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## CALCULATION OF MULTIPLE MIXING RATIOS OF GAMMA RAYS FROM $^{142-150}_{60}Nd(n, n')$ $^{142-150}_{60}Nd$ INTERACTION

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### Abstract

In the current research, multiple mixing ratios of gamma -transitions of the energy levels  $^{142-150}_{60}Nd$  isotopes populated in  $^{142-150}_{60}Nd(n, n')$   $^{142-150}_{60}Nd$  interaction are calculated using the constant statistical tensor (CST) method. The results obtained are, in general, in good agreement or consistent, within the experimental error, with the results published in the previously researches. Existing discrepancies result from inaccuracies in the experimental results of previous works. The current results confirm the validity of the constant statistical tensor method of calculating the values of mixing ratios and its predictability of errors in experimental results.

**Keywords:** Multiple Mixing Ratios, The Constant Statistical Tensor (CST) Method, Electric Quadra Pole  $E_2$ , Magnetic Dipole  $M_2$ ,  $^{142-150}_{60}Nd$ , Gamma –Transition,  $\delta$ .

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### 1. Introduction

The mixing ratios ( $\delta$ ) are defined as the ratio between the matrix elements for electric Quadra pole  $E_2$  and the magnetic dipole  $M_2$  for the transition of gamma from the elementary to final level. The angular distribution measurements of gamma radiation are sensitive to interference between matrix elements for  $E_2$  and  $M_2$ . There are different terms in the literature because of the different formulas proposed for the interpretation of angular distribution [1]. Therefore, the definition used for the mixing ratio should be determined in the angular distribution measurements. The definition of the user in the current research is the definition of Steffen and Alder [2]

$$\delta = \frac{\langle J_f | E_2 | J_i \rangle}{\langle J_f | M_1 | J_i \rangle} \dots \dots \dots \quad (1)$$

Youhana [3] noted that the statistical tensor  $\rho_2(J_i)$  that is associated with  $P(M_i)$  of the relationship:

$$\rho_2(J_i) = \sum_{\substack{M_i=0 \\ \text{or} \\ M_i=\frac{1}{2}}}^{M_i=J_i} \rho_2(J_i, M_i) P(M_i) \dots \dots \dots \quad (2)$$

It must be constant for levels with the same spin as long as the coefficients of the statistical tenser  $\rho_2(J_i, M_i)$  are not dependent on the energy of the level but only on  $J_i$  and  $M_i$ , Youhana therefore proposed the method of constant statistical tenser (CST) and applied it successfully in the calculation of  $\delta$  – values of mixed transitions from excited levels in interactions  $^{90,92,94}\text{Zr}(n, n'\gamma)^{90,92,94}\text{Zr}$  [3],[4] and  $^{150}\text{Nd}(n, n'\gamma)^{150}\text{Nd}$  reaction [5]. This method proved to be a good way, such as the international program (CINDY) for the calculated of mixed gamma transitions, as well as its predictability of any error or inaccuracy in experimental results because the method is based on experimental results and does not depend on any nuclear model, to do all the necessary calculations.

In the current research, the transitions ( $2^+ - 2^+$ ) in  $^{142,144,146,158,150}\text{Nd}$  isotopes were studied and their mixing ratios were calculated due to their importance in examining the nuclear models. Berthier and Berthoumieux [6] mention In their research, that many nuclear models were developed within thirty years for the purpose of describing the nucleus, but so far no nuclear model has been developed to do so due to the lack of empirical data, especially the mixing ratios of gamma transitions ( $2^+ - 2^+$ ) or the discrepancy of measured values in different research .

Demidov and others [7] studied the excited levels of  $^{142,148,150}_{60}\text{Nd}(n, n')$  interactions and Govor and others [8]studied the isotope  $^{142}_{60}\text{Nd}$  from the inflexible scattering reaction of the fast reactor neutrons. They measured the angular distribution of gamma rays emitted from  $^{142}_{60}\text{Nd}$  levels. Al-Janabi and others [9],[10].also studied the angular distribution of gamma rays from the energy levels of the two isotopes  $^{144}_{60}\text{Nd}$  ,  $^{146}_{60}\text{Nd}$  respectively. Youhana [5] studied the angular distribution of gamma rays from the energy levels of the  $^{150}_{60}\text{Nd}$  based on the results of the reference [7] and the results [11].

## 2. Data Reduction and Analysis

For pure transitions and transitions that can be counted as pure, the constant statistical tenser  $\rho_2(J_i)$  can be calculated for the following relationship:

$$a_2(J_i - J_f) = \rho_2(J_i) F_2(J_i J_f \delta) \dots \dots \dots \quad [12] \quad (3)$$

Where  $\rho_2(J_i)$  =constant statistical tenser for primary level  $J_i$   
 $F_k(J_i J_f \delta)$  =Coefficients which contain the information on angular momentum changes and the multipole mixing

$$F_k(J_i J_f \delta) = \frac{F_k(J_f L_1 L_1 J_i) + 2\delta F_k(J_f L_1 L_2 J_i) + \delta^2 F_k(J_f L_2 L_2 J_i)}{(1+\delta^2)} \dots \dots \dots \quad [13] \quad (4)$$

$$L_1 = |J_i - J_f| \neq 0 \dots \dots \dots \quad (5)$$

$$L_2 = L_1 + 1 \dots \dots \quad (6)$$

$$L = l + s \neq 0 \dots \dots \dots \quad (7)$$

*L = Angular momentum of gamma Ray  $\neq 0$  because the following*

*l = Angular orbit momentum = 0, 1, 2, 3, 4 ...*

*s = spin = 1*

If the transition is pure, then  $\delta = 0$  the equation (4) becomes:

$$F_k(J_i J_f \delta) = F_k(J_f L_1 L_1 J_i) \dots \dots \dots \quad (8)$$

Reparation of equation (8) in (3) Produce:

$$a_2(J_i - J_f) = \rho_2(J_i) F_k(J_f L_1 L_1 J_i) \dots \dots \dots \quad (9)$$

From equation (9) we get on  $\rho_2(J_i)$

$$\rho_2(J_i) = a_2(J_i - J_f) / F_k(J_f L_1 L_1 J_i) \dots \dots \dots \quad (10)$$

The values of  $F_k(J_f L_1 L_1 J_i)$ ,  $J_i$  and  $J_f$  are listed in the Appendix (A)

And taking the pure probability ( $2^+ - 0^+$ ) of  $^{142-150}_{60}Nd$  isotopes

$$\rho_2(2) = \frac{a_2(2-0)}{F_2(0222)} = -\frac{a_2(2-0)}{59761} \dots \dots \dots \quad (11)$$

$\rho_2(2)$  Values can be found for the mixed transition ( $2^+ - 2^+$ ) of  $^{142-150}_{60}Nd$  isotopes by  
Reparation of equation (4) in (3) Produce:

$$a_2(J_i - J_f) = \rho_2(J_i) \frac{F_k(J_f L_1 L_1 J_i) + 2\delta F_k(J_f L_1 L_2 J_i) + \delta^2 F_k(J_f L_2 L_2 J_i)}{(1+\delta^2)} \dots \dots \quad (12)$$

$$a_2(2 - 2) = \rho_2(2) \frac{F_2(J_2 L_1 L_1 J_2) + 2\delta F_2(J_2 L_1 L_2 J_2) + \delta^2 F_2(J_2 L_2 L_2 J_2)}{(1+\delta^2)} \dots \dots \dots \quad (13)$$

$$a_2(2 - 2) = \rho_2(2) \frac{-0.41833 - 1.22476 \delta + 0.12806 \delta^2}{(1+\delta^2)} \dots \dots \dots \dots \dots \quad (14)$$

The transition is pure or can be considered pure if the following condition is met:

$$|J_i - J_f| \leq L \leq (J_i + J_f)$$

$$|2 - 0| \leq L \leq (2 + 0) \rightarrow 2 \leq L \leq 2 \rightarrow L = 2 \text{ only}$$

$$\pi_i, \pi_f = (-1)^L \quad (\text{The change in the parity of the electrical radiation (EL)})$$

$\pi_i$  = the parity of initial level= (+)

$\pi_f$  == the parity of final level= (+)

$(+) \times (+) = (-1)^L \rightarrow L = \text{even numbers} = 2, 4, 6, \dots, k \rightarrow$

$E = E2 \text{ because } L = 2 \text{ only}$

$\pi_i, \pi_f = (-1)^{L+1}$  ( The change in the parity of the magnetice radiation (ML) )

$(+) \times (+) = (-1)^{L+1} \rightarrow L = \text{odd numbers} = 1, 2, 3, 5, \dots, k \rightarrow ML = M1, M3, M5, \dots$

$\rightarrow ML \text{ is not found because } L = 2 \text{ only}$

Therefore pure  $E2$  transitions represent transitions  $(2^+ - 0^-)$

While the transition  $(2^+ - 2^+)$  is a mixed transition due to the following:

$$|J_i - J_f| \leq L \leq (J_i + J_f)$$

$$|2 - 2| \leq L \leq (2 + 2) \rightarrow 0 \leq L \leq 4 \rightarrow L = 0, 1, 2, 3, 4$$

For the electrical radiation

$(+) \times (+) = (-1)^L \rightarrow L = \text{even numbers} \rightarrow EL = E2, E4 \text{ but } EL \neq E0 \text{ because } L \neq 0$

$EL = E2 \text{ only because } E2 > E3 > E4 \rightarrow E2 >> E4$

For the magnetice radiation

$(+) \times (+) = (-1)^{L+1} \rightarrow L = \text{odd numbers}, ML = M1, M3$

$ML = M1 \text{ because } M1 > M2 > M3 \rightarrow M1 >> M3$

Therefore the transition  $(2^+ - 2^+)$  is mix transition  $(E2 + M1)$

The  $\rho_2(J_i)$  of pure gamma transitions in addition to the mixed gamma transitions were calculated using the equation (4) and the values of  $a_2(J_i - J_f)$  and  $\delta$ -values measured experimentally for such transitions were considered. If there is more than one value for constant statistical tenser, the weighted average is taken as in relation:

$$\rho_2(J_i) = \frac{\sum \frac{\rho_2(J_i)}{\Delta \rho_2(J_i)_i^2}}{\sum \frac{1}{\Delta \rho_2(J_i)_i^2}} \dots \dots \dots \quad (15)$$

$$\rho_2(J_i) = \sqrt{\frac{1}{\sum \frac{1}{\Delta \rho_2(J_i)_i^2}}} \dots \dots \dots \quad (16)$$

Where  $\Delta \rho_2(J_i)$  = The amount of error in constant statistical tenser

### 3. Results and Discussion

Tables (1), (2),(3),(4),(5) show the energy levels and the gamma- transitions  $(2^+ - 0^+)$  used to calculate  $\rho_2(2^+)$  for the levels  $J_l^\pi = 2^+$  in  $^{142}_{60}Nd$ ,  $^{144}_{60}Nd$ ,  $^{146}_{60}Nd$ ,  $^{148}_{60}Nd$ ,  $^{150}_{60}Nd$  respectively. While the table (6) shows the weighted average for  $\rho_2(2^+)$  values of the isotopes themselves.

Table 1: the energy levels and gamma transitions ( $2^+ - 0^+$ ) in  $^{142}_{60}\text{Nd}$  used in the calculation  $\rho_2(2^+)$

$E_i(\text{KeV})$	$E_\gamma(\text{KeV})$	$a_2$ [8]	$\rho_2(2^+)$
1575.8	1575.8	0.223(17)	$-0.37315 \pm 0.02845$
2384.3	2384.3	0.297(17)	$-0.49698 \pm 0.02845$
2845.9	2845.8	0.325(25)	$-0.54383 \pm 0.04183$
3045.1	3045.1	0.250(19)	$-0.41833 \pm 0.03179$
3128.0	3128.0	0.210(30)	$-0.35140 \pm 0.06693$
3358.7	3358.6	0.210(80)	$-0.35140 \pm 0.13387$
3470.2	3470.3	0.260(60)	$-0.43507 \pm 0.10040$
3579.8	3579.8	0.380(50)	$-0.63587 \pm 0.08367$

Table 2: the energy levels and gamma transitions ( $2^+ - 0^+$ ) in  $^{144}_{60}\text{Nd}$  used in the calculation  $\rho_2(2^+)$

$E_i(\text{KeV})$	$E_\gamma(\text{KeV})$	$a_2$ [9]	$\rho_2(2^+)$
696.3	696.3	0.223(18)	$-0.37315 \pm 0.03012$
1560.3	1560.3	0.361(67)	$-0.60407 \pm 0.11211$
2072.2	2072.1	0.417(52)	$-0.69778 \pm 0.08701$
2270.0	2270.0	0.180(81)	$-0.30120 \pm 0.13554$
2526.7	2526.5	0.466(52)	$-0.77977 \pm 0.08701$

Table 3: the energy levels and gamma transitions ( $2^+ - 0^+$ ) in  $^{146}_{60}\text{Nd}$  used in the calculation  $\rho_2(2^+)$

$E_i(\text{KeV})$	$E_\gamma(\text{KeV})$	$a_2$ [10]	$\rho_2(2^+)$
453.9	453.9	0.231(24)	$-0.38654 \pm 0.04016$
1470.4	1470.4	0.312(34)	$-0.52208 \pm 0.05689$
1787.1	1787.2	0.281(35)	$-0.47021 \pm 0.05857$
1977.8	1977.4	0.322(50)	$-0.53881 \pm 0.08367$
2119.5	2119.4	0.305(87)	$-0.51037 \pm 0.14558$
2208.4	2208.4	0.310(25)	$-0.51873 \pm 0.04183$

Table 4: the energy levels and gamma transitions ( $2^+ - 0^+$ ) in  $^{148}_{60}\text{Nd}$  used in the calculation  $\rho_2(2^+)$

$E_i(\text{KeV})$	$E_\gamma(\text{KeV})$	$a_2$ [7]	$\rho_2(2^+)$
301.6	301.6	0.24(3)	$-0.40160 \pm 0.05020$
1171.0	1171.0	0.36(6)	$-0.60240 \pm 0.10040$
1248.8	1248.9	0.35(3)	$-0.58567 \pm 0.05020$

Table 5: the energy levels and gamma transitions ( $2^+ - 0^+$ ) in  $^{150}_{60}Nd$  used in the calculation  $\rho_2(2^+)$

$E_i(KeV)$	$E_\gamma(KeV)$	$a_2$ [5]	$\rho_2(2^+)$
130.1	130.1	0.185(25)	$-0.30957 \pm 0.04183$
1061.6	1061.7	0.277(12)	$-0.46351 \pm 0.02008$
2260.4	1584.0	0.305(79)	$-0.51037 \pm 0.13219$

Table 6: Average weighted for values  $\rho_2(2^+)$

Isotope	$\rho_2(2^+)$
$^{142}_{60}Nd$	$-0.44625 \pm 0.01482$
$^{144}_{60}Nd$	$-0.44716 \pm 0.02582$
$^{146}_{60}Nd$	$-0.47201 \pm 0.02246$
$^{148}_{60}Nd$	$-0.50572 \pm 0.03347$
$^{150}_{60}Nd$	$-0.43607 \pm 0.01793$

These values are constant for all levels  $J_i^\pi = 2^+$  and used in the calculation of  $\delta$  values of transitions ( $2^+ - 2^+$ ) in even neodymium isotopes. The obtained results are shown in the tables (7,8,9,10,11) for isotopes,  $^{142}_{60}Nd$ ,  $^{144}_{60}Nd$ ,  $^{146}_{60}Nd$ ,  $^{148}_{60}Nd$ ,  $^{150}_{60}Nd$  respectively. In each table, the current results were compared with the previously measured results.

Table 7: Mixing ratios for gamma transitions ( $2^+ - 2^+$ ) in  $^{142}Na$  by using constant statistical tenser (CST) method

$E_i(KeV)$	$E_\gamma(KeV)$	$\frac{a_2}{a_4}[8]$	$\delta$ – values		
			Ref. [8]	Ref. [14]	Present work
2384.3	808.6	0.329(37)	0.16(6)	0.20(5)	$(0.34_{-0.12}^{+?})$
		0.078(50)	.....	1.3(2)	$(1.1_{-?}^{+0.3})$
2583.1	1007.3	0.030(22)	-0.28(3)	-0.28(2)	$-(0.27_{-0.03}^{+0.05})$
		0.020(30)	.....	$(6.5_{-0.9}^{+1.3})$	$(6.5_{-1.3}^{+2.2})$
2845.9	1270.0	-0.220(90)	-0.60(30)	-0.82(8)	$-(1.1_{-0.5}^{+?})$
		-0.160(130)	$-(0.60_{-3}^{+29})$	-3.7(7)	$-(2.2_{-?}^{+4.7})$
3045.1	1469.5	0.290(60)	$0.1 < \delta < 1.5$	0.20(9)	$(0.22_{-0.14}^{+0.23})$
		-0.030(80)	.....	1.4(3)	1.4(5)
3128.0	1552.2	-0.120(24)	-0.69(9)	-0.61(4)	$-(0.60_{-0.06}^{+0.08})$
		-0.030(50)	$-(5.1_{-1.4}^{+2.2})$	-5.8(14)	$-(8.1_{-2.8}^{+5.6})$
3358.7	1782.9	-0.200(40)	$-5 < \delta$	$-(0.84_{-0.10}^{+0.20})$	$-(0.94_{-0.21}^{+?})$
		0.020(50)	$< -0.7$	$-(3.4_{-0.9}^{+1.3})$	$-(2.9_{-?}^{+1.7})$

Table 8: Mixing ratios for gamma transitions ( $2^+ - 2^+$ ) in  $^{144}Na$  by using constant statistical tenser (CST) method

$E_i(KeV)$	$E_\gamma(KeV)$	$\frac{a_2}{a_4}[9]$	$\delta$ – values		
			Ref. [9]	Ref. [14]	Present work
1560.5	864.2	-0.165(30)	-0.73(8)	-0.70(6)	$-(0.75_{-0.11}^{+0.16})$
		0.088(43)	....	$-(4.6_{-0.9}^{+1.5})$	$-(4.3_{-1.3}^{+2.2})$

2072.2	1375.9	0.309(38) -0.005(56)	0.13(7) 1.7(3)	0.12(4) 1.5(2)	( 0.27 <sup>+0.18</sup> <sub>-0.11</sub> ) 1.2(3)
2084.4	1387.9	-0.185(50) -0.040(61)	-(0.70 <sup>+0.20</sup> <sub>-0.12</sub> ) -(5.1 <sup>+4.8</sup> <sub>-2.0</sub> )	-( 0.77 <sup>+0.18</sup> <sub>-0.08</sub> ) -( 3.9 <sup>+2</sup> <sub>-1.1</sub> )	-( 0.85 <sup>?</sup> <sub>-0.21</sub> ) -(3.4 <sup>+3.1</sup> <sub>-?</sub> )
2367.8	1671.7	0.288(36) 0.016(43)	0.15 <sup>+0.09</sup> <sub>-0.06</sub> 1.6(3)	0.18(6) 1.5(2)	( 0.21 <sup>+0.13</sup> <sub>-0.09</sub> ) 1.4(3)
2526.7	1830.5	0.393(59) -0.048(84)	0.50(25) .....	Imaginary roots	Imaginary roots
2591.6	1895.2	0.289(51) 0.067(94)	0.15(10) .....	0.18(7) 1.4(3)	( 0.22 <sup>+0.18</sup> <sub>-0.13</sub> ) 1.4(4)
2828.2	1267.6	0.221(92) -0.106(113)	0.02(15) (2.2 <sup>+1.2</sup> <sub>-0.8</sub> )	(0.06 <sup>+0.22</sup> <sub>-0.16</sub> ) (1.9 <sup>+1.2</sup> <sub>-0.7</sub> )	(0.06 <sup>+0.24</sup> <sub>-0.16</sub> ) (1.9 <sup>+1.2</sup> <sub>-0.7</sub> )
2900.5	1339.9	0.293(47) 0.087(87)	( 0.15 <sup>+0.17</sup> <sub>-0.13</sub> ) 1.6(6)	0.20(8) 1.4(3)	(0.23 <sup>+0.17</sup> <sub>-0.12</sub> ) 1.3(4)

Table 9: Mixing ratios for gamma transitions ( $2^+ - 2^+$ ) in  $^{146}Na$  by using constant statistical tensor (CST) method

$E_i(KeV)$	$E_\gamma(KeV)$	$\frac{a_2}{a_4}[10]$	$\delta - values$		
			Ref. [10]	Ref. [14]	Present work
1470.4	1016.5	0.05(250) -0.020(31)	-0.24(5) (5.4 <sup>+3</sup> <sub>-2.3</sub> )	-0.24(2) (5.6 <sup>+0.8</sup> <sub>-0.6</sub> )	-0.24(5) (5.5 <sup>+1.6</sup> <sub>1.1</sub> )
1787.1	1333.2	-0.111(37) 0.000(47)	-0.64(11) -(10 <sup>+70</sup> <sub>-4</sub> )	-( 0.56 <sup>+0.06</sup> <sub>-0.04</sub> ) -(11 <sup>+18</sup> <sub>-2</sub> )	-(0.56 <sup>+0.11</sup> <sub>-0.09</sub> ) -(11 <sup>+32</sup> <sub>-5</sub> )
1905.4	1451.6	-0.058(33) -0.025(50)	-0.45(7) .....	-0.44(4) .....	-0.44(7) only
1977.8	1523.7	0.177(29) -0.001(38)	-0.07(4) 2.8(4)	-0.05(2) 2.6(2)	-0.03(5) 2.5(4)
2143.1	1689.4	-0.074(12) -0.012(15)	-0.48(3) .....	-0.48(2) .....	-0.47(3) Only
2197.4	1743.5	0.165(26) -0.052(32)	..... 2.9(4)	..... 2.7(2)	-0.05(5) 2.6(4)
2265.9	1812.0	0.374(48) -0.006(59)	( 0.40 <sup>?</sup> <sub>-0.16</sub> ) (0.95 <sup>+0.35</sup> <sub>-?</sub> )	0.45(11) 0.87(20)	( 0.47 <sup>?</sup> <sub>-0.21</sub> ) ( 0.86 <sup>+0.39</sup> <sub>-?</sub> )
2286.0	1831.7	0.083(14) -0.003(17)	-0.19(3) 4.4(5)	-0.19(2) 4.2(3)	-0.19(2) 4.2(5)
2436.8	1982.5	0.089(14) 0.008(17)	-0.18(2) .....	-0.18(1) .....	-0.18(2) 4.0(5)
2457.2	2002.9	0.280(80) 0.042(92)	(0.14 <sup>+0.20</sup> <sub>-0.14</sub> ) 1.6(5)	0.16(9) 1.6(4)	( 0.16 <sup>+0.24</sup> <sub>-0.16</sub> ) (1.5 <sup>+0.7</sup> <sub>-0.5</sub> )
2491.1	2037.2	-0.206(78) 0.111(165)	-( 0.85 <sup>?</sup> <sub>-0.47</sub> ) .....	-0.90(20) .....	-( 0.90 <sup>?</sup> <sub>-0.30</sub> ) -(3.1 <sup>+4.9</sup> <sub>-?</sub> )

Table 10: Mixing ratios for gamma transitions ( $2^+ - 2^+$ ) in  $^{148}Na$  by using constant statistical tensor (CST) method

$E_i(KeV)$	$E_\gamma(KeV)$	$\frac{a_2}{a_4}[7]$	$\delta - values$		
			Ref.[7]	Present work	
1171.0	869.2	0.01(4) 0.02(5)	..... $(8.3_{-2.4}^{+11.7})$	-0.31(7) $(8.6_{-3}^{+9.6})$	
1248.8	947.1	-0.06(4) 0.01(4)	-33(17) .....	-0.44(8) only	
1645.4	1343.8	0.09(6) 0.04(7)	-0.18(10) $(4.2_{-1.3}^{+3.4})$	-0.19(9) $(4.2_{-1.2}^{+2.6})$	
1659.6	1358.0	0.22(6) 0.00(7)	0.02(8) 2.2(5)		$(0.01_{-0.09}^{+0.11})$ 2.2(6)

Table 11: Mixing ratios for gamma transitions ( $2^+ - 2^+$ ) in  $^{150}Na$  by using constant statistical tensor (CST) method

$E_i(KeV)$	$E_\gamma(KeV)$	$\frac{a_2}{a_4}[5]$	$\delta - values$		
			Ref. [ 5 ]	Ref. [11]	Present work
1061.6	931.6	-0144(29) 0.006(35)	$-(0.68_{-0.09}^{+0.12})$ $-(5.5_{-1.7}^{+3.2})$	-0.75(10) .....	$-(0.69_{-0.10}^{+0.12})$ $-(5.4_{-1.7}^{+3.1})$
1391.2	329.7	-0.178(15) -0.017(19)	$-(0.82_{-0.07}^{+0.1})$ $-(3.6_{-0.6}^{+0.8})$	-0.92(10) $-(3.1_{-0.5}^{+0.7})$	$-(0.83_{-0.07}^{+0.10})$ $-(3.5_{-0.6}^{+0.8})$
1434.6	373.3	0.196(36) -0.016(44)	0.02(8) 2.1(4)	0.05(8) .....	0.03(7) 2.1(4)
	234.2	-0.089(46) 0.017(55)	$-(0.52_{-0.10}^{+0.13})$ ....	-0.7(2) .....	$-(0.53_{-0.11}^{+0.13})$ only
1545.2	1414.6	0.103(8) -0.016(10)	-0.14(2) 3.5(2)	-0.13(2) 3.4(2)	-0.14(2) 3.5(2)
1579.4	1449.7	0.054(30) -0.036(37)	-0.23(6) $(5.1_{-1.1}^{+1.9})$	-0.23(6) 5.0(11)	-0.23(5) $(5.1_{-1.1}^{+1.9})$
71986.7	925.2	-0.211(42) -0.095(49)	$-(1.1_{-0.3}^{+?})$ $-(2.4_{-?}^{+1.7})$	$-(1.1_{-0.3}^{+?})$ .....	$-(1.1_{-0.3}^{+?})$ $-(2.4_{-?}^{+1.6})$
2243.0	1181.1	0.140(26) -0.018(31)	-0.08(5) 2.8(5)	-0.08(5) 2.8(5)	-0.08(5) 2.8(5)
2260.4	1198.5	-0.228(38) -0.051(46)	$-(1.3_{-0.4}^{+?})$ $-(1.8_{-?}^{+1.4})$	$-(1.6_{-0.4}^{+?})$ .....	$-(1.4_{-0.5}^{+?})$ $-(1.7_{-?}^{+1.4})$
2532.3	1140.8	0.190(39) 0.034(45)	0.01(8) 2.2(5)	0.00(7) 2.2(5)	0.01(8) 2.2(5)
2580.1	1518.5	0.108(8) -0.009(10)	-0.14(2) 3.4(2)	-0.14(2) .....	-0.14(2) 3.4(2)

From the table (7), we observe that the calculated  $\delta$  values for the transition 1007.3 KeV from the level 2583.1 KeV are very agreement with the  $\delta$  values published in the two references [8,14]. As well as the calculated  $\delta$  values for the transition 1552.2 KeV from the level 3128.0 and the

calculated  $\delta$  values for the two transitions 1469.5 KeV and 1782.9 KeV of the two levels 3045.1 KeV and 3358.7 KeV respectively agreement with the reference [14], and within the range in reference [8] to each of these two transitions. As for the two transitions 808.6 and 1270.0 KeV respectively, from the levels 2384.3 and 2845.9 KeV there is a difference in  $\delta$  values. This indicates the inaccuracy of the Coefficients  $a_2$  and  $a_4$  measured in the reference [8]. This is clear from the value of  $a_4$  – coefficient of the first transition, which should have been negative signal within the experimental error. As for the second transition, it is clear from the relatively large error associated with each of  $a_2$  and  $a_4$  for this transition.

Note from Table (8) that the  $\delta$  values calculated in the current search using the CST method are consistent with  $\delta$  values within the experimental error limits published in the references [9,14], for all transitions except for the transition 1830.5 KeV from the level 2526.7 KeV. The imaginary roots obtained in the present research and in the reference [14] indicate inaccuracy  $a_2$  and  $a_4$  published in the reference [9] for this transition.

Note from the table (9) that the values calculated in the current search in agreement with the published  $\delta$  values in the references [10, 14] for all transitions.

The table (10) shows that the current  $\delta$  values are fully consistent with the reference values [7] for all transitions except the transition 947.1 KeV from the level 1248.8 KeV where we observe a clear contradiction. This indicates that the published  $\delta$  value in the reference [7] is incorrect. In this reference, the authors did not mention why they ignored the value of  $\delta$  small.

The table (11) shows that the  $\delta$  values calculated in the current search are fully consistent or within the experimental error limits with references  $\delta$  values [ 5, 11]. This indicates that the measured  $a_2$  and  $a_4$  values in the reference [11] for these transitions are correct.

#### 4. Conclusions

It was ascertained that the constant statistical tenser (CST) method of calculating the values of the mixing ratios as well as its ability to predict the existence of any error of the experimental results.

#### APPENDIX A

$J_i$	$L_1$	$L_2$	$J_f$	$F_2$	$F_4$
1	1	1	0	0.70711	0
1	1	1	1	-0.35355	0
1	1	2	1	-1.06067	0
1	2	2	1	-0.35355	0
1	1	1	2	0.07071	0
1	1	2	2	0.47434	0
1	2	2	2	0.35355	0
1	2	2	3	-0.10101	0
1	2	3	3	0.37796	0
1	3	3	3	0.53034	0
1	3	3	4	-0.17678	0
2	2	2	0	-0.59761	-1.06904

2	1	1	1	0.41833	0
2	1	2	1	-0.93542	0
2	2	2	1	-0.29881	0.71269
2	1	1	2	-0.41833	0
2	1	2	2	-0.61238	0
2	2	2	2	0.12806	-0.30544
2	1	1	3	0.11952	0
2	1	2	3	0.65466	0
2	2	2	3	0.34149	0.07636
2	2	2	4	-0.17075	-0.00848
2	2	3	4	0.50507	-0.06274
2	3	3	4	0.44822	-0.02970
2	3	3	5	-0.29881	0.00405
3	3	3	0	-0.86603	0.21320
3	2	2	1	-0.49487	-0.44670
3	2	3	1	-0.46290	1.04463
3	3	3	1	-0.64953	0.03553
3	1	1	2	0.34641	0
3	1	2	2	-0.94869	0
3	2	2	2	-0.12372	0.67006
3	1	1	3	-0.43301	0
3	1	2	3	-0.43301	0
3	2	2	3	0.22682	-0.44670
3	1	1	4	0.14434	0
3	1	2	4	0.72169	0
3	2	2	4	0.30929	0.14890
3	2	2	5	-0.20620	-0.02030
3	2	3	5	0.54554	-0.13430
3	3	3	5	0.36085	-0.05492
3	3	3	6	-0.36085	0.00969
4	3	3	1	-0.78349	0.14527
4	2	2	2	-0.44770	-0.30438
4	2	3	2	-0.52972	0.90036
4	3	3	2	-0.47009	-0.04842
4	1	1	3	0.31339	0
4	1	2	3	-0.94018	0
4	2	2	3	-0.04477	0.60876
4	1	1	4	-0.43875	0
4	1	2	4	-0.33541	0
4	2	2	4	0.26455	-0.49807
4	1	1	5	0.15955	0
4	1	2	5	0.75679	0
4	2	2	5	0.28490	0.19370
4	2	2	6	-0.22792	-0.02980
4	2	3	6	0.56407	-0.184337

4	3	3	6	0.29915	-0.06874
4	3	3	7	-0.39887	0.01422
5	3	3	2	-0.73599	0.11589
5	2	2	3	-0.42056	-0.24281
5	2	3	3	-0.55634	0.80301
5	3	3	3	-0.36799	-0.07726
5	1	1	4	0.29439	0
5	1	2	4	-0.93095	0
5	2	2	4	0	0.56556
5	1	1	5	-0.44159	0
5	1	2	5	-0.27386	0
5	2	2	5	0.28307	-0.52297
5	1	1	6	0.16984	0
5	1	2	6	0.77832	0
5	2	2	6	0.26689	0.22413
5	2	2	7	-0.24263	-0.03736
5	2	3	7	0.57416	-0.22100
5	3	3	7	0.25476	-0.07726
5	3	3	8	-0.42461	0.01783
6	3	3	3	-0.70510	0.09967
6	2	2	4	-0.40291	-0.20883
6	2	3	4	-0.56980	0.73833
6	3	3	4	-0.30219	-0.09018
6	1	1	5	0.28204	0
6	1	2	5	-0.92319	0
6	2	2	5	0.02878	0.53699
6	1	1	6	-0.44320	0
6	1	2	6	-0.23146	0
6	2	2	6	0.29355	-0.53699
6	1	1	7	0.17728	0
6	1	2	7	0.79283	0
6	2	2	7	0.25326	0.24613
6	2	2	8	-0.25326	-0.04343
6	2	3	8	0.58028	0.24879
6	3	3	8	0.22160	-0.08292
6	3	3	9	-0.44321	0.02073

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