OPTIMIZATION OF SINGLY REINFORCED RC BEAMS

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Abstract

The main objective of this paper is to achieve an optimal design for the reinforced concrete beams. Optimization of beams results in saving in cost. The objective function is to minimize the total cost of the beam. The cost of each beam includes the cost of concrete, reinforcement and formwork. The optimization process is done for different grades of concrete and steel to determine the best grade of concrete and steel. The optimal design is carried out using MATLAB's (The Mathworks, Inc.) software. Optimization problem is formulated as a Nonlinear constrained minimization problem. This was solved using the fmincon SQP Algorithm. Many problems were solved and it was found that the solutions give the most economical design.

Keywords: Optimization; Nonlinear Programming Problem; Constrained Nonlinear Minimization; Matlab; SQP Algorithm.

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1. Introduction

Optimization is the process of finding a minimum or maximum value of a function subject to some constraints. Optimization techniques play an important role in structural design, the very purpose of which is to find the best solutions from which a designer or a decision maker can derive a maximum benefit from the available resources. The basic requirement for an efficient structural design is that the response of the structure should be acceptable as per various specifications. There can be large number of feasible designs, but it is desirable to choose the best from these several designs. The best design could be in terms of minimum cost, minimum weight or maximum performance or a combination of these.

In several studies, the optimum cost design of the reinforced concrete (RC) elements has been investigated. Coello et.al.optimized RC beams [1] using a search technique employing genetic algorithm (GA). Ferreira et.al. studied on the optimal design of T-shaped RC beams according to

various design codes [2]. The simulated annealing algorithm was employed by Leps and Sejnoha to find optimum values of continuous steel reinforced beams [3]. Cost optimization of singly and doubly RC beams was investigated by Barros et.al. [4] Govindaraj and Ramasamy made a detailed investigation on optimum design of RC continuous beams using GA. Different groups of reinforcements were considered to find the solution with the optimum cost [5]. During the present study, optimization of reinforced concrete beam is carried out. The objective function was the total cost of the concrete element which includes the cost of concrete, steel and formwork. Optimization process was done for different grades of concrete and steel and the effect of diameter of rods on optimal cost was also studied. The optimization problem formulated is a Constrained Nonlinear minimization problem and was solved by using fmincon SQP Algorithm of matlab. While framing the constraints, for the design of RC beam elements, it was ensured that all codal provisions of IS 456-2000 were taken into account. Many singly reinforced beams were optimized using the matlab algorithm. The efficiency of the SQP algorithm was examined and found to be good. All the solved problems proved that the results are economical and give minimum construction cost.

2. Optimal Design of Ric Beams

The general form of an optimization problem consists of the following steps
1) Identification of design variables
2) Formulation of objective function
3) Formulation of constraints
4) Selection of suitable algorithm
5) Getting solution.

In an optimization problem, some of the parameters can be considered as pre-assigned and others are considered as design variables. The design variables are determined in such a way that the value of an objective function, which is often the cost of the structure, becomes minimum. Some restrictions, called design constraints, may limit the acceptable values of the design variables.

3. Optimal Design of Singly Reinforced Beams

3.1. Constant Parameters

In this work, optimal design of singly reinforced beam was done for different material combinations of M20, M25, M30, M35, M40, M45grades of concrete and Fe 415, Fe 500,Fe 550D grades of steel.

The cost of concrete, reinforcement including the cost of bar bending and the cost of formwork were worked out as per the latest market rates and the cost coefficients were worked out for 1m length of beam.

The value of cost coefficients for different grades of steel, concrete and for the formwork are given in Table 1.

Table 1: Values of cost co-efficient

grade of concrete	cost coefficient in rupees			
	Cost co-efficient for concrete	Grade of steel	Cost co-efficient for steel	cost co- efficient for formwork
M20	0.0057	Fe 415	0.43175	0.43
M25	0.0065			
M30	0.0072	Fe 500	0.4687	
M35	0.0078			
M40	0.0084	Fe550D	0.5119	
M45	0.0088			

3.2. Design Variables

Breadth of the beam, depth of the beam and the area of tension reinforcement were considered as design variables.

Width of beam = b = x_1
Overall Depth of beam = d= x_2
Area of steel in tension = Ast = x_3

3.3. Objective Function

The objective function to be minimized is given below:
$$f = (x_1 * x_2 - x_3) * C_c + x_3 * C_s + (2 * x_2 + x_1) * C_f$$

C_c – cost coefficient of concrete /m length of beam
C_s – cost coefficient of steel /m length of beam
C_f – cost coefficient for formwork / m^2

3.4. Design Constraints

i.   Maximum reinforcement percentage is incorporated as a constraint as given below
     $x_3 / (x_1 * x_2) \leq 0.04$
ii. Minimum reinforcement percentage is incorporated as a constraint as given below
     $x_3 / (x_1 * x_2) \geq 0.85/f_y$
iii. Moment constraints are given below
     $Mu \leq 0.36 * f_{ck} * x_{umax} \, b * (d - 0.42 * x_{umax})$
     $Mu \leq 0.87 * f_y * A_{st} * (d - 0.42 * x_{umax})$
     $x_2 \geq \sqrt{(Mu / (f_{ck} * 0.134 * x_1))}$
iv. The minimum width of beam is taken as 230mm
     $x_1 \geq 230$

3.5. Starting Solution

Starting solution is nothing but initial point where solver begins its search for a minimum value within the feasible range. The starting solution considered is a scalar which is given below

x_0 = [200 200 600]

where:
M_u = Factored Moment
A_{st} = Area of longitudinal reinforcement
f_{ck} = Characteristic compressive strength of the concrete
f_y = Characteristic strength of the steel

A matlab program was written using the SQP algorithm available in the matlab tool box.

3.5.1.   Effect of Grade of Concrete

In order to determine the effect of grade of concrete on the cost of singly reinforced beams, the optimal design of beams was found for various grade of concrete. The details of the optimal designs of beams carrying moments of 75kNm, 150kNm, 200kNm and 500kNm are given in Table 2, Table 3, Table 4 and Table 5 respectively.

Table 2: Optimal results for the beam with a factored moment of 75kNm, 150kNm, 200kNm and 500kNm are given in Table 2, Table 3, Table 4 and Table 5 respectively.

Table 2: Optimal results for the beam with a factored moment of 75kNm

Grades of concrete	b (mm)	d (mm)	Ast (mm^2)	Cost/m (Rs)
M20	230	350	750	1172.51
M25	230	310	840	1188.05
M30	230	280	930	1199.55
M35	230	260	1000	1218.91
M40	230	245	1065	1234.37
M45	230	230	1135	1243.34

Table 3: Optimal results for the beam with a factored moment of 200kNm

grades of concrete	b (mm)	d (mm)	Ast (mm^2)	Cost/m (Rs)
M20	230	490	1065	1617.39
M25	230	440	1185	1638.97
M30	230	400	1305	1655.63
M35	230	370	1400	1683.01
M40	230	345	1500	1704.87
M45	230	325	1600	1717.56

Table 4: Optimal results for the beam with a factored moment of 200kNm

grades of concrete	b (mm)	d (mm)	Ast (mm²)	Cost/m (Rs)
M20	230	580	1200	1852.37
M25	230	500	1390	1877.29
M30	230	465	1500	1896.53
M35	230	430	1620	1928.14
M40	230	400	1750	1953.38
M45	230	375	1855	1968.03

Table 5: Optimal results for the beam with a factored moment of 500kNm

grades of concrete	b (mm)	d (mm)	Ast (mm²)	Cost/m (Rs)
M20	230	900	1930	2871.63
M25	230	800	2175	2911.04
M30	230	735	2360	2941.45
M35	230	680	2560	2991.43
M40	230	635	2735	3031.35

From the results obtained, it can be concluded that M20 grade concrete is preferred as it gives the least cost. It can be seen that lower grade concrete results in larger depths, lower area of steel and least cost. Hence, lower grade concrete can be preferred wherever depth restriction is not a criterion for the design.

3.5.2. Effect of Diameter of Reinforcement on the Cost of Beams

Designers normally determine A_{st} and determine the number of rods assuming the diameter of rods. In case of beams, the designers can select the diameter from a set of {12, 16,20,25,32,40mm} which are available in the market. Only one of them will give the least cost.

In order to determine the optimal diameter of reinforcement, the optimal design of beams were carried out for various diameters of reinforcement.

The details of the optimal design for the beam with a factored moment of 75kNm are given in Table 6.

Table 6: Effect of diameter of reinforcement on optimal cost of beam with a moment of 75kNm

Diameter (mm)	12	16	20	25
No of bars	7	4	3	2
Area provided (mm2)	791.68	804.25	942.48	981.75
Cost/m (Rs)	1196.05	1201.40	1260.29	1277.02

It can be seen from the Table that 12mm diameter of reinforcement gives the least area. Therefore, providing 12mm diameter rod will give the least cost. When 25mm diameter rods are used, the percentage increase with respect to least cost is around 6.7%

The details of the optimal design for the beam with a factored moment of 150 kNm are given in Table 7.

Table 7: Effect of diameter of reinforcement on optimal cost of beam with a moment of 150kNm

Diameter (mm)	16	20	25	32
No of bars	6	4	3	2
Area provided (mm^2)	1206.37	1256.64	1472.62	1608.49
Cost/m (Rs)	1676.66	1698.08	1790.10	1847.99

It can be seen from the Table that 16mm diameter of reinforcement gives the least area. Therefore, providing 16mm diameter rod will give the least cost. When 32 mm diameter rods are used, the percentage increase with respect to least cost is around 10.21 %.

The details of the optimal design for the beam with a factored moment of 200 kNm are given in Table 8.

Table8: Effect of diameter of reinforcement on optimal cost of beam with a moment of 200kNm

Diameter (mm)	16	20	25	32
No of bars	6	4	3	2
Area provided (mm^2)	1206.4	1256.6	1472.62	1608.941
Cost/m (Rs)	1872.1	1893.5	1985.49	2043.569

It can be seen that 16mm diameter of reinforcement gives the least area. Therefore, providing 16mm diameter rod will give the least cost. When 32 mm diameter rods are used, the percentage increase with respect to least cost is around 9.15%

The details of the optimal design for the beam with a factored moment of 500 kNm are given in Table 9.

Table 9: Effect of diameter of reinforcement on optimal cost of beam with a moment of 500kN

Diameter (mm)	20	25	32	40
No of bars	7	4	3	2
Area provided (mm2)	2199.11	1963.49	2412.74	2513.27
Cost/m (Rs)	2989.73	2889.35	3080.75	3123.58

It can be seen that 25mm diameter of reinforcement gives the least area. Therefore, providing 16mm diameter rod will give the least cost.

4. Effect of Grade of Steel on Cost of Beam

In order to study the effect of grade of steel on the cost of beam, optimal designs were found for various grades of steel. The grade of concrete assumed was M20.The optimization was done with Fe 415, Fe 500 and Fe 550D grades of steel. Even though IS 456-2000 permits the use of only three grades namely Fe 250, Fe 415 and Fe500, Fe 550D which has a higher ductility is also tried, since Fe550D is included in IS 1786-2008and it satisfies the ductility requirements with enhanced benefits.

The details of beams with different factored moments designed with Fe415 rods are given in Table 10.

Table 10: Design details with Fe 415 rods

Moment (kNm)	d (mm)	Ast (mm^2)	Cost/m (Rs)
75	350	750.00	1172.51
100	400	860.00	1338.67
125	450	965.00	1485.05
150	490	1065.00	1617.39
200	580	1200.00	1852.37
250	630	1380.00	2059.39
300	700	1500.00	2246.55
400	850	1640.00	2578.86
500	900	1930.00	2871.63

The details of beams with factored moments designed with Fe500 rods are given in Table 11.

Table 11: Design details with Fe 500 rods

Moment (kNm)	d (mm)	Ast (mm^2)	Cost/m (Rs)
75	350	610.40	1139.14
100	400	704.84	1300.14
125	450	788.03	1441.98
150	500	863.25	1570.21
200	575	996.79	1797.89
250	640	1114.44	1998.48
300	700	1220.82	2179.82
400	800	1409.68	2501.81
500	931	1576.07	2785.49

The details of beams with factored moments designed with Fe550D rods are given in Table 12.

Table 12: Design details with Fe 550D rods

Moment (kNm)	d (mm)	Ast (mm^2)	Cost/m (Rs)
75	356.11	539.91	1142.87
100	411.2	623.44	1304.44
125	459.745	697.02	1446.78

150	503.62	763.55	1575.47
200	581.53	881.67	1803.97
250	650.17	985.73	2005.28
300	712.23	1079.82	2187.27
400	822.41	1246.87	2510.41
500	919.49	1394.04	2795.10

From Tables 10, 11 and 12, it is observed that Fe 500 grade steel and yields the optimum design values for the singly reinforced beam with least cost. The percentage reduction in cost when Fe 500 steel is around 3% with respect to Fe 415 steel.

5. Conclusions

The main conclusions drawn from the current research can be summarized as follows:

- The efficiency of the SQP algorithm optimization was examined and found to be good.
- Optimization of reinforced concrete beams indicate that minimum percentage of steel must be used as reinforcement. Higher percentage of steel results in higher cost.
- The effect of grade of concrete on the cost of beams was studied and it was found that grade of concrete has an impact on the cost of R.C beams. M20 grade of concrete is found to give least cost.
- The effect of grade of steel on the cost of beams was studied and it was found that grade of steel has an impact on the cost of R.C beams. Fe 500 grade of steel is found to give least cost.
- The effect of diameter of reinforcement on the cost of beam were also studied and it was found that diameter of concrete has an impact on the cost of reinforced concrete beams. Instead of selecting the diameter of the reinforcement arbitrarily, diameter has to be found through optimization.

Acknowledgement

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