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STUDY ON SYNCHRONIZATION STABILITY OF COMPLEX NETWORKS BASED ON IMPULSE CONTROL THEORY

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Abstract

A complex network is a complex set composed of several elementary units which have certain characteristics and functions and interact with each other. In recent years, the complex network has become one of the research hotspots of nonlinear science. Many scholars in the fields of control, mathematics, computer, biology and economy have devoted themselves to the research of complex networks. This study focuses on the stability of pinning impulse synchronization for the directed complex dynamic networks. From the Impulse Control Theory, a simple and general synchronization criterion for complex dynamic networks is obtained. Furthermore, the obtained results are applied to a small-world network consisting of a convolution neural network (CNN) and a Hodgkin-Huxley model neuron oscillator as power nodes. The numerical simulation shows the correctness of the obtained theoretical results and the validity of the control method.

Keywords: Pinning Control; Impulse Control; Complex Network Synchronization; CNN; Neuron Oscillator.

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1. Introduction

In recent years, the pinning control and synchronization of complex dynamic networks have attracted wide attention from various fields, such as science and engineering technology [1-2]. In particular, the pinning impulse synchronization of complex dynamic networks has become a hot topic. There are many important results for pinning impulse control of complex dynamic networks with different types and topologies [3-8].

Pinning impulse control technology is an easy-to-implement control technology which is superior to some successive pinning control schemes and only controls some nodes in the network at some discrete times. Generally speaking, it is very difficult to estimate the impulse gain of the coupled dynamic networks, that is, the synchronous convergence analysis of the controlled networks is a

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very challenging task. Recently, in the literature [4, 7], a new analysis method has been proposed on the stability of pinning impulse of complex (time-lag) dynamic networks. Some simple and general robust synchronization criteria for complex (time-lag) dynamic networks are obtained based on local linearization technology. In addition, it is proved that a given network can be synchronized to its homogeneous solution by a single impulse controller. In the literature [5], based on the observation of the state of all the nodes in the network, the overall synchronization convergence criterion of the complex dynamic network is obtained by using the impulse control method. Obviously, the control method described above may control all nodes of the entire network. Therefore, in essence, this control method is not a real pinning control technology. What's more, in most previous work, most of the networks are undirected, that is, the coupled matrix is symmetric. However, in practice, most of the networks, such as telephone communication networks and interpersonal networks are directed.

Based on the above comments, this study deals with the stability of pinning impulse synchronization of complex dynamic networks, and gives a simple and general synchronization convergence criterion for complex dynamic networks to achieve synchronization. Furthermore, the obtained results are applied to a small-world network composed of CNN and Hodgkin-Huxley neuron oscillators as power nodes. The numerical simulation verifies the correctness of the obtained theoretical results and the validity of the control method.

In the Introduction section, present clearly and briefly the problem investigated, with relevant references.

2. Complex Dynamic Network Model

We consider a general model of a complex dynamic network, which contains N identical nodes of linear diffusion coupled oscillators. Each node is an n-dimensional non-autonomous dynamic system. The state equation of the entire network is described by the following differential equation:

$$\dot{x}_i(t) = f\left(t, x_i(t)\right) + c\sum_{j=1}^{N} l_{ij}\Gamma x_j(t) \tag{1}$$

Where, $i = 1,2,\dots,N$, $x_i(t) = \left(x_{i1}(t),\dots,x_{n1}(t)\right)^T \in R^n$ is the state variable of the ith dynamic node, $f: R \times R^n \to R^n$ is a continuous vector equation of the parameters of the second variable. The constant $c > 0$ is the strength of coupling and its internal relation matrix $\Gamma > 0$ is positive definite and its coupling matrix $L = \left(l_{ij}\right)_{N\times N}$ represents the Laplacian graph of the whole network, where l_{ij} has the following definition:

$$l_{ij} = \begin{cases} a_{ij}, & i \neq j \\ -\sum_{k=1,k\neq i}^{N} a_{ik}, & j = i \end{cases} \tag{2}$$

If the network is connected, the Laplacian graph L is an asymmetric irreducible matrix. $L^T = SJS^{-1}$ Is Jordan decomposition of L, where $J = \mathrm{diag}\{J_1,\dots,J_l\}$ is a diagonal block matrix. If $\lambda_k, k = 1,\dots,l$, are different eigenvalues of matrix L and J_1 is a 1×1 matrix, then the first column of the matrix S is the right eigenvector $[1,\dots,1]^T$when the eigenvalue is 0.

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Our main work is to design a single impulse controller to deal with the pinning synchronization in complex dynamic networks so that all states of the controlled dynamic networks are uniformly synchronized to a special solution $s(t)$ of a homogeneous system, such as $\dot{s}(t) = f(t, s(t))$, then

$$\lim_{t\to+\infty} \|x_i(t) - s(t)\| = 0, i = 1, \dots, N, \tag{3}$$

Where $s(t)$ may be a balance point, a periodic orbit, or a chaotic attractor?
In general, the ith node is selected as the pinning point, and the node sequence of the network (1) is rearranged so that the first node is pinned or controlled.

Then, this pinning control network can be described by the following measure differential equation:

$$\begin{cases} \dot{x}_1(t) = f\big(t, x_1(t)\big) + c\sum_{j=1}^{N} l_{1j}\Gamma x_j(t) + U, \\ \quad \dot{x}_i(t) = f\big(t, x_i(t)\big) + c\sum_{j=1}^{N} l_{ij}\Gamma x_j(t), \\ \qquad\qquad\qquad\qquad i = 2, \dots, N, \end{cases} \tag{4}$$

Where, $U = \sum_{k=1}^{+\infty} b_k(x_1(t_k^-) - s(t))\delta(t - t_k)$ is an n-dimensional impulse controller whose control increment $b_k (k \in Z^+)$ is a constant, where $\delta(t)$ is a Dirac function.

From the nature of the Dirac function, equation (4) can be expressed in the following form:

$$\begin{cases} \dot{x}_i(t) = f\big(t, x_i(t)\big) + c\sum_{j=1}^{N} l_{ij}\Gamma x_j(t), \\ \qquad\qquad i = 1,2, \dots, N, t \neq t_k \\ \Delta x_1(t) = b_k\big(x_1(t^-) - s(t)\big), \qquad t = t_k, \\ \Delta x_i(t) = 0, \qquad\qquad i = 2, \dots, N, t = t_k, \end{cases} \tag{5}$$

Where the time series $\{t_k\}_{k=1}^{+\infty}$ satisfies$t_{k-1} < t_k$ and $\lim_{k\to+\infty} t_k = +\infty$, $\Delta x_i = x_i(t_k^+) - x_i(t_k^-)$ is control law that satisfies $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$ and$x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$. Generally, we assume that $x_i(t_k^+) = x_i(t_k)$ represents that the solution $x_i(t)$ is right continuous. It is assumed that equation (4) has a unique solution to the initial value.

Then the pinning impulse synchronization in a complex dynamic network (1) is translated into the stability analysis of the synchronous manifold of the controlled dynamic network (4). And we can get the following theorem:
Theorem 1:
If $k \in Z^+$ satisfies the following conditions:

a) $\sigma + 2c\alpha_2\lambda_{min}(\Gamma) + 2c\lambda_{max}(\Gamma) < 0$,
b) $\sigma(t_{k+1} - t_k) + \ln d_k < 0$,

Where, $\sigma \geq \lambda_{max}\Big(D^T f(t, s) + Df(t, s)\Big), d_k = (1 + b_k)^2 < 1$.

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Then the controlled network (4) may be uniformly exponentially synchronized to s (t) by a single point impulse controller, where s (t) may be a balance point, a periodic orbit, or a chaotic attractor.

3. Numerical Simulation of Pinning Network

In order to verify the validity of the method, we consider a small-world network consisting of CNN with 50 nodes and Hodgkin-Huxley neuron oscillator as power nodes. For this small-world neural network, we set the parameters$N = 50, K = 3, p = 0.3$, and then the Laplacian graph $L = L_{sw}$ can be generated randomly by the small-world model in the directed graph. The pinning points of the single point impulse controller are loaded for random selection.

3.1. CNN Neuron Oscillators

CNN neuron oscillator can be described by the following three-dimensional nonlinear differential equation:

$$\dot{x}(t) = -Cx(t) + Af\big(x(t)\big), \tag{6}$$

Where, $x(t) = \big(x_1(t), x_2(t), x_3(t)\big)^T$ is state vector of network, $f(x) = \big(f(x_1), f(x_2), f(x_3)\big)^T$satisfies$f_i(x) = f(x) = \frac{1}{2}(|x+1| - |x-1|)$,where

$A = \begin{pmatrix} 1.2500 & -3.200 & -3.200 \\ -3.200 & 1.100 & -4.400 \\ -3.200 & 4.400 & 1.000 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Obviously, the network is a typical CNN neuron oscillator.

If the controlled synchronization state $s(t) = \big(s_1(t), s_2(t), s_3(t)\big)^T$ is the special solution of CNN neuron oscillator, the three different pinning situations are as follows:

- Pinning the network to a balance point

The model (5) has three unstable balance points:
$\overline{x_+} = (1.1971, 0.7273, -0.7107) \in \Sigma_+$,
$\Sigma_+ = \{(x_1, x_2, x_3) | |x_1| > 1, |x_2| \le 1, |x_3| \le 1\}$,
$\overline{x_0} = (0,0,0) \in \Sigma_0$,
$\Sigma_+ = \{(x_1, x_2, x_3) | |x_1| \le 1, |x_2| \le 1, |x_3| \le 1\}$,
$\overline{x_-} = (-1.1971, -0.7273, 0.7107) \in \Sigma_-$,
$\Sigma_+ = \{(x_1, x_2, x_3) | x_1 \le 1, |x_2| \le 1, |x_3| \le 1\}$,
Generally, the numerical simulation is to pin the network to one of the balance points $s = (1.1971, 0.7273, -0.7107)^T$, then σ = 3.6974, c = 5.0, the internal connection matrix $\Gamma = I_3$, constant step h = 0.02. For simplicity, we consider equal impulse intervals $t_k - t_{k-1} = \Delta t$ and impulse control increment for any $k \in Z^+, b_k \equiv b \in (-2,0)$. Then it can be easily obtained if the following condition is satisfied:

$$3.6974\Delta t + 2\ln|1 + b| < 0 \tag{7}$$

Then all the conditions of Theorem 1 are satisfied, which means that the small world neural network (6) can be uniformly exponentially synchronized to the balance point s. In this simulation, the impulse control increment $b = -0.5$ is made to satisfy the in equation (7) with an equal impulse interval $\Delta t = 0.06$. Figure 1 intuitively describes the evolution process of the state variable of the controlled small-world network (6).

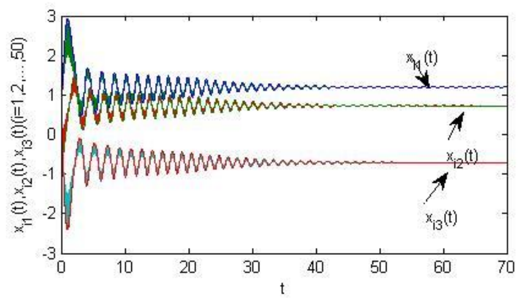

Figure 1: Pinning the CNN neuron oscillator to a balance point

- Pinning the network to a chaotic attractor

The CNN neuron oscillator has a chaotic attractor of the initial value $(x_1(0), x_2(0), x_3(0))^T = (0.1,0.1,0.1)^T$, as shown in Figure 2. In this simulation, all parameters are as follows: $\sigma \leq 9.3537, c = 11, \Delta t = 0.01, h = 0.001$, so that it satisfies the condition of Theorem 1. Accordingly, Figure 3 intuitively describes the evolution process of the state variable of the controlled small-world network (6).

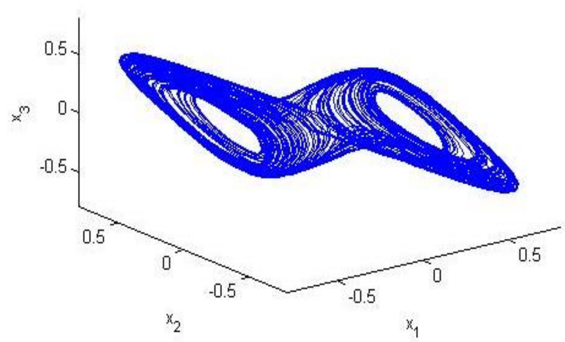

Figure 2: Chaotic attractor of CNN neuron oscillator

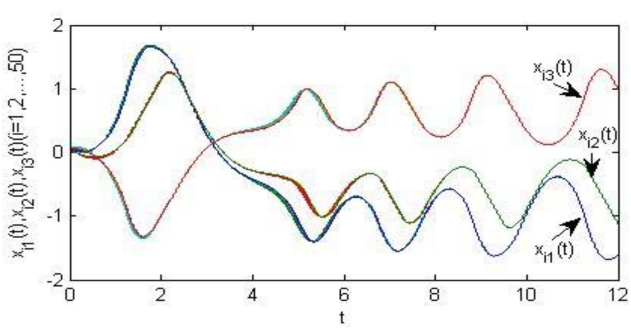

Figure 3: Pinning the CNN neuron oscillator to a chaotic orbit

- Pinning the network to a periodic orbit

A three-dimensional CNN neuron oscillator in the following form is considered:

$$\dot{x}(t) = -Cx(t) + Af\big(x(t)\big) + \sin x(t), \tag{8}$$

The structure of the equation (8) indicates that the CNN neuron oscillator has a periodic behavior, as shown in Figure 4, where the initial value is:

$$\big(x_1(0), x_2(0), x_3(0)\big)^T = (0.1,0.1,0.1)^T$$

In this simulation, all parameters are consistent with the test parameters in Figure 1, $\sigma \leq 3.9449$ Figure 5 is a phase diagram based on the results of this numerical simulation, which intuitively demonstrates that the network is pinned to the periodic orbit $s(t)$ by the single point impulse controller. Accordingly, Figure 6 intuitively describes the evolution process of the state variable of the controlled small-world network (6).

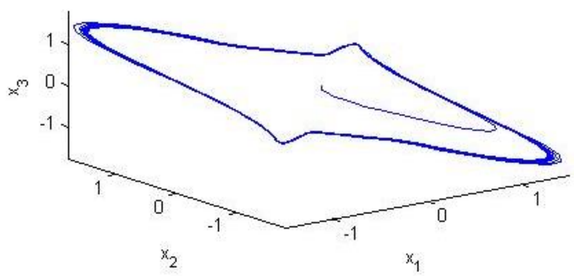

Figure 4: Periodic behavior of CNN neuron oscillator

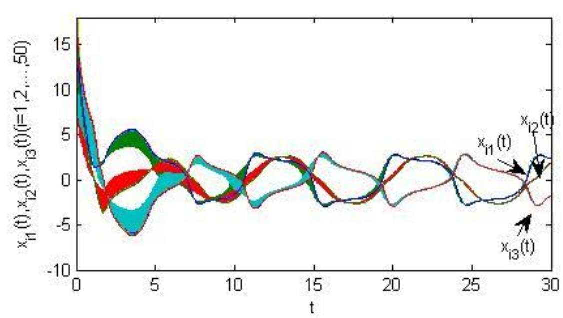

Figure 5: Phase diagram of controlled CNN neuron oscillator

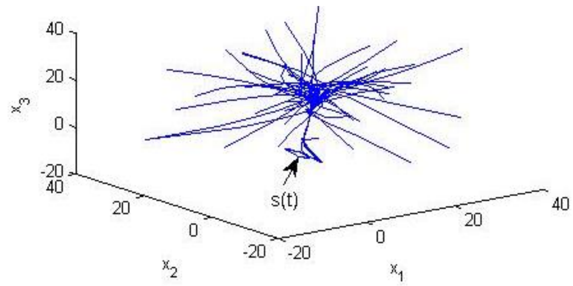

Figure 6: Pinning the CNN neuron oscillator to a periodic orbit

3.2. Hodgkin-Huxley Neuron Oscillator

The Hodgkin-Huxley neuron oscillator [9] is described by the following differential equation:

$$\begin{cases} C\frac{dV}{dt} = I_0 - gN_a m^3 h\left(V - V_{N_a}\right) \\ \quad -g_k n^4 (V - V_K) - g_L (V - V_L) \\ \frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m \\ \frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h \\ \frac{dn}{dt} = \alpha_n(V)(1-h) - \beta_n(V)n \end{cases} \tag{9}$$

Where, $g_{N_a} = 120\frac{mS}{cm^2}, g_K = 36\frac{mS}{cm^2}$,
$g_L = 0.3\frac{mS}{cm^2}, V_{N_a} = 50\,mV, V_K = -11\,mV$,
$V_L = -54.4\,mV, C = 1\frac{\mu F}{cm^2}$, and

$$\begin{cases} \alpha_m(V) = \frac{0.1(V+40)}{1-\exp[-(V+40)/10]} \\ \beta_m(V) = 4\exp[-(V+65)/18] \\ \alpha_h(V) = 0.07\exp[-(V+65)/20] \\ \beta_h(V) = \left\{1 + \exp\left[-\frac{V+35}{10}\right]\right\}^{-1} \\ \alpha_n(V) = \frac{0.01(V+55)}{1-\exp[-(V+55)/10]} \\ \beta_n(V) = 0.125\exp[-(V+65)/80]. \end{cases}$$

When $I_0 = 7\ \mu A/cm^2$, the initial value $\left(V(0), m(0), h(0), n(0)\right)^T = (-25, 0.38, 0.57, 0.08)^T$ has a stable limit cycle, as shown in Figure 7 and Figure 8.

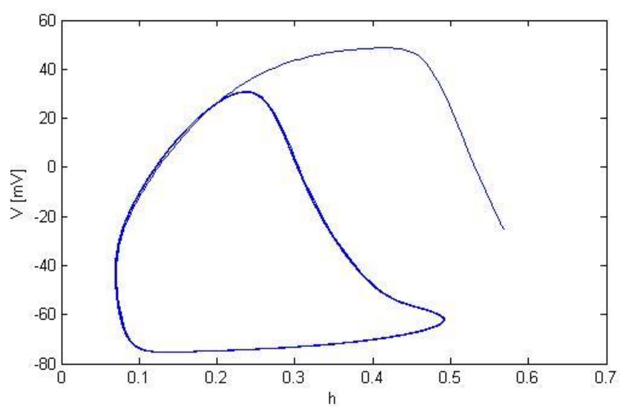

Figure 7: Stability limit cycle of Hodgkin-Huxley neuron oscillator (phase diagram)

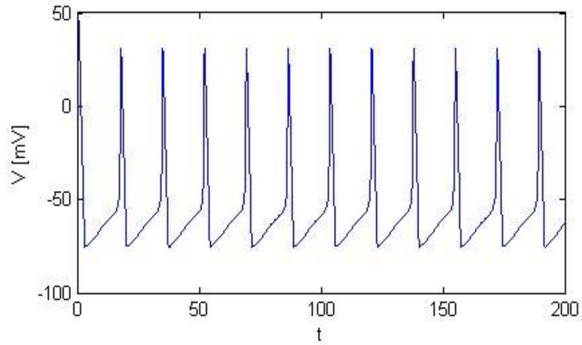

Figure 8: Stability limit cycle of Hodgkin-Huxley neuron oscillator (time sequence diagram)

In this simulation, all parameters are as follows: $\sigma \leq 2.5153, c = 5, \Delta t = 0.02, h = 0.06$ so that it satisfies the condition of Theorem 1. Accordingly, Figure 9 is a phase diagram based on the numerical simulation results, which intuitively demonstrates that the network is pinned to the periodic orbit $s(t)$ by the single point impulse controller. Accordingly, Figure 10 intuitively describes the evolution process of the state variable of the controlled small-world network (5).

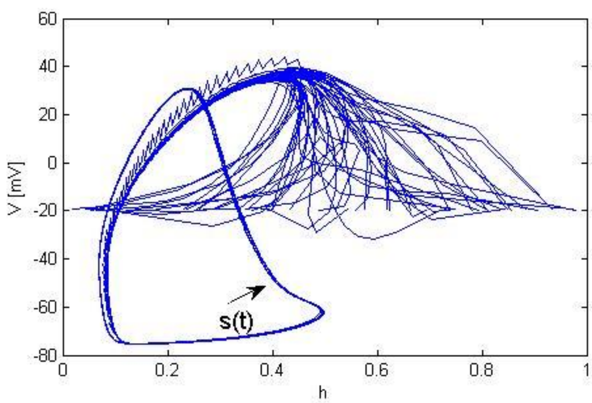

Figure 9: Phase diagram of controlled Hodgkin-Huxley neuron oscillator

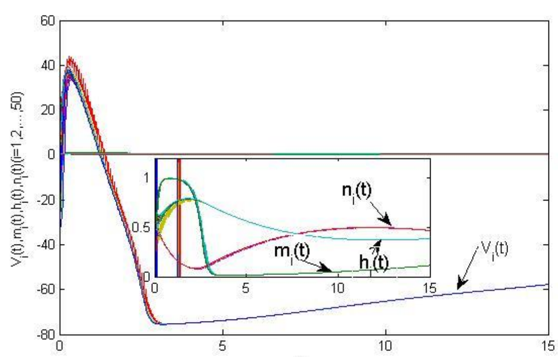

Figure 10: Pinning the Hodgkin-Huxley neuron oscillator to a periodic orbit

The results section should provide details of all of the experiments that are required to support the conclusions of the paper. The section may be divided into subsections, each with a concise subheading.

It is advised that this section be written in past tense. It is a good idea to rely on charts, graphs, and tables to present the information. This way, the author is not tempted to discuss any conclusions derived from the study. The charts, graphs, and table should be clearly labeled and should include captions that outline the results without drawing any conclusions. A description of statistical tests as it relates to the results should be included.

4. Conclusions

This study mainly discusses the stability of pinning impulse synchronization of directed complex dynamic networks. Based on the Impulse Control Theory, simple and general synchronization criteria for complex dynamic networks are obtained. Furthermore, the obtained results are applied to a small-world network composed of CNN and Hodgkin-Huxley neuron oscillators as power nodes. The numerical simulation shows the correctness of the obtained theoretical results and the validity of the control method.

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