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ON HOMOGENEOUS QUINARY QUADRATIC DIOPHANTINE EQUATION $x^2 + y^2 + 4(z^2 + w^2) = 24t^2$

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ABSTRACT

The homogeneous quadratic Diophantine equation with five unknowns given by $x^2 + y^2 + 4(z^2 + w^2) = 24t^2$ is analyzed for determining its non-zero distinct integer solution through employing linear transformations.

Keywords: Homogeneous Quadratic, Quadratic with Five Unknowns, Integer Solutions

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1. INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous, or non-homogeneous quadratic Diophantine equations with two or more variables have been an interest to mathematicians since antiquity Dickson (1971), Mordell (1969), Andre (1984), Datta and Singh (1938). In this context, one may refer Gopalan and Srividhya (2012), Gopalan et al. (2013), Vijayasankar et al. (2017), Vidhyalakshmi et al. (2018), Adiga (2020) for different choices of quadratic Diophantine equations with four unknowns. In Anbuselvi and Rani (2017), Anbuselvi and Rani (2018), Gopalan et al. (2013) the quadratic Diophantine equation with five unknowns are analysed for obtaining their non-zero distinct integer solutions.

This motivated me for finding integer solutions to other choices of quadratic equations with five unknowns. This paper deals with the problem of determining non-zero distinct integer solutions to the quadratic Diophantine equation with five unknowns given by $x^2+y^2+4(z^2+w^2)=24t^2$

2. METHOD OF ANALYSIS

The second-degree Diophantine equation with five unknowns to be solved is

$$x^2+y^2+4(z^2+w^2)=24t^2 \quad \textbf{Equation 1}$$

The process of obtaining different sets of non-zero distinct integer solutions To Equation 1 is exhibited below:

Set 1:

The substitution of the linear transformations

$$x=4t,y=2t \quad \textbf{Equation 2}$$

in Equation 1 leads to the Pythagorean equation

$$t^2=z^2+w^2 \quad \textbf{Equation 3}$$

which is satisfied by

$$w=a^2-b^2,z=2ab,t=a^2+b^2 \quad \textbf{Equation 4}$$

In view of Equation 2, one has

$$\begin{aligned} x&=4(a^2+b^2),\\ y&=2(a^2+b^2) \end{aligned} \quad \textbf{Equation 5}$$

Thus, Equation 4 and Equation 5 represent the integer solutions to Equation 1

Set 2:

Introducing the linear transformations

$$x = 4u, y = 4v, z = u + v, w = u - v \quad \textbf{Equation 6}$$

in Equation 1 it simplifies to the Pythagorean equation

$$t^2 = u^2 + v^2 \quad \textbf{Equation 7}$$

whose solutions may be taken as

$$t = p^2 + q^2, u = p^2 - q^2, v = 2pq \quad \textbf{Equation 8}$$

In view of Equation 6 the integer solutions to Equation 1 are given by

$$x = 4(p^2 - q^2), y = 8\,pq, z = (p^2 - q^2 + 2pq), w = (p^2 - q^2 - 2pq)\;, t = (p^2 + q^2)$$

Set 3:

Taking

$$x = 4t, y = 2(z - 2\alpha), w = 2\alpha \quad \textbf{Equation 9}$$

in Equation 1 it reduces to

$$z^2 - 2\alpha z + 4\alpha^2 - t^2 = 0 \quad \textbf{Equation 10}$$

Treating Equation 10 as a quadratic in z and solving for z, it is seen that Equation 10 is satisfied by

$$t = 3r^2 + s^2,$$
$$\alpha = 2rs,$$
$$z = 2rs \pm (3r^2 - s^2)$$

In view of Equation 9 it is seen that the corresponding values of *x, y, w* satisfying Equation 1 are

$$x = 4(3r^2 + s^2)$$
$$y = -4rs \pm 2(3r^2 - s^2),$$
$$w = 4rs$$

Set 4:

Taking

$$x = 4(z + w), y = 2Y, t = z + w \quad \textbf{Equation 11}$$

in Equation 1 it reduces to

$$z^2 + 4zw + w^2 - Y^2 = 0 \quad \textbf{Equation 12}$$

Treating Equation 12 as a quadratic in z and solving for z, it is seen that Equation 12 is satisfied by

$$Y = 3r^2 - s^2,$$
$$w = 2rs,$$
$$z = -4rs \pm (3r^2 + s^2)$$

In view of Equation 11 it is seen that the corresponding values of satisfying Equation 1 are

$$x = -8rs \pm 4(3r^2 + s^2)$$
$$y = 2(3r^2 - s^2),$$
$$t = -2rs \pm (3r^2 + s^2)$$

3. CONCLUSION

In this paper, an attempt has been made to obtain non-zero distinct integer solutions to the quadratic Diophantine equation with five unknowns given by $x^2 + y^2 + 4(z^2 + w^2) = 24t^2$ The readers of this paper may search for finding integer solutions to other choices of quadratic Diophantine equations with five or more unknowns.

CONFLICT OF INTERESTS

None.

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