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COMPARISON OF THREE OPTION PRICING MODELS FOR INDIAN OPTIONS MARKET

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ABSTRACT

Black-Scholes option pricing model is used to decide theoretical price of different Options contracts in many stock markets in the world. In literature we can find many generalizations of BS model by modifying some assumptions of classical BS model. In this paper we compared two such modified Black-Scholes models with classical Black-Scholes model only for Indian option contracts. We have selected stock options form 5 different sectors of Indian stock market. Then we have found call and put option prices for 22 stocks listed on National Stock Exchange by all three option pricing models. Finally, we have compared option prices for all three models and decided the best model for Indian Options.

Motivation/Background: In 1973, two economists, Fischer Black, Myron Scholes and Robert Merton derived a closed form formula for finding value of financial options. For this discovery, they got a Nobel prize in Economic science in 1997. Afterwards, many researchers have found some limitations of Black-Scholes model. To overcome these limitations, there are many generalizations of Black-Scholes model available in literature. Also, there are very limited study available for comparison of generalized Black-Scholes models in context of Indian stock market. For these reasons we have done this study of comparison of two generalized BS models with classical BS model for Indian Stock market.

Method: First, we have selected top 5 sectors of Indian stock market. Then from these sectors, we have picked total 22 stocks for which we want to compare three option pricing models. Then we have collected essential data like, current stock price, strike price, expiration time, rate of interest, etc. for computing the theoretical price of options by using three different option pricing formulas. After finding price of options by using all three models, finally we compared these theoretical option price with market price of respected stock options and decided that which theoretical price has less RMSE error among all three model prices.

Result: After going through the method described above, we found that the generalized Black-Scholes model with modified distribution has minimum

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RMSE errors than other two models, one is classical Black-Scholes model and other is Generalized Black-Scholes model with modified interest rate.

Keywords: BlackScholes Model, Modified BlackScholes Model, Truncated Normal Distribution, Interest Rate, Options

1. INTRODUCTION

Derivatives are becoming most important financial instruments in current days. The most common derivatives are options, forwards, futures, swaps. Among all different types of derivatives, options are the most common derivatives traded in exchange-traded market all over the world. In very broad sense, there are mainly two types of options, one is call option and other is put option. Further these options can be classified into European options and American options.

Black-Scholes(B-S) Option Pricing Model gives the theoretical value of European call and put options by using complex mathematical concepts. This model was first introduced by Fischer Black and Myron Scholes in 1973, and then Robert Merton generalized this model to its current form. The B-S model has not been able to portrait the real market situations by mathematical equations. This model heavily used in almost all markets as a useful approximation for theoretical value of options. But without considering the limitations of model, if we apply it in real market, we might face big loss in market. B-S model also has some limitations. To overcome from B-S model's limitations, several different generalizations in B-S model have been done by researchers as well as practitioners. In this paper we have studied two such modified B-S models and compare them with classical B-S model. We then compare theoretical option prices computed by all three models for different stock options of top five Indian stock market sectors.

2. LITERATURE REVIEW

Black and Scholes (1973) proposed the Black–Scholes (B-S) model for pricing European options, which is still commonly used to value a wide range of derivative securities. Though this model has enormous uses world-wide, it has some well-known deficiencies. This deficiency always attracts Researchers and Academicians to revise/-modify Black-Scholes model. For example, observed returns of the underlying asset from financial markets are not normally distributed and they are usually skewed shown by Peiro (1999) and fat-tailed shown by Rachev et al. (2005). There are many modified B-S model in literature in which log- returns of underlying asset not taken as normally distributed. Generalized beta distribution of the second kind was used by McDonald & McDonald and Bookstaber (1991) while Burr-3 distribution was adopted by Sherrick et al. (1996). Other examples include Weibull distribution used by Savickas (2002), g-and-h distribution studied by Dutta and Babbel (2005) and generalized gamma distribution taken by Fabozzi et al. (2009).

All the pricing models in the existing literature, assume that the underlying stock's price is not bounded, that is, the price of underlying may take values from zero to infinity. But in practice there is no chance that any underlying price could reach infinity. A modification to the B–S formula, which considers that option traders often have their own expected (finite) range of the underlying price in mind, which is very reasonable and attractive idea to refine B-S model. By Zhu and He (2017), such a modification was carried out and assumed that the log-returns of the underlying asset follow a truncated normal distribution within a certain period with fixed upper and lower bounds.

Khan et al. (2012) suggested modification in risk-free interest rate used in B-S model, without given its practical implementation. One of the important outcomes of the B-S model is that the expected return on the underlying assets, does not appear in the equation. Khan et al. (2013) modified the B-S model by adding some new variables based on given assumption related to risk-free interest-rate and shows the calculation process of new risk-free interest rate based on modified variable. Further they have tested empirically this modified B-S model for BANKNIFTY call and put options.

3. CLASSICAL BLACK-SCHOLES MODEL (B-S)

The option price can be computed by B-S model as given below. The formula for computing theoretical price of European style call options first given in Black and Scholes (1973) which is given below.

S_0= Today's stock price

X = Strike price

T = expiration time

σ = volatility of underlying stock

r = risk-free interest rate

Then the value, V_c of call today is given by,

$$V_c = S_0 N(d_1) - X e^{-rT} N(d_2) \tag{1}$$

In this formula, $N(x)$ denotes the standard normal distribution function.

And,

$$d_1 = \frac{In\left(\frac{s_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sqrt{T}^{\sigma}} \; and \; d_2 = d_1 - \sigma\sqrt{T} \tag{2}$$

Further, a European call price is related to a European put price. We can find European put price of any Stock option if we have European call price of that Stock option by using Put-Call parity formula. The formula for finding European put price is,

$$V_p = V_c - S_0 + e^{-rT} X$$

Were,

V_p = Value of Put option
V_c = Value of Call option
S_0= Today's stock price
X = Strike price
T = expiration time
r = risk-free interest rate

4. MODIFIED B-S MODEL BY CHANGING INTEREST RATE (MBS$_i$)

In the B-S model, risk free interest rate is taken as constant. If we compute the interest rate given by following method and then use the computed rate as an interest rate in B-S model, we get Modified B-S model by changing Interest rate (MBS_i). This modification in interest rate was carried out in Khan et al. (2013).

Basically, we include three type of Risk factor which replaces the risk-free interest rate i.e.

- Yield curve Risk (R_{YC});
- Basis Risk (R_B); and
- Reprising Risk (R_{RP});

After finding all three risks, new Interest rate can be computed by following formula.

$$R_{IIR} = \frac{\sum_{i=1}^{n} R_{YC_i}}{n} + \frac{\sum_{i=1}^{n} R_{B_i}}{n} + \frac{\sum_{i=1}^{n} R_{RP_i}}{n} \tag{3}$$

After calculating new interest rate R_{IIR}, we use this value in place of risk-free interest rate in B-S model.

To compute the above three types of risks, we have selected top 10 banks namely, SBI, ICICI, HDFC, PNB, CANARA, AXIS, BOB, IDFC, BOI, CORPORATION. We denote, the Long-term interest rate of bank i by L_{TIR_i}, and the Short-term interest rate of bank i by S_{TIR_i}. For example, $L_{TIR_{SBI}}$ denotes the long-term interest rate of SBI, and similarly $S_{TIR_{ICICI}}$ denotes the Short-term interest rate of ICICI. Long term and Short-term interest rates of each bank are given in following Table 1 .

Table 1 Short-term and Long-term rates for selected banks

Bank(i)	S_{TIR_i} (%)	L_{TIR_i} (%)	Bank(i)	S_{TIR_i} (%)	L_{TIR_i} (%)
SBI	2.9	5.4	AXIS	2.5	5.5
ICICI	2.5	5.5	BOB	2.9	5.3
HDFC	2.5	5.5	IDFC	2.75	5.75
PNB	3	5.3	BOI	3.25	5.3
CANARA	2.95	5.5	CORPORATION	3	5.4

Yield Curve Risk - R_{YC}

For Yield curve risk, first compute difference between long term and short-term risk for each bank and then take its average.

$$R_{YC_{SBI}} = L_{TIR_{SBI}} - S_{TIR_{SBI}}$$

$$R_{YC_{ICICI}} = L_{TIR_{ICICI}} - S_{TIR_{ICICI}}$$

$$*$$

$$*$$

$$*$$

$$R_{YC_{COR}} = L_{TIR_{COR}} - S_{TIR_{COR}}$$

Finally, $R_{YC} = \frac{\sum_{i=1}^{10} R_{YC_i}}{10}$, where i denotes corresponding banks.

$Basis\ Risk$ - R_B

For Basis risk, first compute difference between long- and short-term rates each pair of banks.

$$\begin{matrix} L_{TIR_{SBI}} - S_{TIR_{ICICI}} & L_{TIR_{ICICI}} - S_{TIR_{HDFC}} & & \\ L_{TIR_{SBI}} - S_{TIR_{HDFC}} & L_{TIR_{ICICI}} - S_{TIR_{PNB}} & & \\ * & * & & \\ * & * & *\ *\ * & L_{TIR_{BOI}} - S_{TIR_{COR}} \\ * & * & & \\ L_{TIR_{SBI}} - S_{TIR_{COR}} & L_{TIR_{ICICI}} - S_{TIR_{COR}} & & \end{matrix}$$

After computing all the above differences, the average of these values is called R_B.

Reprising Risk - R_{RP}

For Reprising risk, first we observe last 10 changes in repo rates from 2016 to 2020 by Reserved Bank of India (RBI) and then take average of these changes. The last 10 changes in repo rates from 2016 to 2020 has been listed in following Table 2
.

Table 2 Last 10 Repo rate changes

Date	Repo rate (%)	Change
04-10-2016	6.25	0.25
02-08-2017	6	0.25
06-06-2018	6.25	0.25
01-08-2018	6.5	0.25
07-02-2019	6.25	0.25
04-04-2019	6	0.25
06-06-2019	5.75	0.35
07-08-2019	5.4	0.25
04-10-2019	5.15	0.75
27-03-2020	4.4	0.4

5. MODIFIED B-S MODEL BY CHANGING DISTRIBUTION (MBS$_d$)

As we discussed in literature review section, Zhu and He (2017) proposed the modified B-S formula for pricing European call options with truncated normally distributed underlying. Following is the closed form option pricing formula for value, V_C of call option.

$$V_c = S_0 \frac{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right) - \Phi\left(\frac{\ln\frac{K}{S_0} - \mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right)}{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right) - \Phi\left(\frac{a-\mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right)} - Ke^{-rt} \frac{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{\ln\frac{K}{S_0} - \mu t}{\sigma\sqrt{t}}\right)}{\Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{a-\mu t}{\sigma\sqrt{t}}\right)} \tag{4}$$

Were,

S_0 : - Stock price, K :- Strike price, r :- riskless interest rate, t :- time to maturity, a :- lower bound for truncated distribution, b :- upper bound for truncated distribution, μ :- mean of log return σ :- Volatility of underlying asset, Φ :- Cumulative density function for standard normal distribution. We will find prices of Indian stock options by using all above three option pricing models to check which model best suits for Indian options.

Here also we can find value of put options using MBS_d model by help of Put-Call parity formula,

$$V_p = V_c - S_0 + e^{-rt}K$$

6. DATA COLLECTION

Five most active sectors are selected for this study namely, IT, FMCG, ENERGY, FINANCIAL SERVICES and PRIVATE BANKS. We select total 22 stocks from these sectors in this study. The last trading price (LTP) of an option has been taken as market price of that option contract for that date. We select 10 option contracts for Call options and 05 option contracts for Put options with different strike prices available on 19th March 2021 expiry date 25th March 2021 & 29th March 2021 for each stock. By this way, we are comparing these models for total 330 options prices. Risk free interest rate is taken as 3.47% for BS & GBS_d. The historical data and Annualized volatilities have been collected from NSE website.

To compute interest rate for MBS_i model, fist we collect long term interest rates and Short-term interest rates for all 10 selected banks. After this we can calculate three types of risk by using these data as shown in previous section. Yield curve Risk (R_{YC}) is 2.62, Basis Risk (R_B) is 0.3522 and Reprising Risk (R_{RP}) is 0.325. By using these three types of risks we can finally get our modified interest rate $R_{IIR} = R_{YC} + R_B + R_{RP} = 2.62 + 0.3522 + 0.325 =$ 3.2972. Different bounds a and b have been selected for different stocks to calculate option price using MBS_d model.

As discussed earlier we know that option traders often have their own expected (finite) range of the underlying price in mind. We assume here the underlying price is bounded above and bounded below by some finite numbers. This bound may vary person by person. The price of option is dependent on this bound if one changes the bound, theoretical option price of our modified model will also change. Therefore, the importance of theses bounds is not negligible.

7. COMPARISON OF TWO MODELS

We have used following terms in each table.

- **Observed date**: A date when market price of option contact has taken.
- **Spot price**: A market piece of underlying stock at time t=0
- **Strike price**: A fixed price of an underlying stock agreed at the time of contract.
- **Vol.**: Historical volatility of underlying stock.
- **Market price**: A price of a call and put option traded in market (LTP)
- **BS price**: A price of option calculated through B-S model.
- **MBS_d price**: A price of option calculated through modified model by changing distribution.
- **MBS_i price**: A price of option calculated through modified model by changing interest rate.

In the following Tables 3 and 4 we have demonstrated the option pricing calculation for TCS stock (Call options) and HCLTECH stock (Put options). We can get the BS price by putting required values in equation (1), MBS_d price by equation (4) and MBS_i price by equation (1) taking the interest rate computed by formula (3). Similarly, we can calculate option prices for all 22 stocks.

Table 3 Call Option prices for TCS Stock

Observed Date	Spot price	Strike price	Vol.	Market price	BS price	MBS_d price	MBS_i price
19-Mar-21	3054.8	3100	0.3565	28.5	41.4273	36.6985	41.3863
19-Mar-21	3054.8	3200	0.3565	5.95	14.7285	10.5045	14.7093
19-Mar-21	3054.8	3080	0.3565	37.25	49.5010	44.8130	49.4550
19-Mar-21	3054.8	3060	0.3565	48.5	58.6060	54.0071	58.5547
19-Mar-21	3054.8	3120	0.3565	21.1	34.3460	29.6253	34.3100
19-Mar-21	3054.8	3040	0.3565	62.3	68.7642	64.3016	68.7077
19-Mar-21	3054.8	3300	0.3565	0.75	4.08386	1.31273	4.07717
19-Mar-21	3054.8	3160	0.3565	11.35	22.9371	18.3757	22.9102
19-Mar-21	3054.8	3140	0.3565	15.6	28.2042	23.5397	28.1730
19-Mar-21	3054.8	3100	0.3565	28.5	41.4273	36.6985	41.3863

Table 4 Put Option prices for HCLTECH Stock

Observed Date	Spot price	Strike price	Vol.	Market price	BS price	MBS_d price	MBS_i price
19-Mar-21	962	950	0.4207	8.7	16.45709	10.71748	16.47079
19-Mar-21	962	940	0.4207	5.85	12.59327	7.153702	12.60458
19-Mar-21	962	900	0.4207	1.3	3.328766	0.292579	3.332743
19-Mar-21	962	960	0.4207	12.7	21.02556	15.12722	21.04175
19-Mar-21	962	910	0.4207	1.6	4.844005	1.054878	4.849409
19-Mar-21	962	920	0.4207	2.5	6.844048	2.392863	6.851163
19-Mar-21	962	930	0.4207	3.5	9.405308	4.399125	9.414403
19-Mar-21	3054.8	3100	0.3565	28.5	41.4273	36.6985	41.3863

8. RESULTS

In the following Tables 5 and 6 we have demonstrated the RMSE error difference between model price and market price of each stock options for both Call options and Put options.

Table 5 Error table for Call Options

STOCKS	RMSE for BS	RMSE for MBS_d	RMSE FOR MBS_i
TCS	7.976086	*4.775548	7.949856
WIPRO	1.319988	*0.677165	1.316946
HCLTECH	3.945692	*2.069776	3.936871
INFY	10.61631	*9.6698	10.62128

Continued on next page

Table 5 continued

TECHM	3.055824	*1.019779	3.047755
ITC	1.733277	*1.644262	1.734029
HINDUNILVR	11.22057	*9.925248	11.18832
BRITANNIA	17.78487	*15.76001	17.75883
GODREJCP	3.771274	3.860605	*3.768607
DABUR	1.448482	*1.383376	1.443864
RELIANCE	13.98319	*7.473577	13.957
NTPC	*0.669087	0.817199	0.669537
ONGC	0.758547	*0.180173	0.757302
POWERGRID	0.382504	*0.208789	0.379967
IOC	1.383608	*1.267203	1.38234
HDFCBANK	*7.337846	8.104768	7.337968
HDFC	14.94323	*6.068009	14.9183
ICICIBANK	4.150286	*3.946224	4.146804
KOTAKBANK	6.112423	*1.565582	6.09723
SBIN	2.326853	*2.248894	2.322897
INDUSINDBANK	12.05381	*2.526378	12.04533
AXISBANK	6.68257	*5.9323	6.675468

Table 6 **Error table for Put Options**

Stocks	RMSE for BS	RMSE for MBS_{d}	RMSE FOR MBS_{i}
TCS	15.04286	*3.136084	15.06583
WIPRO	1.58161	*0.465709	1.584314
HCLTECH	3.899856	*0.92848	3.906708
INFY	6.844729	*6.748978	6.850387
TECHM	3.594823	*0.713173	3.60125
ITC	0.633079	*0.241034	0.634187
HINDUNILVR	4.9234	*2.656332	4.93197
BRITANNIA	16.37341	*2.714004	16.39874
GODREJCP	2.643915	*1.993582	2.644962
DABUR	1.782209	*0.597065	1.786355
RELIANCE	6.670019	*1.55547	6.679658
NTPC	0.041698	0.041741	*0.041264
ONGC	0.408745	*0.10776	0.409598
POWERGRID	0.584837	*0.273975	0.586586
IOC	0.161917	*0.161887	0.16209
HDFCBANK	3.598188	*1.39565	3.611126
HDFC	11.96195	*2.037434	11.97862
ICICIBANK	2.495526	*0.509561	2.498856
KOTAKBANK	9.75421	*1.002749	9.772224
SBIN	1.693482	*0.411333	1.695935
INDUSINDBANK	9.867259	*1.54839	9.87399
AXISBANK	4.704487	*0.958686	4.709226

9. CONCLUSIONS

We have compared three models, BS model, MBS_d model and MBS_i model by comparing RMSE errors between market price of options and theoretical price computed by all three models. From the error tables we can observe that,

For Call options, only for NTPC and HDFCBANK, BS model outperforms other two models. For only one stock GODREJCP, MBS_i model outperforms other two models. For remaining 19 stocks, MBS_d model outperforms other two models.

For Put options, only for NTPC, MBS_i model outperforms other two models, for remaining 21 stocks, MBS_d model outperforms other two models.

Therefore, we can suggest that MBS_d model could be used to obtain better theoretical option prices as far as Indian stock options are concerned.

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