   - Interest rate
   - Time to expiration
   - Strike price
   - Volatility

Risk-Free Interest Rate is measured in percentage per year. In a particular trade it is the rate at which cash over the life of the option is deposited or borrowed. Call option value increases with the risk-free rate. Put option value decreases as the risk-free rate increases. Its sensitivity with option price is termed as RHO.

MIBOR (Mumbai interbank offer rate) is the rate at which a bank lends loan to another bank on short-term. For a continuous developed market of India, a reference rate is all required for its debt market, which evolved MIBOR. It is used in conjunction with forward rates and the Mumbai interbank bid (MIFOR and MIBID). NSE (2022).

1.3. THE BLACK-SCHOLES' MODEL Santra A. and Chakrabarti, B. (2017). Comparison of Black-Scholes and Heston Models for Pricing Index Options, Indian Institute of Management, 796,2-6., Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

This model has number of assumptions.

- Random walk.
- Interest rate remains constant.
- Stock pays no dividends.
- No transaction cost.
- Option can only be exercised upon expiration.
- Stock returns are normally distributed; thus, the volatility remain constant throughout.

In Black F. and Scholes M. (1973). The Pricing of Options and Corporate Liabilities, Journal of Political Economy,81(3), 637-644. proposed a European Call option pricing model which is based on Geometric Brownian motion.

$$dS_t = \mu S_t dt + \sigma_t S_t dW_t$$

S_t - asset price,

μ - drift (that is constant),

σ_t- return volatility(constant) and

W_t - Brownian motion.

Black F. and Scholes M. (1973). The Pricing of Options and Corporate Liabilities, Journal of Political Economy,81(3), 637-644. makes use of the risk neutral probability instead of true probability to compute option price.

The risk neutral dynamics on asset is given by.

$$dS_t = rS_t dt + \sigma_t S_t dW_t$$

Here, r is the risk-free rate and Geometric Brownian Motion is the solution to the above stochastic differential equation.

$$S_t = S_0 \, exp \left[\sigma W_t + \left(\mu - \frac{\sigma^2}{2} \right) t \right]$$

Geometric Brownian Motion (GBM) model is the lognormal of the above equation for stock prices.

$$ln\left(\frac{S_t}{S_0}\right) = \sigma W_t + \left(\mu - \frac{\sigma^2}{2}\right)t$$

Here, R.H.S. equation is a normal random variable whose mean is $\left(\mu - \frac{\sigma^2}{2}\right)t$
And variance is $\sigma^2 t$.
The Black-Scholes Formula for European call price is,

$$C = S_0 N(d_1) - Ke^{-rt}N(d_2)$$

Where, $d_1 = \frac{ln(S_0/K) + (r+\sigma^2/2)t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$

K – strike price, S_0 – current stock price, t – time to expiration, r – riskless interest rate (constant), σ – volatility of stock (constant). Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42.

1.4. THE HESTON MODEL Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26., Ye, Z. (2013). The Black-Scholes and Heston Models for Option Pricing, Waterloo University, 23-25.

In Heston S.L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options,The Review of Financial Studies, 6(2),327-343. developed a Stochastic Volatility option pricing Model. Consider at time t the underlying asset S_t which obeys a diffusion process with volatility being treated as a latent stochastic process of Feller as proposed by Cox, Ingersoll, and Ross (CIR): Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42.

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_t^1$$

$$dV_t = k[\theta - V_t]dt + \sigma\sqrt{V_t} dW_t^2$$

Here, W_t^1 and W_t^2 are two separate Brownian motion which are correlated with a correlation coefficient $\rho > 0$:

$$dW_t^1 dW_t^2 = \rho dt$$

Here, S_t - asset price, r - risk free rate, V_t - variance at time t, $\theta > 0$ is the long term mean variance, $k > 0$ is variance mean-reversion speed, $\sigma \geq 0$ is the volatility of the variance.

European call option price is given by.

$$C = S_0 \Pi_1 - e^{-rt} K \Pi_2$$

Here, Π_1- delta of the option and Π_2 - risk-neutral probability of exercise (i.e., when $S_t > K$)

Heston characteristic function for j=1, 2 is given as; Indian Institute of Management Calcutta (n.d.). Website -

$$f_j(x, v, \tau; \emptyset) = e^{C(\tau;\emptyset)+D(\tau;\emptyset)v+i\emptyset x}$$

Were,

$$C(\tau; \emptyset) = r\emptyset i\tau + \frac{a}{\sigma^2}\left\{\left(b_j - \rho\sigma\emptyset i + d\right)\tau - 2ln\left[\frac{1 - ge^{d\tau}}{1 - g}\right]\right\}$$

$$D(\tau; \emptyset) = \frac{b_j - \rho\sigma\emptyset i + d}{\sigma^2}\left[\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}\right]$$

$$g = \frac{b_j - \rho\sigma\emptyset i + d}{b_j - \rho\sigma\emptyset i - d}$$

$$d = \sqrt{\left(\rho\sigma\emptyset i - b_j\right)^2 - \sigma^2\left(2u_j\emptyset i - \emptyset^2\right)}$$

$$u_1 = \frac{1}{2}, u_2 = -\frac{1}{2}, a = k\theta, b_1 = k - \rho\sigma, b_2 = k$$

The required probabilities can be obtained by inverting the characteristic functions Indian Institute of Management Calcutta (n.d.). Website -

$$\Pi_j(x, v, T; \ln[K]) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\emptyset \ln[K]} f_j(x, v, T; \emptyset)}{i\emptyset}\right] d\emptyset$$

2. MATERIALS AND METHODS

Data: The live market data has been gathered from NSE India website Indian Institute of Management Calcutta (n.d.). Website -. Five different companies are considered randomly for calculation of European call option.

Aurobindo Pharma limited, Biocon limited, Cipla limited, Zydus Cedilla Healthcare limited and Glenmark are considered for the period from January 01 to January 31, 2021. February 25, 2021 is considered as the maturity date.

Parameter Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

Moneyness - Percentage difference between the current underlying price and the strike price:

- Moneyness (%) = $S/K + 1$

The result has been bifurcated in terms of moneyness option and maturity time.

- ITM (In the money) - In call options if the strike price is lower than the underlying stock price.
- ATM (At the money) - In call options when the strike price is similar to the underlying stock price.
- OTM (Out of the money) - In call options if the strike price is more than the underlying stock price.
- An error estimator UMBRAE (Unscaled Mean Bounded Relative Absolute Error) = $\frac{MBRAE}{1-MBRAE}$

$$MBRAE = \frac{1}{n}\sum_{t=1}^{n}(BRAE)$$

$$BRAE = \frac{|r_t|}{|r_t| + |r_t^*|}$$

$$r_t = y_t - f_t$$

$$r_t^* = y_t - f_t^*$$

Where, y_t is observed price of the model, f_t is the actual forecasted value of the market and, f_t^* is the forecasted value of the market from Naive Method.

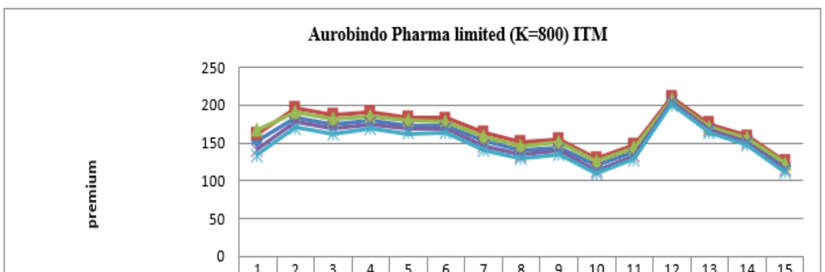
- We have used MATLAB software to run the Black-Scholes model for calculation of European call option value.
- Risk –Free Interest rate: During the life of an option, the amount of money lends or borrowed at the particular rate is called Risk-Free Interest rate. Value of call option increases with increase in rate.
- Volatility: It is defined as the standard deviation of the continuously compounded return of the stock. Value of call option is high for the higher volatility.
- We have used MATLAB software to run the Heston model for calculation of European call option value.
- Initial Variance bounds of 0 and 1 have been considered.
- Long-term Variance bounds of 0 and 1 have been considered.
- Correlation: Correlation takes values from -1 to 1 between the stochastic processes.

- Volatility of Variance: It gives positive value. As the volatility of assets may increase in short term, a broad range of 0 to 5 has been considered.
- Mean-Reversion Speed: This is dynamically set with the help of a non-negative constraint (Feller, 1951). The constraint $2k\theta - \sigma^2 > 0$ guarantees that the variance in CIR process is always strictly positive.
- Initial Variance = 0.28087
- Long-term Variance = 0.001001
- Volatility of Variance = 0.1
- Correlation Coefficient = 0.5
- Mean Reversion Speed = 2.931465

Interest rate is the supreme parameter in the overall study of the paper. To observe the effect of change of interest rate in option pricing models, two different rates have been considered, R_1 = 10 % (arbitrary) and R_2 = 3.47% (MIBOR- Mumbai Interbank offer Rate)

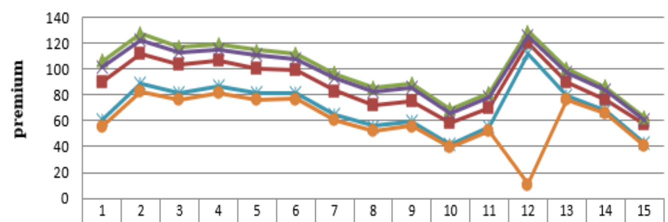
3. RESULTS AND DISCUSSIONS

Randomly five different stocks have been chosen and its theoretical premium value of European call option is computed for B-S model and Heston model Sisodia, N. and Gor, R. (2020). Effect of implied volatility on option prices using two option pricing models, NMIMS ManagementReview, 31-42.. Two different interest rates i.e., R_1 and R_2 have considered for different option of moneyness i.e. In-the-money (ITM), Out-of-the-money (OTM) and At-the-money (ATM). Error estimation UMBRAE is evaluated and compared for both the models.



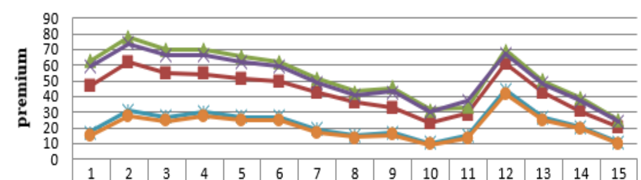
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	153.6	184.15	175.3	180.1	173.4	173.25	152.55	140.3	144.6	120.8	138.2	204.95	167.8	153.6	118.8
Black-Scholes' model at R1	162.24	196.43	186.93	190.97	184.27	182.6	162.93	150.44	155	129.68	147.17	210.69	175.21	159.18	126.34
Black-Scholes' mode at R2	167.84	190.75	181.64	185.95	179.28	177.94	158.49	146.44	151.13	126.21	143.21	206.91	171.64	156.07	123.52
Heston model at R1	141.24	177.39	168.49	175.32	168.52	168.73	146.2	134.29	139.74	113.18	133.13	205.21	166.79	151.39	114.74
Heston model at R2	134	170.35	161.95	169.24	162.47	163.18	140.81	129.47	134.87	109.05	128.8	201.06	162.74	147.9	111.5

Aurobindo Pharma limited (K = 900) ATM



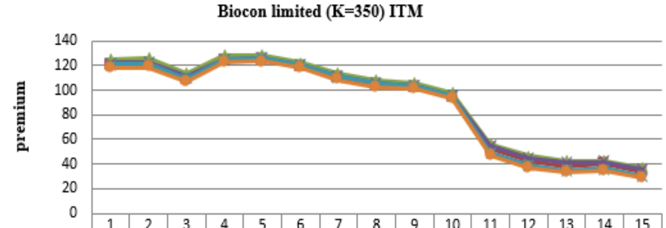
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	89.35	112.15	103.95	106.15	100.45	99.45	83.2	72.5	75.1	58	69.8	120.4	89.55	76.6	57.6
Black-Scholes' model at R1	106.07	127.71	117.48	119.4	114.63	111.79	96.41	85.35	88.91	68.55	81.21	129.36	100.62	86.1	62.85
Black-Scholes' mode at R2	101.45	122.8	112.97	115.08	110.41	107.87	92.8	82.2	85.69	65.98	78.35	125.98	97.64	83.61	60.8
Heston model at R1	60.71	88.48	81.26	86.73	81.26	81.4	64.22	55.77	59.57	42.05	54.96	111.92	79.75	67.92	42.84
Heston model at R2	55.45	82.39	75.82	81.51	76.21	76.74	60.14	52.33	55.99	39.46	51.88	10.97	76.35	65.17	40.76

Aurobindo Pharma limited (K = 1000) OTM



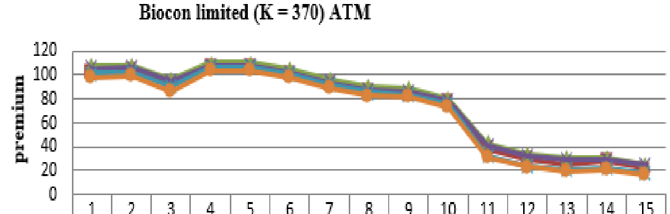
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	46.85	61.5	54.85	54	50.95	49.5	42.85	36.2	32.45	23.35	28.9	60.9	42.4	30.6	20.75
Black-Scholes' model at R1	62.7	77.42	69.75	69.91	65.18	61.94	51.43	43.01	45.29	31.5	33.13	69.52	49.95	39.07	25.21
Black-Scholes' mode at R2	59.31	73.66	66.4	66.12	62.09	59.11	48.95	40.93	43.14	29.95	37.31	67.02	47.94	37.51	24.08
Heston model at R1	17.41	30.73	26.99	29.81	26.99	27.06	18.94	15.36	16.94	10.23	15.04	44.13	26.23	20.59	10.46
Heston model at R2	15.25	27.55	24.28	27.09	24.46	24.72	17.17	13.95	15.45	9.3	13.81	41.64	24.52	19.33	9.7

Biocon limited (K=350) ITM

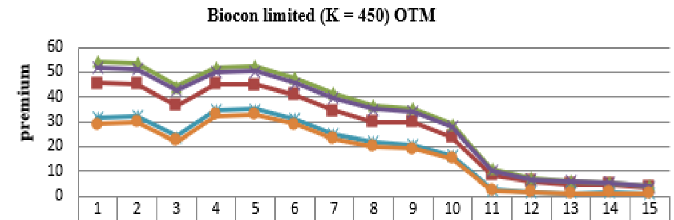


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	120.3	121.15	109.05	124.25	124.65	119.25	110.1	104.35	102.5	94.85	52.1	42.55	38	40	33.55
Black-Scholes' model at R1	125.23	125.55	113.45	127.75	128.23	122.45	113.6	107.66	105.98	97.61	56.29	46.36	42.55	42.84	36.89
Black-Scholes' mode at R2	122.22	122.71	110.85	125.23	125.7	120.13	111.32	105.59	103.91	95.75	54.8	45.1	41.35	41.74	35.87
Heston model at R1	120.96	121.86	109.33	125.42	125.92	120.28	110.89	105.1	103.31	95.15	48.62	38.68	34.45	36.25	29.71
Heston model at R2	117.54	118.54	106.37	122.68	123.18	117.76	108.38	102.82	101.03	93.1	46.82	37.21	33.06	35.01	28.57

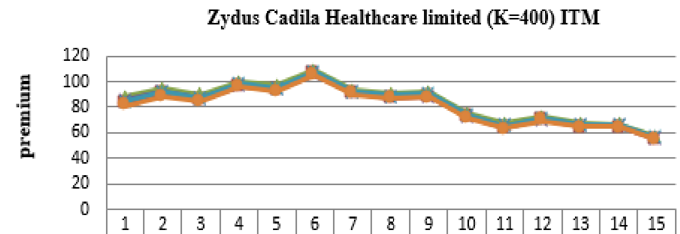
Biocon limited (K = 370) ATM



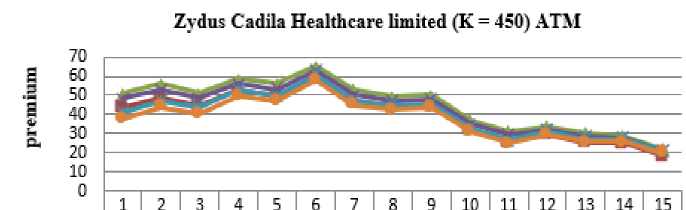
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	102.5	103.15	91.4	105.75	106.05	100.7	91.8	86.05	84.2	76.85	38.1	29.7	25.75	27.9	21.65
Black-Scholes' model at R1	108.4	108.48	96.7	110.01	110.46	104.65	96.19	90.25	88.65	80.32	42.89	33.93	30.77	30.52	25.63
Black-Scholes' mode at R2	105.42	105.66	94.14	107.48	107.93	102.34	93.93	88.2	86.62	78.5	41.55	32.85	29.74	29.59	24.8
Heston model at R1	101.31	102.19	89.71	105.7	106.2	100.56	91.23	85.48	83.71	75.67	32.46	24.28	20.97	22.37	17.41
Heston model at R2	97.72	98.84	86.63	102.82	103.31	97.92	88.61	83.12	81.35	73.58	30.91	23.1	19.89	21.37	16.56



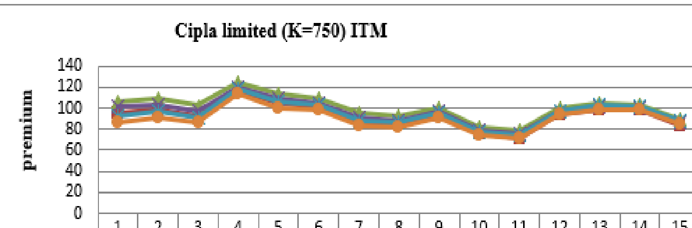
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	45.5	45.25	36.6	45.05	44.95	40.7	34.55	29.8	30	23.7	8.35	5.55	4.6	4.75	3.4
Black-Scholes' model at R1	54.2	53.37	44.25	52.07	52.4	47.22	41.46	36.46	35.45	28.81	10.9	6.93	5.93	5.24	3.94
Black-Scholes' model at R2	51.87	51.15	42.36	50.07	50.39	45.44	39.81	35.03	34.04	27.64	10.33	6.56	5.6	4.96	3.72
Heston model at R1	31.64	32.24	24.04	34.71	35.07	31.04	24.93	21.51	20.51	16.31	2.64	1.49	1.13	1.27	0.79
Heston model at R2	28.94	29.68	22.05	32.39	32.74	29.03	23.18	20.05	19.1	15.22	2.39	1.36	1.02	1.17	0.73



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	83.2	90.35	86	96.55	93.2	105.8	90.5	87.6	88.5	72.35	63.85	69.85	64.55	64.65	54.45
Black-Scholes' model at R1	88.7	95.14	90.24	100.2	97.07	108.9	93.95	90.77	91.71	75.33	67.16	72.55	67.64	66.97	57.19
Black-Scholes' model at R2	85.35	91.9	87.22	97.32	94.19	106.1	91.29	88.34	89.28	73.22	65.12	70.65	65.78	65.31	55.61
Heston model at R1	84.73	92.2	87.7	98.64	95.33	107.9	92.43	89.49	90.47	73.86	65.32	71.48	66.42	66.29	56.32
Heston model at R2	80.97	88.64	84.41	95.56	92.26	105.1	89.62	86.94	87.92	71.64	63.17	69.52	64.49	64.6	54.71

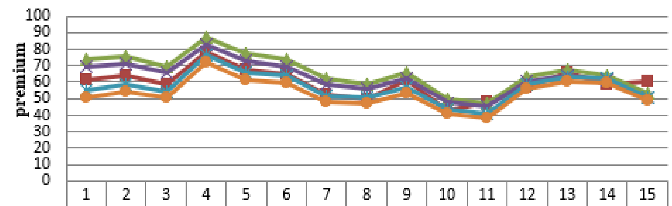


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	43.5	48.7	44.7	52.7	49.75	60.15	47.1	44.15	44.65	32.05	25.85	29.35	25.2	25	18.25
Black-Scholes' model at R1	51.27	55.85	50.96	58.47	55.86	65.17	52.66	49.38	50.14	36.64	30.76	33.72	30.22	28.79	22.3
Black-Scholes' model at R2	48.47	53.06	48.4	55.88	53.33	62.62	50.34	47.29	48.03	34.98	29.26	32.26	28.85	27.57	21.25
Heston model at R1	41.08	47.2	43.46	52.67	49.82	60.86	47.34	44.88	45.69	32.65	26.61	30.9	27.35	27.25	20.79
Heston model at R2	37.95	44.06	40.64	49.82	47.03	58.08	44.84	42.65	43.44	30.94	25.1	29.43	25.97	26.04	19.77



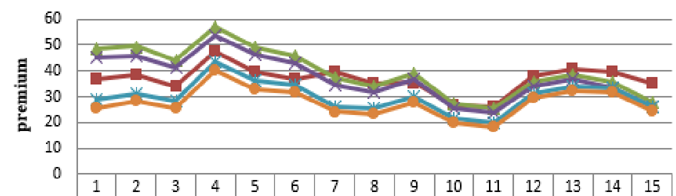
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	95.15	98.6	92.7	117	104.7	102	87.2	84.75	93	76	72	94.75	99.1	98.2	83.7
Black-Scholes' model at R1	106.2	108.78	102.35	124.81	113.17	108.84	94.92	92.32	100.77	82.51	79.05	100.2	105.12	102.67	88.86
Black-Scholes' model at R2	100.92	103.7	97.67	119.97	108.51	104.55	90.87	88.6	96.9	79.24	75.84	96.95	101.82	99.73	86.07
Heston model at R1	92.86	97.23	91.6	118.36	105.75	103.08	87.94	86.27	95.24	77.57	73.95	97.51	102.59	101.41	87.4
Heston model at R2	86.83	91.48	86.37	113.06	100.64	98.42	83.56	82.31	91.11	74.15	70.61	94.16	99.18	98.43	84.59

Cipla limited (K = 800) ATM



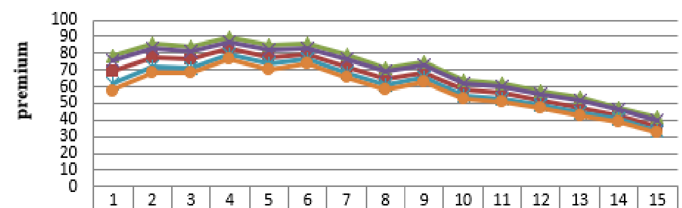
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	61.7	63.95	58.6	77.85	67.4	64.7	52.7	50	61	42.95	48	56.45	66.05	58.35	60.15
Black-Scholes' model at R1	73.81	75.36	69.4	87.13	77.27	73.41	61.89	58.67	65.47	50.12	47.47	63.14	67.16	64.07	53.01
Black-Scholes' mode at R2	69.33	71.02	65.47	82.85	73.25	69.73	58.55	55.63	62.23	47.54	44.97	60.42	64.36	61.58	50.75
Heston model at R1	55.37	58.85	54.38	76.37	65.78	63.58	51.51	50.22	57.24	43.64	40.98	59.04	63.15	62.18	51.07
Heston model at R2	50.62	54.22	50.26	71.76	61.51	59.72	48.1	47.14	53.29	41.09	38.53	56.3	60.31	59.7	48.86

Cipla limited (K = 850) OTM

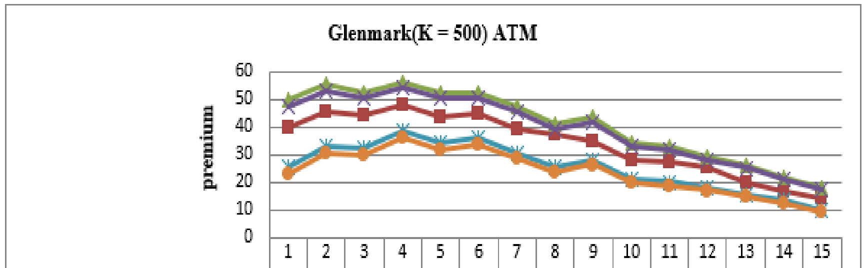


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	37	38.2	33.8	47.3	39.3	36.95	39.4	34.95	35.35	26.4	25.9	37.7	40.9	39.45	35.25
Black-Scholes' model at R1	48.72	49.39	44.27	56.99	49.3	45.67	37.11	34.12	39.04	27.47	25.68	35.71	38.6	35.38	27.75
Series4	45.19	45.97	41.23	53.54	46.15	42.84	34.64	31.92	36.63	25.69	23.99	33.73	36.53	33.58	26.22
Series5	28.87	31.22	28.22	43.68	36.03	34.48	26.34	25.5	30.14	21.34	19.71	31.36	34.19	33.51	26.06
Series6	25.76	28.12	25.53	40.28	33.04	31.8	24.13	23.52	27.9	19.76	18.22	29.49	32.2	31.7	24.61

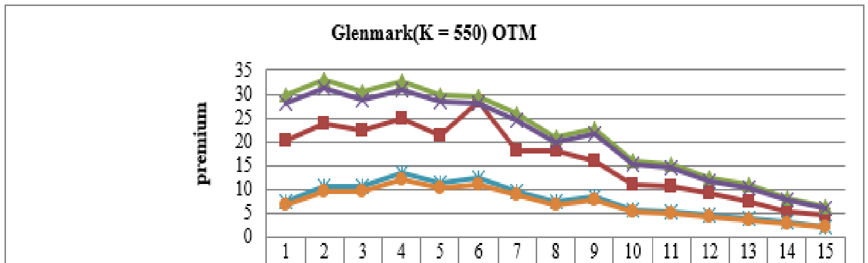
Glenmark(K=450) ITM



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	69.25	77.85	76.65	82.9	77.45	79.5	72.2	64.65	68.05	58.2	56.25	52.2	47.25	43	36.1
Black-Scholes' model at R1	78.34	86.06	83.83	89.58	84.66	85.72	78.99	71.25	74.92	63.51	61.98	57.35	53.27	47.59	41.22
Black-Scholes' model at R2	75.41	83.13	81.07	86.9	82.04	83.24	76.61	69.13	72.75	61.63	60.13	55.71	51.68	46.23	39.96
Heston model at R1	61.53	72.18	71.55	79.6	73.87	76.43	68.66	61.2	65.42	54.48	52.75	49.22	44.7	40.3	33.47
Heston model at R2	57.77	68.47	68.11	76.32	70.65	73.45	65.77	58.66	62.82	52.28	50.59	47.34	42.9	38.79	32.11



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	39.55	45.45	43.95	47.9	43.6	44.65	39	37	34.95	28	27	25.4	19.95	16.85	14.15
Black-Scholes' model at R1	49.69	55.14	52.47	56.31	52.44	52.52	47.33	40.73	43.46	34.04	32.97	28.91	26.19	21.65	17.81
Black-Scholes' mode at R2	47.33	52.73	50.21	54.08	50.29	50.5	45.42	39.1	41.76	32.66	31.62	27.75	25.11	20.78	17.05
Heston model at R1	25.6	32.89	32.43	38.35	34.09	35.96	30.38	25.37	28.16	21.15	20.12	18.07	15.59	13.3	10.05
Heston model at R2	23.15	30.18	29.93	35.78	31.69	33.68	28.32	23.69	26.36	19.8	18.82	16.99	14.61	12.54	9.44



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Market Price	20.35	23.8	22.4	25	21.4	28.4	18.1	18.1	16	11	10.45	9.2	7.25	5.15	4.55
Black-Scholes' model at R1	29.66	33.11	30.47	32.63	29.9	29.37	25.82	20.91	22.67	16.01	15.37	12.49	11.01	8.17	6.35
Black-Scholes' model at R2	27.94	31.32	28.83	31	28.36	27.93	24.51	19.84	21.54	15.17	14.56	11.84	10.42	7.73	6
Heston model at R1	7.52	10.75	10.53	13.41	11.32	12.22	9.6	7.43	8.61	5.76	5.37	4.63	3.78	3.05	2.09
Heston model at R2	6.54	9.51	9.39	12.13	10.19	11.12	8.68	6.75	7.84	5.25	4.9	4.25	3.46	2.81	1.92

Above graph's shows that the model price's at R_1 are higher than at R_2. This indicates that higher the interest rate so is the premium value. Also, it is seen that Black-Scholes' model gives closer result at MIBOR i.e., R_2 as compared to Heston model.

Further, UMBRAE (Unscaled Mean Bounded Relative Absolute Error) is computed, and comparison is done for various option of moneyness. Interest Rate is considered as the principal parameter in both Black-Scholes' and Heston Option pricing model. Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.

Table 1: Aurobindo Pharma Limited

Model	**Error estimation UMBRAE**					
	Call Option at R_1			Call Option at R_2		
	ITM K=800	ATM K=900	OTM K=1000	ITM K=800	ATM K=900	OTM K=1000
Black-Schole	0.7	1	1	0.45	0.78	1
Heston	0.41	1	1	0.69	1	1

According to the above table, at R_1 Heston model outperforms Black-Scholes' model at ITM while other shows maximum error value 1. At R_2 Black-Schole model outcompete Heston model at ITM and ATM options while OTM options shows no difference.

Table 2: Biocon Limited

Model	Error estimation UMBRAE					
	Call Option at R_1			Call Option at R_2		
	ITM K=350	ATM K=370	OTM K=450	ITM K=350	ATM K=370	OTM K=450
Black-Schole	0.83	1	1	0.47	0.69	0.88
Heston	0.37	0.28	1	0.57	0.86	1

According to the above table, at R_1 Heston model outcompete Black-Schole model at ITM and ATM options. At R_2 Black-Scholes model outcompete Heston for all the three moneyness option.

Table 3: Zydus Cedilla Healthcare Limited

Model	Error estimation UMBRAE					
	Call Option at R_1			**Call Option at R_2**		
	ITM K=400	ATM K=450	OTM K=500	ITM K=400	ATM K=450	OTM K=500
Black-Schole	0.65	1	1	0.23	0.77	1
Heston	0.41	0.31	1	0.41	0.48	1

According to the above table, at R_1 Heston model outcompete Black-Schole model at ITM and ATM options. At R_2 Black-Scholes' model is better than Heston model at ITM while vice-versa in ATM option.

Table 4: Cipla Limited

Model	Error estimation UMBRAE					
	Call Option at R_1			Call Option at R_2		
	ITM K=750	ATM K=800	OTM K=850	ITM K=750	ATM K=800	OTM K=850
Black-Schole	0.96	0.91	0.75	0.5	0.74	0.94
Heston	0.63	0.51	1	0.63	1	1

According to the above table, at R_1 Heston model is better than Black-Scholes' model in ITM and ATM options and vice-versa in OTM option. At R_2 Black-Scholes' model outcompete Heston model in all moneyness option.

Table 5: Glenmark

Model	Error estimation UMBRAE					
	Call Option at R_1			Call Option at R_2		
	ITM K=450	ATM K=500	OTM K=550	ITM K=450	ATM K=500	OTM K=550
Black-Schole	1	1	1	0.93	1	1
Heston	0.72	1	1	1	1	1

According to the above table, at R_1 Heston model outperforms Black-Scholes' model in ITM option and vice-versa at R_2. Rest all other shows maximum error value 1.

4. CONCLUSIONS AND RECOMMENDATIONS

We conclude the following from the above study.

R_1 = 10% (Arbitrary):

- Heston model is far better than Black-Scholes' model at In-the-money and At-the-money option for all cases.
- Black-Scholes' model is better than Heston only in Cipla limited while; others have similar error value maximum 1 at Cut-off–the-money option.

R_2 = 3.47 (MIBOR):

- Black-Scholes' model proves to be better than Heston model in all except Zydus Cedilla limited at At-the-money option.
- Black-Scholes' model performs better than Heston model in all chosen companies' data at In-the-money option.
- Black-Scholes' outcompete Heston model in the case of Biocon and Cipla limited. Remaining all gives similar error value indicating no such difference at Out-of-the-money option.

We have considered one-month different stock data from five different companies for two different interest rates R_1 and R_2. We have computed the error estimation for both the Black-Scholes' model and Heston model for various option of moneyness i.e. In-the money (ITM), out-of-the-money (OTM) and At-the-money (ATM) Sisodia, N. and Gor, R. (2021). A Study on the effect of historical volatility using two option pricing models, IOSR Journal of Economics and Financce (IOSR-JEF), 12(1), 19-26.. The comparison is then done in both the models. At last, we could conclude that, the Black-Scholes' model is more reliable for interest rate at MIBOR while, Heston model must be used for other higher rates. Because as the rate increases the higher the premium value we get. Thus, the Black-Scholes' model fails with the increasing interest rates. This kind of study is always useful to derivative market investors in both short term and long-term options [18]. In future, we could work on the large number of data, compute many more results, and forecast much accurately.

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