INTERNATIONAL JOURNAL OF RESEARCH –
GRANTHAALAYAH
A knowledge Repository

Science

AN ESTIMATION METHOD OF THE NATURAL FREQUENCY OF A CUBE FORM MICRO SATELLITE

Kei-ichi OKUYAMA[*1], Shigeru HIBINO [1], Misuzu MATSUOKA [1], Aleksander LIDTKE [1]
[1] Applied Sciences and Integrated System Engineering, Kyushu Institute of Technology,1-1 Sensui-cho, Tobata Ward, Kitakyushu, Fukuoka Prefecture 804-0015, Japan

Abstract

Micro satellites must survive severe mechanical conditions during their launch phase. One design requirement for rockets is the stiffness requirement, i.e. the natural frequencies requirement. In the early stages of satellite development, presumption of the natural frequency of a satellite may be difficult.

The material used for the structure of many micro satellites is an aluminum alloy. The structure subsystem occupies a large portion of the satellite mass, and the elastic modulus of this aluminum alloy is larger than that of other subsystems. Therefore, the mechanical property of the aluminum alloy cannot be used to represent the mechanical property of the whole satellite. The density of an actual satellite differs from the density of the aluminum alloy. Therefore, when estimating the minimum natural frequency, the size and the elastic modules of an actual satellite structure must be used. When using an actual satellite structure, the estimated minimum natural frequencies of the lateral direction and the longitudinal direction during the ascent phase are in agreement with the measured values acquired by the vibration tests.

In order to shorten a process of satellite development, this paper describes a practical method for estimating the natural frequency of a micro satellite.

Keywords: Micro Satellite; Natural Frequency; Stiffness Requirement; Structure Designing; Structure Development.

Cite This Article: Kei-ichi OKUYAMA, Shigeru HIBINO, Misuzu MATSUOKA, and Aleksander LIDTKE. (2018). "AN ESTIMATION METHOD OF THE NATURAL FREQUENCY OF A CUBE FORM MICRO SATELLITE." *International Journal of Research - Granthaalayah*, 6(3), 88-97. https://doi.org/10.29121/granthaalayah.v6.i3.2018.1502.

1. Introduction

When a spacecraft such as a probe, which explores a planet or the sun, or a satellite which goes the around the earth, is launched with a rocket, they are exposed to severe mechanical conditions. Launch environments consist of a series of events, each of which has several independent sources of load for a probe and a satellite. Some loads are relatively steady-state; or constant over time, such as thrust while a rocket motor burns. Some are transient, such as thrust when a

rocket motor ignites or shuts down. Acoustic loads are sound pressure waves. Because most acoustics include waves with many different frequencies, they cause structures to vibrate randomly. Pyrotechnic shock, or pyro shock for short, is high-intensity, high-frequency vibration caused by the explosives commonly used to separate stages.

One of the types of spacecraft is a small satellite, and they are differentiated by mass. Small satellites which are 100 kg or less, 10 kg or less, and 1 kg or less are called micro satellites, nanosatellites, and picosatellites, respectively. In this paper, all of these three satellites are treated as micro satellites.

Since these micro satellites must survive these severe mechanical conditions, the design requirements of the rocket are the strength requirement and the stiffness requirement. Significant stresses are generated on a micro satellite structure according to the static loads and the dynamic loads during the ascent phase. These stresses must not deviate from the allowable yield strength or the ultimate strength of the structure materials. The margin of safety (MS) is defined as the ratio between the allowable yield strength, or the ultimate strength, and the actual stresses multiplied by a safety factor minus one. This means that the value of the MS must be greater than or equal to zero. Furthermore, we have to satisfy a rocket requirement of the natural frequencies for a satellite. When using an H-IIA rocket which is a Japanese key rocket, the natural frequencies in the lateral direction and in the longitudinal direction must be larger than 100Hz and 50Hz respectively [1].

Usually, micro satellite design is performed through a procedure which designs and manufactures a bread board model, an engineering model. These models used for evaluation have satisfied the strength requirement and the stiffness requirement by using a vibration test. The design of these models is performed by CAD and the internal stress analysis and the natural frequency analysis of a micro satellite structure during the ascent phase are conducted in a finite-element-method analysis. In an early stage of development, which has a finite-element-method model of a satellite in the process of creation, presumption of the natural frequency of a satellite may be difficult. In order to shorten the process of satellite development, this paper describes a practical estimation method of the natural frequency of a micro satellite.
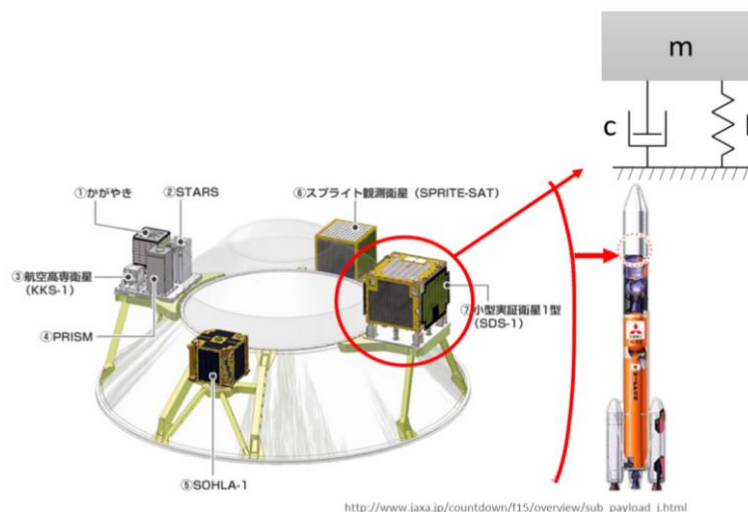

Figure 1: micro satellites in an H-IIA rocket and a Single-Degree-Of-Freedom system

2. Prediction of Minimum Natural Frequency

A satellite by which the fitting was carried out in a rocket is expressed in Fig. 1 [2]. This is called a single-degree-of-freedom system, i.e. a SDOF system. This SDOF system is loaded by the force $F_0 \sin \omega t$ at the mass m, were ω is the angular frequency and t is time.

The mass is suspended by a linear spring with a spring stiffness k and a damper with a damping constant c.

The equation of motion of the SDOF system is:

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t \tag{1}$$

Was $x(t), \dot{x}(t), \ddot{x}(t)$ which represent displacement, velocity and acceleration.

The general solution of Eq. 1 is defined by following equation:

$$x = e^{-\zeta \omega_n t}(C_1 \sin \omega_d t + C_2 \cos \omega_d t)$$
$$+ \frac{(F_0/k)\omega_{n^2}}{\left(\omega_{n^2} - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2} [(\omega_{n^2} - \omega^2)\sin \omega t - 2\zeta\omega_n\omega \cos \omega t]. \tag{2}$$

C_1 and C_2 of Eq.2 are constants decided based on an initial condition. For example, when the initial displacement is $x(t) = 0$, the initial velocity $\dot{x}(t) = 0$, the constants C_1 and C_2 are:

$$C_1 = \frac{(F_0/k)\omega_{n^2}}{\left(\omega_{n^2} - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2} \frac{\omega}{\omega_d}(\omega^2 - \omega_{d^2} + \zeta^2\omega_{n^2}) \ , \tag{3}$$

$$C_2 = \frac{(F_0/k)\omega_{n^2}}{\left(\omega_{n^2} - \omega^2\right)^2 + (2\zeta\omega_n\omega)^2} \frac{\omega}{\omega_d} 2\zeta\omega_n\omega_d \ . \tag{4}$$

The natural angular frequency of a non-damping system, the natural angular frequency of a damping system and the damping ratio are ω_n, ω_d and ζ respectively. Thus ω_n, ω_d and ζ are expressed as:

$$\omega_n = \sqrt{k/m}, \ \omega_d = \sqrt{1 - \zeta^2}\omega_n, \ \zeta = {}^c/_{2\sqrt{mk}} \ . \tag{5}$$

Since the natural angular frequency ω can be indicated as $\omega=2\pi f$ using the natural frequency f, f_n and f_d are expressed as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \ \ f_d = \frac{1}{2\pi}\sqrt{1 - \zeta^2}f_n \ . \tag{6}$$

The form of the micro satellites which are used in this paper take the form of a cube; which is expressed as a rod, as shown in Fig. 2. This is *a cantilever beam*, the upper end of the rod is free to deflect but the lower end is rigidly clamped.
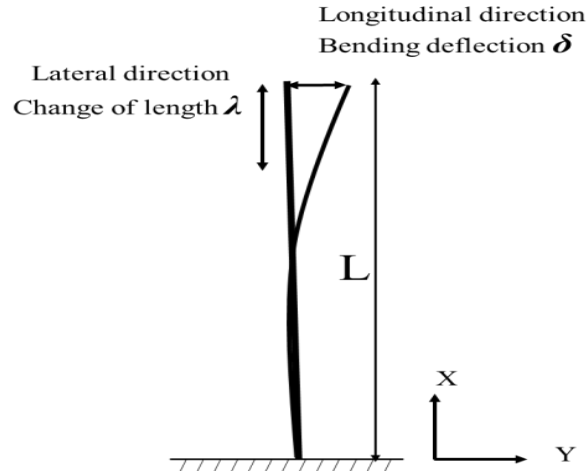
Figure 2: rod from which length changes in lateral direction (*x-direction*) and longitudinal direction(*y-direction*)

In the development of micro satellites, the value of ζ being used is 0.05 or below its value [3-6]. Therefore, the damping natural frequency f_d is almost the same as the non-damping natural frequency f_n. There are some approximate solutions of a non-damping vibration of a one-dimensional distribution mass system. Rayleigh's method is one which can estimate for the minimum natural frequency.

A force P applies to this rod in *x-direction*. The rod has a cross-section A and a length L. The elongation of the rod, due to the applied force P, is denoted with λ. The stiffness of the rod k is defined with:

$$k = {P}/{\lambda} \ . \tag{7}$$

The total strain ε is:

$$\varepsilon = {\lambda}/{L} \ . \tag{8}$$

The strain ε is the ratio between the occurring stress σ and the elastic modulus E by Hooke's Law as:

$$\varepsilon = {\sigma}/{E} = {P}/{AE} \ . \tag{9}$$

Thus the elongation λ of the rod now becomes,

$$\lambda = \varepsilon L = {PL}/{AE} \ , \tag{10}$$

And therefore the stiffness k is shown as:

$$k = {AE}/{L} \ . \tag{11}$$

The minimum natural frequency of *x-direction*, which is the lateral direction of the launching phase $f_{n,lateral}$ can be estimated with

$$f_{n,lateral} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{AE}{mL}} = \frac{1}{2\pi}\sqrt{\frac{E}{\rho A}} \ . \tag{12}$$

When a load acts on this rod with the unit volume weight *w* in *y-direction*; displacement *y(x,t)* is expressed by:

$$y(x,t) = X(t) \cdot coswt \ . \tag{13}$$

Where *X(t)* is a suitable displacement function with which it satisfies a boundary condition. Kinetic energy *K* is expressed by Eq. 14.

$$K = \frac{1}{2}\int_0^L wAy^2 dx = \frac{w^2}{2}\int_0^L wAX^2(x)dx \cdot sin^2\omega t = K_{max} \cdot sin^2\omega t. \tag{14}$$

Furthermore, the strain energy *E* which arose according to a bending load is expressed by Eq. 15.

$$E = \frac{1}{2}\int_0^L \frac{M^2}{EI} dx = \frac{1}{2}\int_0^L EI\left(\frac{d^2X}{dx^2}\right)^2 dx \cdot cos^2\omega t = E_{max} \cdot cos^2\omega t, \tag{15}$$

Where the moment of inertia of area is *I*.

Supposing K_{max} and E_{max} are equal here,

$$\frac{w^2}{2}\int_0^L wAX^2(x)dx = \frac{1}{2}\int_0^L EI\left(\frac{d^2X}{dx^2}\right)^2 dx. \tag{16}$$

Therefore, the minimum natural angular frequency is indicated by:

$$\omega^2 = \frac{1}{2}\int_0^L EI\left(\frac{d^2X}{dx^2}\right)^2 dx \ / \ \frac{w^2}{2}\int_0^L wAX^2(x)dx \ . \tag{17}$$

If *X(x)* is set to $= \left(1 - cos\frac{\pi}{2L}x\right)$,

$$\int_0^L \left(\frac{d^2X}{dx^2}\right)^2 dx = \int_0^L \left[\delta\left(\frac{\pi}{2L}\right)^2 cos\frac{\pi}{2L}x\right]^2 dx = \frac{\pi^4}{32}\frac{\delta^2}{L^3}, \tag{18}$$

$$\int_0^L X^2(x) dx = \delta^2 \int_0^L \left[1 - cos\frac{\pi}{2L}x\right]^2 dx = \delta^2 L\left(\frac{2}{3} - \frac{4}{\pi}\right) \ , \tag{19}$$

Where the bending deflection is *δ*.

Eq. 19 and Eq. 20 are substituted for Eq. 17,

$$\omega^2 = \frac{\pi^5}{16(3\pi-8)L^4}\frac{EI}{wA} \ , \tag{20}$$

$$\omega = \frac{1}{L^2} \sqrt{\frac{\pi^5}{16(3\pi-8)} \frac{EI}{wA}} \; . \tag{21}$$

Therefore, the minimum natural frequency is expressed by:

$$f_n = \frac{1}{2\pi}\omega = \frac{1}{2\pi L^2}\sqrt{\frac{\pi^5}{16(3\pi-8)}\frac{EI}{wA}} \; . \tag{22}$$

The structure material used for many micro satellites is an aluminum alloy. The structure subsystem occupies a substantial portion of the entire satellite mass; the elastic modulus of this aluminum alloy is larger than that of other subsystems. Therefore, the mechanical property of the main structure material cannot be used to represent the mechanical property of the whole satellite. The density ρ_{main} of a material which mainly constitutes the satellite differs from the actual density of the satellite $\rho_{satellite}$. A length of one side of a cube-shaped satellite, namely specific length L_s, is expressed by:

$$L_s = \left[\left(\rho_{satellite}/\rho_{main}\right)V_{satellite}\right]^{1/3}. \tag{23}$$

From the elastic modulus of other subsystems being very small as compared with the elastic modulus of structure material E_{main}, the elastic modulus of an actual satellite E_s is indicated by Eq. 24,

$$E_s = E_{main}\left[\left(\rho_{satellite}/\rho_{main}\right)\right]^3. \tag{24}$$

Therefore, Eq. 12 and Eq. 15 can be rewritten as:

$$f_{n,lateral} = \frac{1}{2\pi}\sqrt{\frac{E_s}{\rho_{satellite}L_s^2}} \; , \tag{25}$$

Because the satellite form that is dealt with in this paper is a cube which has a length of one side is L, the moment of inertia of area I can be calculated by:

$$I = \frac{L^4}{12}. \tag{26}$$

Thus the minimum natural frequency of the *y-direction*, which is the longitudinal direction $f_{n,longitudinal}$ can be estimated using Eq. 22:

$$f_{n,longitudinal} = \frac{1}{2\pi L^2}\sqrt{\frac{\pi^5}{16(3\pi-8)}\frac{EI}{wA}} = \frac{1}{2\pi}\sqrt{\frac{\pi^5}{16(3\pi-8)}\frac{E}{\rho A}} = \frac{1}{2\pi}\sqrt{\frac{\pi^5}{16(3\pi-8)}\frac{E_s}{\rho_{satellite}L_s^2}}. \tag{27}$$

Where the unit volume weight w = ρ.
Using Eq. 25 and Eq. 27, the minimum natural frequency of the lateral direction is expressed as:

$$f_{n,lateral} = \sqrt{\frac{\pi^5}{16(3\pi-8)}}f_{n,lateral}. \tag{28}$$

3. Vibration Test

3.1. Test Facility

In order to acquire the minimum natural frequencies of micro satellites, vibration tests were carried out. These vibration tests were mainly done in the center for nanosatellite testing (CeNT) [7] at the Kyushu Institute of Technology using a vibration testing facility which generates vibration according to the parameters such as arbitrary force, acceleration and frequency. The completeness of a satellite structure can be evaluated by arbitrary oscillating loads. The vibration testing equipment of the CeNT is the 35kN type of EMIC Corporation [8]. For this FH-35K/60 model, the maximum exciting force in a sine wave is 35,000N, the frequency range is up to 2,200Hz, the maximum acceleration 1,000m/s^2 and the maximum payload mass is 400kg. The vibration test facility is illustrated in Fig. 3.
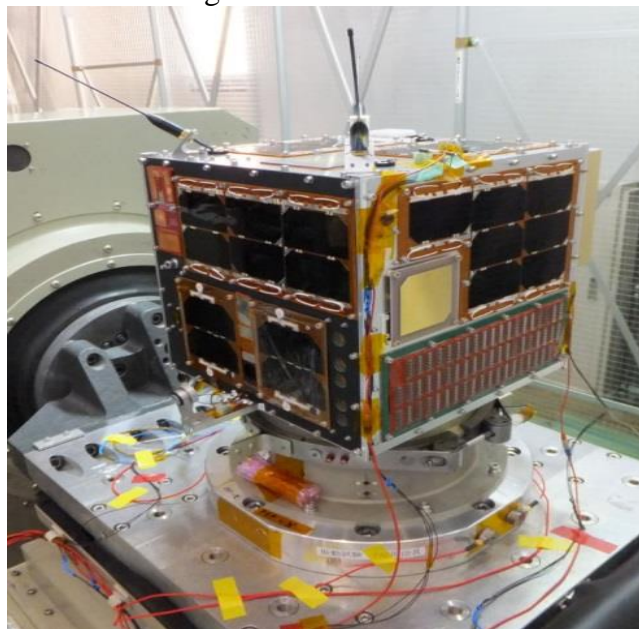

Figure 3: vibration testing facility at the center for nanosatellite testing (CeNT) at the Kyushu Institute of Technology

3.2. Test Conditions and Specimens

The vibration tests were carried out based on the standard ISO 17025. This ISO-17025 is a standard certified by an authoritative third-party accreditation body to determine whether laboratories / calibration laboratories have the ability to generate accurate measurement / calibration results. Testing laboratories that conduct product inspection, analysis, measurement, etc., require certification. Requirements for calibration organizations conducting the calibration work of measuring instruments are specified in this standard. Certified organizations must have management ability in product management / quality control and technology for generating reliable tests / calibration results. The authority of this standard is recognized internationally. There are two main fields of testing and calibration, and the CeNT is concerned with the test item "Vibration test based on ISO-19683 and JAXA JERG - 2 - 130 - HB 003 of artificial satellite and satellite components." The CeNT had already received certification.

Each micro satellite was vibrated in the *x-direction* and the *y-direction*, and the minimum natural frequencies for each were measured.

The specifications of the micro satellites used for this testing are shown in Table 1. All the forms of the micro satellites with which the testing were provided are all cubes; the masses range from approximately 1 kg to approximately 60 kg. Satellite structures are primarily constructed from an aluminum alloy.

Table 1: Specifications of the micro satellites used for this testing

Satellite structure			Satellite structure specifications							
Main material	Density	Modulus of elasticity	Size			Volume	Mass	Density	Density ratio	Specific length
	ρ_{main}	E	W	D	H	$V_{satellite}$	m	$\rho_{satellite}$	$\rho_{satellite}/\rho_{main}$	$((\rho_{satellite}/\rho_{main})V_{satellite})^{1/3}$
	[kg/m^3]	[GPa]	[mm]	[mm]	[mm]	[m^3]	[kg]	[kg/m^3]	[-]	[m]
Aluminium alloy	2770	72.0	0.10	0.10	0.11	1.14E-03	0.723	637	0.230	0.064
	2770	72.0	0.10	0.10	0.11	1.14E-03	1.13	996	0.359	0.074
	2770	72.0	0.39	0.39	0.42	6.21E-02	21.7	349	0.126	0.199
	2770	72.0	0.50	0.50	0.55	1.38E-01	57.6	419	0.151	0.275
	2770	72.0	0.50	0.50	0.70	1.75E-01	64.8	370	0.134	0.286
	2770	72.0	0.50	0.50	0.50	1.25E-01	45.7	366	0.132	0.255
	2770	72.0	0.50	0.50	0.49	1.24E-01	43.9	355	0.128	0.251

4. Results and Discussions

The vibration test results are shown in Table 2. The masses of micro satellites which were used for this vibration test ranged from approximately 720g up to 65 kg. Those forms were cubes mostly and the lengths of one side ranged from approximately 10 cm to 50 cm. The main material of these satellite structures is an aluminum alloy, the ratio of the satellites densities to this aluminum alloy density were from approximately 0.36 to 0.13.

Table 2: Comparison of minimum natural frequencies between vibration test results and simple estimated results

Satellite structure			Satellite structure specifications				Vibration test results		Estimation results	
Main material	Density	Modulus of elasticity	Size			Mass	Minimum natural frequency		Minimum natural frequency	
	ρ_{main}	E	W	D	H	m	lateral direction	longitudinal direction	lateral direction	longitudinal direction
	[kg/m^3]	[GPa]	[mm]	[mm]	[mm]	[kg]	[Hz]	[Hz]	[Hz]	[Hz]
Aluminium alloy	2770	72.0	0.10	0.10	0.11	0.723	1390		1790	488
	2770	72.0	0.10	0.10	0.11	1.13	860	560	2800	763
	2770	72.0	0.39	0.39	0.42	21.7	324	94	258	71
	2770	72.0	0.50	0.50	0.55	57.6	186	47	238	65
	2770	72.0	0.50	0.50	0.70	64.8	144	43	194	53
	2770	72.0	0.50	0.50	0.50	45.7	209	55	214	58
	2770	72.0	0.50	0.50	0.49	43.9	207	63	209	57

The measurement results are shown in Fig. 4 and Fig. 5. Although the minimum natural frequencies of the satellites where the density ratio is small, i.e., the satellites whose mass are approximately 50-kg class, is around 200 Hz. The minimum natural frequencies of the 1-kg class satellites are around 1,000 Hz. This can be understood from Eq. 6.

The calculation results of the minimum natural frequencies using Eq. 27 and Eq. 28 were also shown in Fig. 4 and Fig. 5. The measured values and the calculated values of the domain where the density ratio is small are in agreement mutually from Fig. 4 and Fig. 5. However, in the domain where the density ratio is large, both are not strictly in agreement. Since a small satellite size with a mass of around 1 kg is very small, amounts of the bending deflections which arise by vibration primarily are very small.

In this research, vibration tests for several micro satellites were carried out. A simple estimation method of the minimum natural frequency was established using these results, which are shown below:

1) In this research, a small satellite carried in a rocket presupposed that it can be expressed as a SDOF system. Past experience shows that the dumping factor ζ is smaller than 0.05. Therefore, the natural frequency of the non-damping vibration is almost equal to the natural frequency of the damping vibration.
2) The minimum natural frequency of a small satellite of the longitudinal direction, and the lateral direction during the ascent phase can be shown by Eq. 6 and Eq.12.
3) The structure material used for many micro satellites is an aluminum alloy. The structure subsystem occupies a large portion of the whole satellite mass. The elastic modulus of this aluminum alloy is larger than that of other subsystems. Therefore, the mechanical property of the main structure material cannot be used as a mechanical property of the whole satellite.
   The density of an actual satellite differs from the density of the main structure material. Therefore, when estimating minimum natural frequency, the size and the elastic modules of an actual satellite structure must be used. These are expressed by in Eq.24 and Eq.25.
4) Using the size and the elastic modules of an actual satellite structure, the minimum natural frequency of the longitudinal direction and the lateral direction during the ascent phase can be shown by Eq. 27 and Eq.28.
5) The measured values and the calculated values of the domain where the density ratio is small are in agreement mutually. However, in the domain where the density ratio is large, both are not strictly in agreement. Since a satellite size with a mass of around 1 kg is very small, the amount of bending deflections which arise primarily through vibrations are very small.
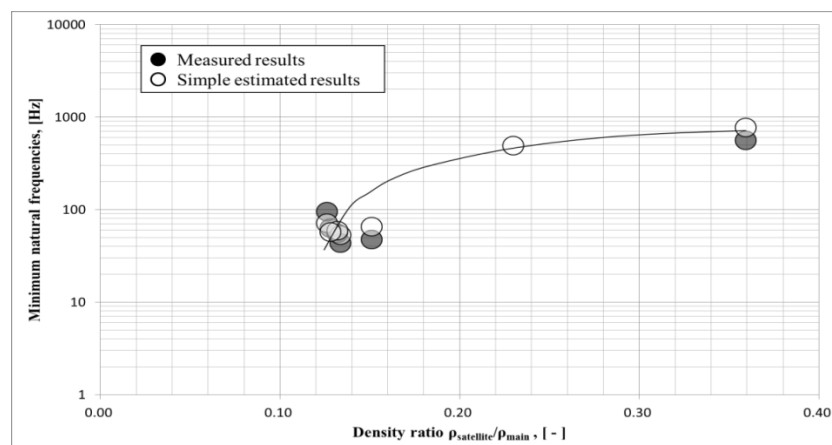

Figure 4: comparison of minimum natural frequencies between vibration test results and simple estimated results in lateral direction
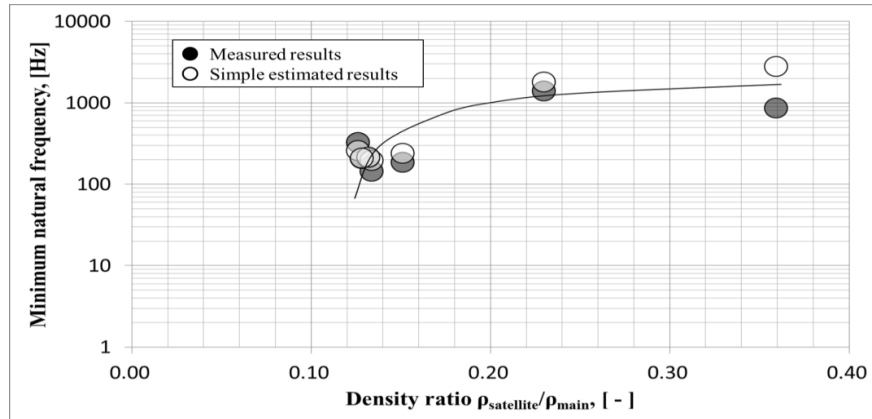
Figure 5: comparison of minimum natural frequencies between vibration test results and simple estimated results in longitudinal direction

References

[1] Japan Aerospace Exploration Agency, User's manual of H-IIA rocket (in Japanese), JERG-4-011A, 2015.

[2] Mitsubishi Heavy Industries, Ltd., User's manual of H-IIA rocket, YET04001, Vol.4, 2015, Japan.

[3] Japan Aerospace Exploration Agency, Structural design standard (in Japanese), JERG-2-320A, 2011.

[4] M. J. Casiano, Extracting Damping Ratio from Dynamic Data and Numerical Solutions, NASA TM-2016-218227, 2016.

[5] L. E. Kinsler, A. R. Frey, A. B. Coppens and J. V. Sanders, Fundamentals of Acoustics, John Wiley & Sons, 4th edition, 2000.

[6] J. JaapWijker, Spacecraft Structures, Springer, 2008 edition, 2008.

[7] Mengu CHO, Hirokazu MASUI, Yasuhiro AKAHOSHI, Koju HIRAKI, Minoru IWATA and Shinji HATTA, Initial Operations of Center for Nanosatellite Testing, 2nd Nano- Satellite Symposium, 14th-16th March, 2011, the University of Tokyo, Japan.

[8] EMIC Corporation, User's manual of vibration testing system (Model FH-35K/60), 2018.

*Corresponding author.
E-mail address: okuyama@ ise.kyutech.ac.jp