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(k, d)-MEAN LABELING OF SOME DISCONNECTED GRAPHS

B. Gayathri[1], V. Sulochana[2]
[1]PG and Research Department of Mathematics, Periyar E.V.R. College, Tiruchirappalli, India
[2]Department of Mathematics, Seethalakshmi Ramaswami College, Tiruchirappalli, India

Abstract

The concept of mean labeling was introduced by Somasundaram and Ponraj. K-odd mean, (k,d)-odd mean labeling were introduced and discussed by Gayathri and Amuthavalli. K-mean, k-even mean and (k,d)-even mean labeling were further studied by Gayathri and Gopi. We have obtained (k,1)-mean labeling for some new families of graphs. We have introduced (k,d)-mean labeling and obtained results for some family of trees and for some special graphs. In this paper, we investigate (k,d)-mean labeling for some disconnected graphs. Here k and d denote any positive integer greater than or equal to 1.

Keywords: **(**k,d)-Mean Labeling; (k,d)-Mean Graph.

AMS Subject Classification: 05C78

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1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [8]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G.

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling).

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [9]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, Crystallography, radar, Coding theory, astronomy, circuit design and Communication network addressing. Particular interesting applications of graph labeling can be found in [2].

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The concept of mean labeling was introduced by Somasundaram and Ponraj [10]. *k*-odd mean, (*k*, *d*)-odd mean labeling were introduced and discussed by Gayathri and Amuthavalli [1]. *k*-mean, *k*-even mean and (*k*, *d*)-even mean labeling were further studied by Gayathri and Gopi [7]. We have obtained (*k*, 1)-mean labeling for some new families of graphs in [6]. We have introduced (*k*, *d*)-mean labeling and obtained results for some family of trees and for some special graphs [4,5].

In this paper, we investigate (*k*, *d*)-mean labeling for some disconnected graphs. Here *k* and *d* denote any positive integer greater than or equal to 1.
For brevity, we use (*k*, *d*)-ML for (*k*, *d*)-mean labeling and (*k*, *d*)-MG for (*k*, *d*)-mean graph.

2. Main Results

Definition 2.1

A (p, q) graph G is said to have a mean labeling if there is an injective function f from the vertices of G to $\{0, 1, 2, ..., q\}$ such that the induced map f^* defined on E by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection from E to $\{1, 2, ..., q\}$.
A graph that admits a mean labeling is called a mean graph.

Definition 2.2

A (p, q) graph G is said to have a (k, d)-mean labeling if there is an injective function f from the vertices of G to $\{0, 1, 2, ..., k + (q - 1)d\}$ such that the induced map f^* defined on E by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection from E to $\{k, k+d, k+2d, ..., k+(q-1)d\}$.
A graph that admits a (*k*, *d*)-mean labeling is called a (*k*, *d*)-mean graph.

Observation 2.3

Every (*k*, 1)-mean labeling is a *k*-mean labeling.
Every (1, 1)-mean labeling is a mean labeling.

Definition 2.4

A **comb graph** P_n^+ is a tree obtained from a path by attaching exactly one pendant edge to each vertex of the path.

Definition 2.5

A **bistar** $B_{m,n}$ is a tree obtained by joining the center vertices of the copies of $K_{1,m}$ and $K_{1,n}$ with an edge.

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Theorem 2.6

The graph $P_m \cup P_n^+$ is a (k, d)-mean graph for all k and $d \geq 2$.

Proof

Let $V(P_m \cup P_n^+) = \{u_i, 1 \leq i \leq m, v_i, v_i^{'}, 1 \leq i \leq n\}$ be the vertices and $E(P_m \cup P_n^+) = \{u_iu_{i+1}, 1 \leq i \leq m-1, v_iv_{i+1}, 1 \leq i \leq n-1, v_iv_i^{'}, 1 \leq i \leq n\}$ be the edges which are denoted as in figure 2.1.

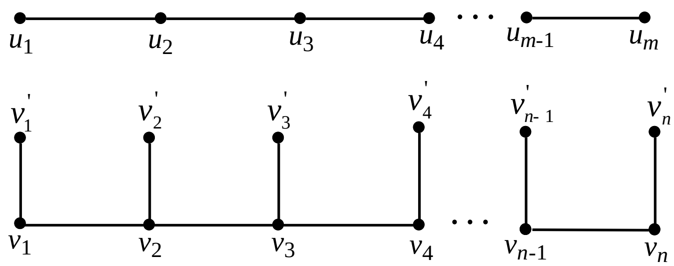

Figure 2.1: Ordinary labeling of $\boldsymbol{P_m \cup P_n^+}$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 2, ..., k + (q-1)d\}$ by

Case 1:
a) when *m* is odd and *n* is even (or)
b) when *m* and *n* both odd

For $1 \leq i \leq m$,

$$f(u_i) = \begin{cases} k + d(i-1) - 1, & i \text{ odd} \\ k + d(i-2), & i \text{ even} \end{cases}$$

$$f(v_1) = [k + (m-1)d] + 1$$

$$f(v_1^{'}) = [k + (m-1)d] - 2$$

$$f(v_2) = [k + (m+1)d] - 1$$

$$f(v_2^{'}) = k + (m+1)d$$

$$f(v_i) = k + d(2i + m - 3), \quad 3 \leq i \leq n$$

$$f(v_i^{'}) = k + d(2i + m - 3) - 1, \quad 3 \leq i \leq n$$

Case 2:
a) when *m* is even and *n* is odd (or)
b) when *m* and *n* both even

For $1 \leq i \leq m$,

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$$f(u_i) = \begin{cases} k + d(i-1) - 1, & i \text{ odd} \\ k + d(i-2), & i \text{ even} \end{cases}$$

$f(v_i) = k + d(2i + m - 3)$, $\quad 1 \le i \le n$

$f(v_i') = k + d(2i + m - 3) - 1$, $\quad 1 \le i \le n$

Then the induced edge labels in both the cases are:

$f^*(u_i u_{i+1}) = k + d(i - 1)$, $\quad 1 \le i \le m - 1$

$f^*(v_i v_{i+1}) = k + d(2i + m - 2)$, $\quad 1 \le i \le n - 1$

$f^*(v_i v_i') = k + d(2i + m - 3)$, $\quad 1 \le i \le n$

The above defined function f provides (k, d)-mean labeling of the graph.
So, the graph $P_m \cup P_n^+$ is a (k, d)-mean graph for all k and $d \ge 2$.

Illustration 2.7

(10, 2)-mean labeling of $P_5 \cup P_8^+$ is shown in figure 2.2.

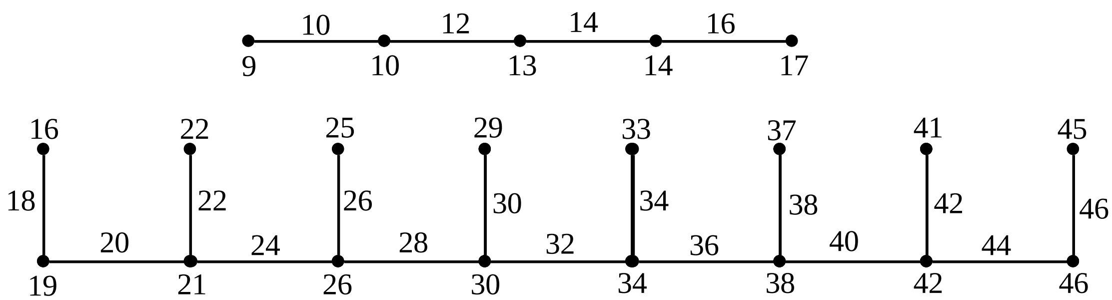

Figure 2.2: (10, 2)-ML of m $P_5 \cup \boldsymbol{P_8^+}$

(10, 2)-mean labeling of $P_3 \cup P_5^+$ is shown in figure 2.3.

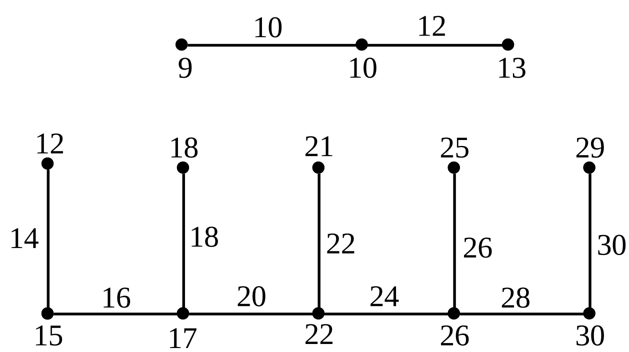

Figure 2.3: (10, 2)-ML of $P_3 \cup \boldsymbol{P_5^+}$

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(8, 10)-mean labeling of $P_4 \cup P_5^+$ is shown in figure 2.4.

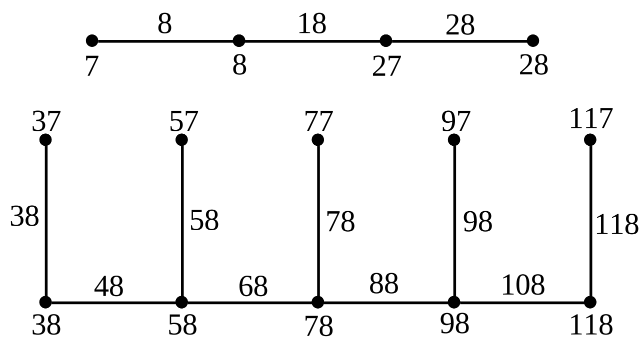

Figure 2.4: (8, 10)-ML of $P_4 \cup \boldsymbol{P_5^+}$

(4, 20)-mean labeling of $P_6 \cup P_4^+$ is shown in figure 2.5.

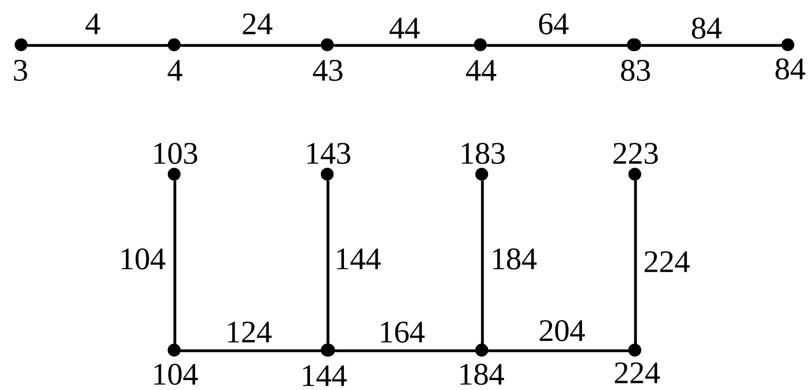

Figure 2.5: (4, 20)-ML of $P_6 \cup \boldsymbol{P_4^+}$

Theorem 2.8

The graph $P_m^+ \cup P_n^+$ is a (k, d)-mean graph for all k and $d \geq 2$.

Proof

Let $V(P_m^+ \cup P_n^+)$ = {u_i, $u_i^{'}$, $1 \leq i \leq m$, v_i, $v_i^{'}$, $1 \leq i \leq n$} be the vertices and $E(P_m^+ \cup P_n^+)$ = {u_iu_{i+1}, $1 \leq i \leq m-1$, $u_iu_i^{'}$, $1 \leq i \leq m$, v_iv_{i+1}, $1 \leq i \leq n-1$, $v_iv_i^{'}$, $1 \leq i \leq n$} be the edges which are denoted as in figure 2.6.

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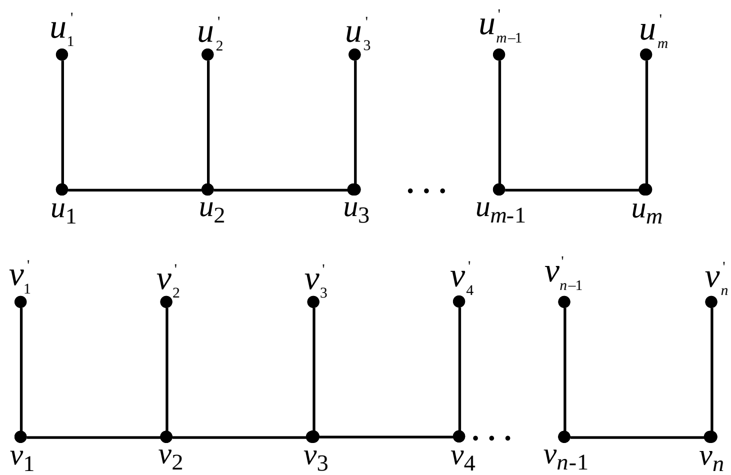

Figure 2.6: Ordinary labeling of $\boldsymbol{P_m^+}$ È $\boldsymbol{P_n^+}$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 2, ..., k + (q - 1)d\}$ by

$f(u_i) = k + 2d(i - 1), \quad 1 \le i \le m$

$f\left(u_i^{'}\right) = k + 2d(i - 1) - 1, \quad 1 \le i \le m$

$f(v_i) = k + d[2(m + i) - 3], \quad 1 \le i \le n$

$f\left(v_i^{'}\right) = k + d[2(m + i) - 3] - 1, \quad 1 \le i \le n$

Then the induced edge labels are:

$f^*(u_iu_{i+1}) = k + d(2i - 1), \quad 1 \le i \le m - 1$

$f^*\left(u_iu_i^{'}\right) = k + 2d(i - 1), \quad 1 \le i \le m$

$f^*(v_iv_{i+1}) = k + 2d(m + i - 1), \quad 1 \le i \le n - 1$

$f^*\left(v_iv_i^{'}\right) = k + d[2(m + i) - 3], \quad 1 \le i \le n$

The above defined function f provides (k, d)-mean labeling of the graph.
So, the graph $P_m^+ \cup P_n^+$ is a (k, d)-mean graph for all k and $d \ge 2$.

Illustration 2.9
(22, 2)-mean labeling of $P_6^+ \cup P_{12}^+$ is shown in figure 2.7.

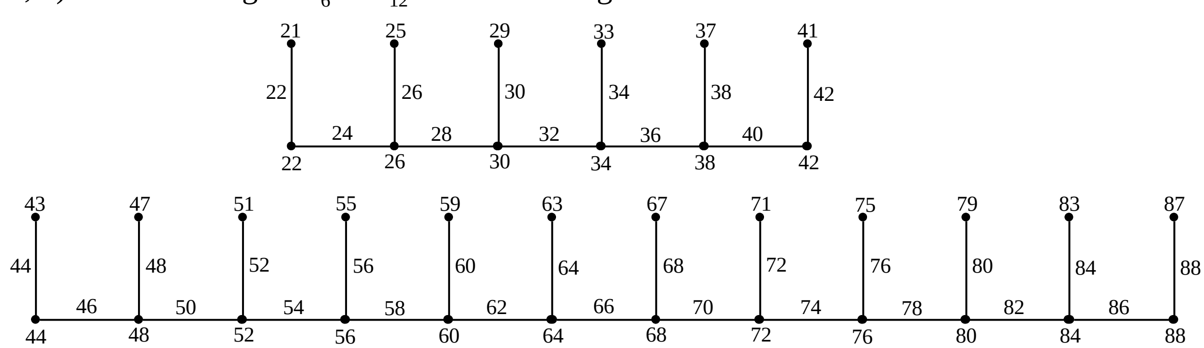

Figure 2.7: (22, 2)-ML of $\boldsymbol{P_6^+}$ È $\boldsymbol{P_{12}^+}$

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Theorem 2.10

The graph $B_{n,n} \cup B_{n,n}$ is a (k, d)-mean graph for all k and $d \geq 2$.

Proof

Let $V\left(B_{n,n} \cup B_{n,n}\right)$ = $\{u, u', v, v', u_i, u_i', v_i, v_i', 1 \leq i \leq n\}$ be the vertices and $E\left(B_{n,n} \cup B_{n,n}\right)$ = $\{uu', vv', uu_i, u'u_i', vv_i, v'v_i', 1 \leq i \leq n\}$ be the edges which are denoted as in figure 2.8.

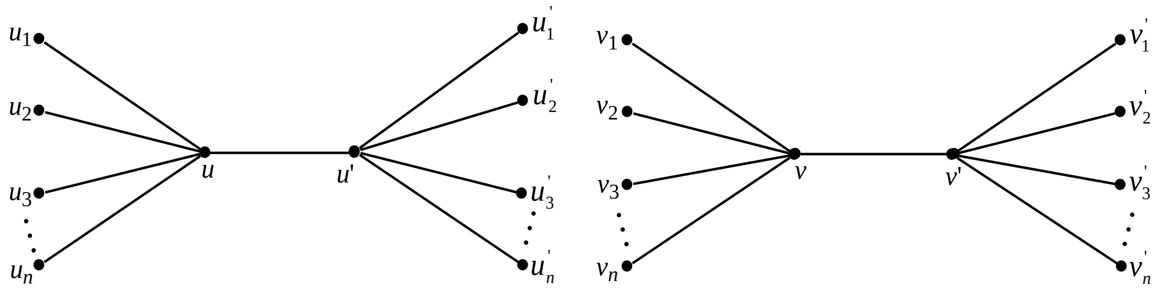

Figure 2.8: Ordinary labeling of $B_{n,n} \cup B_{n,n}$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 2, ..., k + (q - 1)d\}$ by

$$f(u) = k$$
$$f(u_i) = k + 2d(i - 1) - 1, \quad 1 \leq i \leq n$$
$$f(u') = k + 2nd - 1$$
$$f\left(u_i'\right) = k + 2id, \quad 1 \leq i \leq n$$
$$f(v) = k + (2n + 1)d$$
$$f(v_i) = k + d[2(n + i) - 1] - 1, \quad 1 \leq i \leq n$$
$$f(v') = k + (q - 1)d - 1$$
$$f\left(v_i'\right) = k + d[2(n + i) + 1], \quad 1 \leq i \leq n$$

Then the induced edge labels are:

$$f^*(uu_i) = k + d(i - 1), \quad 1 \leq i \leq n$$
$$f^*(uu') = k + nd$$
$$f^*\left(u'u_i'\right) = k + d(n + i), \quad 1 \leq i \leq n$$
$$f^*(vv_i) = k + d(2n + i), \quad 1 \leq i \leq n$$
$$f^*(vv') = k + (3n + 1)d$$
$$f^*\left(v'v_i'\right) = k + d(3n + i + 1), \quad 1 \leq i \leq n$$

The above defined function f provides (k, d)-mean labeling of the graph.
So, the graph $B_{n,n} \cup B_{n,n}$ is a (k, d)-mean graph for all k and $d \geq 2$.

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Illustration 2.11

(7, 2)-mean labeling of $B_{6,6} \cup B_{6,6}$ is shown in figure 2.9.

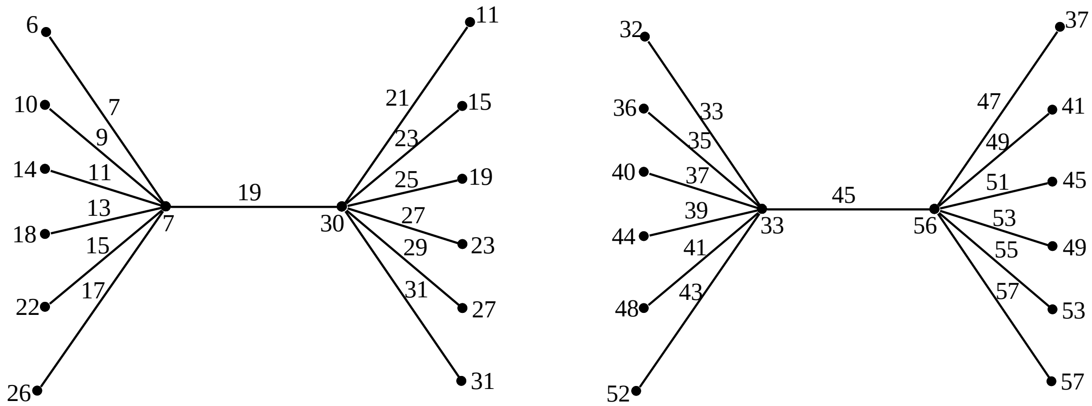

Figure 2.9: (7, 2)-ML of $B_{6,6} \cup B_{6,6}$

Theorem 2.12

The graph $B_{n,n} \cup P_n^+$ is a (k, d)-mean graph for all k and $d \geq 2$.

Proof

Let $V\left(B_{n,n} \cup P_n^+\right) = \{u, u', u_i, u_i', v_i, v_i', 1 \leq i \leq n\}$ be the vertices and $E\left(B_{n,n} \cup P_n^+\right) = \{uu', uu_i, u'u_i', v_iv_i', 1 \leq i \leq n, v_iv_{i+1}, 1 \leq i \leq n-1\}$ be the edges which are denoted as in figure 2.10.

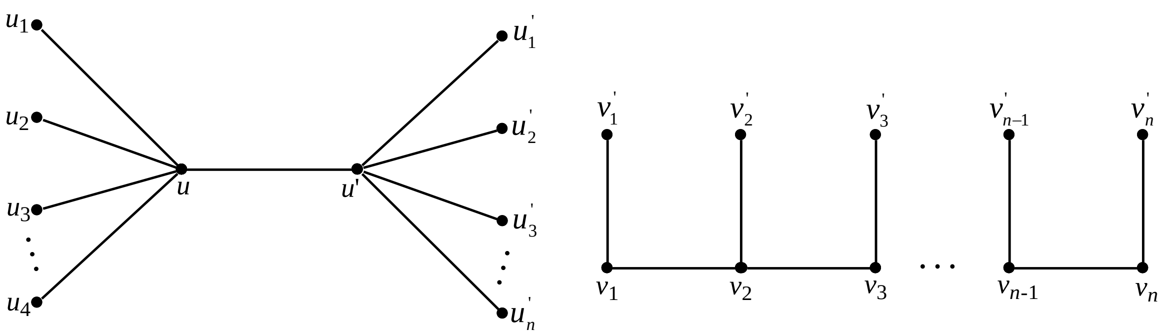

Figure 2.10: Ordinary labeling of $B_{n,n} \cup \boldsymbol{P_n^+}$

First we label the vertices as follows:

Define $f : V \rightarrow \{0, 1, 2, ..., k + (q-1)d\}$ by

$f(u) = k - 1$

$f(u_i) = k + 2d(i-1) + 1, \quad 1 \leq i \leq n$

$f(u') = k + 2nd$

$f\left(u_i'\right) = k + 2di - 1, \quad 1 \leq i \leq n$

$f(v_i) = k + d[2(n+i) - 1], \quad 1 \leq i \leq n$

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$f(v_i') = k + d[2(n + i) – 1] – 1, \quad 1 \le i \le n$

Then the induced edge labels are:

$f^*(uu_i) = k + d(i – 1), \quad 1 \le i \le n$

$f^*(uu') = k + nd$

$f^*(u'u_i') = k + d(n + i), \quad 1 \le i \le n$

$f^*(v_iv_i') = k + d[2(n + i) – 1], \quad 1 \le i \le n$

$f^*(v_iv_{i+1}) = k + 2d(n + i), \quad 1 \le i \le n – 1$

The above defined function f provides (k, d)-mean labeling of the graph.

So, the graph $B_{n,n} \cup P_n^+$ is a (k, d)-mean graph for all k and $d \ge 2$.

Illustration 2.13

(3, 10)-mean labeling of $B_{6,6} \cup P_6^+$ is shown in figure 2.11.

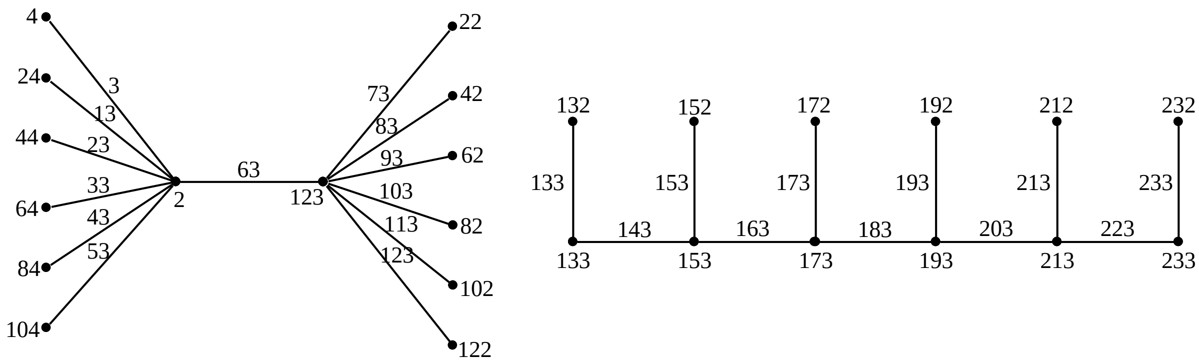

Figure 2.11: (3, 10)-ML of $B_{6,6} \cup \boldsymbol{P_6^+}$

Theorem 2.14

The graph $K_{1,n} \cup K_{1,n}$ is a (k, d)-mean graph for all k and $d \ge 2$.

Proof

Let $V(K_{1,n} \cup K_{1,n}) = \{u, v, u_i, v_i, 1 \le i \le n\}$ be the vertices and $E(K_{1,n} \cup K_{1,n}) = \{uu_i, vv_i, 1 \le i \le n\}$ be the edges which are denoted as in figure 2.12.

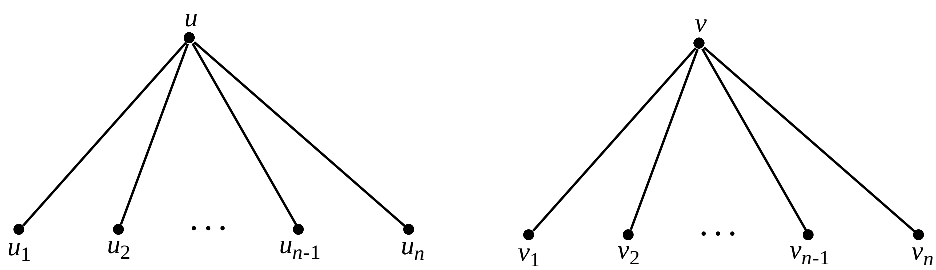

Figure 2.12: Ordinary labeling of $K_{1,n} \cup K_{1,n}$

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First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 2, ..., k + (q-1)d\}$ by

$$f(u) = k - 1$$
$$f(u_i) = k + 2d(i-1), \quad 1 \le i \le n$$
$$f(v) = k + (q-1)d$$
$$f(v_i) = k + d(2i-1), \quad 1 \le i \le n-1$$
$$f(v_n) = k + (q-1)d - 1$$

Then the induced edge labels are:

$$f^*(uu_i) = k + d(i-1), \quad 1 \le i \le n$$
$$f^*(vv_i) = k + d(n+i-1), \quad 1 \le i \le n$$

The above defined function f provides (k, d)-mean labeling of the graph.
So, the graph $K_{1,n} \cup K_{1,n}$ is a (k, d)-mean graph for all k and $d \ge 2$.

Illustration 2.15

(10, 20)-mean labeling of $K_{1,8} \cup K_{1,8}$ is shown in figure 2.13.

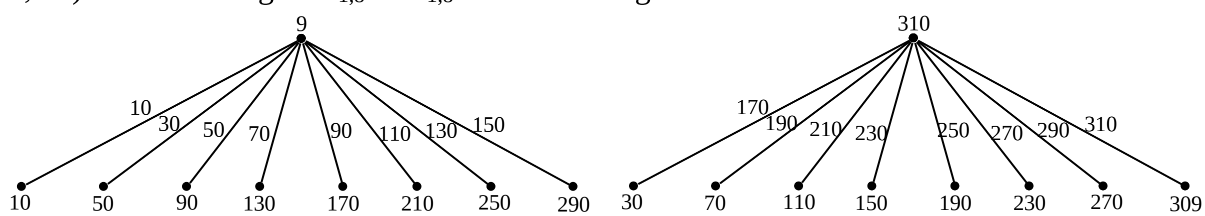

Figure 2.13: (10, 20)-ML of $K_{1,8} \cup K_{1,8}$

3. Conclusion

As every graph is not a (k, d)-mean graph, it is very interesting to investigate graph or graph families which admits (k, d)-mean labeling. In this paper, we investigated (k, d)-mean labeling for some disconnected graphs. For the better understanding of the proofs of the theorems, labeling pattern defined in each theorem is demonstrated by illustration.

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