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# RESOLUTION OF RICCATI EQUATION BY THE METHOD DECOMPOSITIONAL OF ADOMIAN

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## ABSTRACT

***In this work, we will use the method of decompositional for Adomian solve the Riccati equation in the form:***

$$\boldsymbol{u' = a(t) + b(t)u + c(t)u^2} \qquad (\mathbf{1})$$

**Keywords:**
*Adomian decomposition method, Adomian's polynomials, Riccati equation, Development limited.*

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## 1. INTRODUCTION

The Riccati equation it is named in honor of Jacopo Francesco Riccati (1676-1754) and his son Vincenzo Riccati (1707-1775).
In general equation (1) is not solvable by quadratures, if he knows a particular solution up, the Riccati equation (1) reduces to a Bernoulli equation.
And in the 80 G. Adomian proposed a new method to solve differential equations of different types.
This method is to look for the solution in the form of a series, and decompose the non-linear operator in a series of function (polynomials Adomian) [4, 5]
K .Abbaoui and Y.Cherrault, place assumptions on the convergence of series of Adomian to the exact solution [1, 2, 3, 6].
This work mainly concerns the resolution of the Riccati equation by the Adomian method, with application examples.

## 2. ADOMIAN METHOD

We consider the following problem:

$$Fu = Lu + Ru + Nu = f(t) \qquad (2)$$

as N is a nonlinear operator, and L the invertible linear portion of F.
Equation (2) it gives:

$$u = g(t) - L^{-1}Ru - L^{-1}Nu \quad (3)$$

Or

$$Nu = \sum_{n\geq 0} A_n t^n \quad (4)$$

with Ai are called Adomian polynomials.
and the terms of the standard solution defined by:

$$u_0 = g(t)$$
$$u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n \quad (5)$$

with:

$$u = \sum_{n\geq 0} u_n t^n$$

## 3. RICCATI DIFERENTIAL EQUATION

is an ordinary differential equation of first order of the form [7]:

$$u' = a(t) + b(t)u + c(t)u^2 \quad (6)$$

or a , b and c are continuous functions defined on an open interval I of R.
In general there is no solution by quadrature, but if he knows a particular solution, a Riccati equation is reduced by substitution in a Bernoulli equation.

### *3.1. RESOLUTION KNOWING A PARTICULAR SOLUTION*

If it is possible to find a particular solution $u_p$,
So the general solution is of the form:

$$u = u_p + y \quad (7)$$

By replacing u by $u_p + y$ in equation (6)
We obtain:

$$(u_p + y)' = a(t) + b(t)(u_p + y) + c(t)(u_p + y)^2 \quad (8)$$

and as ($u_p$ is a particular solution):

$$u_p' = a(t) + b(t)u_p + c(t)u_p^2$$

In was :

$$y' - (b(t) + 2c(t)u_p)y = c(t)y^2 \quad (9)$$

is a Bernoulli equation, transformation is used: $z = 1/y$
So:

$$z' + (b(t) + 2c(t)u_p)z = -c(t) \quad (10)$$

It is a non-homogeneous linear equation.
We solve the homogeneous equation, then use the method of variation of constants, we find the general solution of the Bernoulli equation.

$$y = \frac{1}{z} = e^{\int (b(s)+2c(t)u_p)ds}(k - \int c(t)\, e^{\int (b(s)+2c(t)up)ds} dt)^{-1} \quad (11)$$

the more general solution of the Riccati equation given by:

$$u = u_p + \frac{1}{z}$$
$$u = u_p + e^{\int (b(s)+2c(t)u_p)ds}(k - \int c(t)\, e^{\int (b(s)+2c(t)up)ds} dt)^{-1} \quad (12)$$

### *3.2. RESOLUTION BY THE METHOD ADOMIAN*

Consider the following problem:

$$u' = a(t) + b(t)u + c(t)u^2 \quad (13)$$
$$u(0) = u_0$$

the Adomian method is used to solve the Riccati equation in the problem (13).
We have:

$$u' = a(t) + b(t)u + c(t)u^2$$

and we ask:

$$L = D^1 = \frac{d}{dt} \Longrightarrow L^{-1} = \int_0^t . ds$$
$$Ru = b(t)u \quad (14)$$
$$Nu = u^2$$

such as:

$$Nu = u^2 = \sum_{n\geq 0} A_n t^n$$

With $A_i$: polynomials Adomian of the function $u^2$[4,5].

$$L^{-1}u' = L^{-1}a(t) + L^{-1}b(t)u + L^{-1}c(t)u^2$$
$$u(t) - u(0) = L^{-1}a(t) + L^{-1}b(t)u + L^{-1}c(t)u^2$$

Using the D,L in the neighborhood of 0 of functions a, b and c:

$$a(t) = \sum_{n\geq 0} a_n t^n$$
$$b(t) = \sum_{n\geq 0} b_n t^n$$
$$c(t) = \sum_{n\geq 0} c_n t^n$$

and we obtain:

$$\sum_{n\geq 0} u_n t^n = u(0) + L^{-1}\sum_{n\geq 0} a_n t^n + L^{-1}\sum_{n\geq 0} b_n t^n . \sum_{n\geq 0} u_n t^n + L^{-1}\sum_{n\geq 0} c_n t^n . \sum_{n\geq 0} A_n t^n$$

$$= u(0) + \sum_{n\geq 0}\frac{a_n}{n+1}t^{n+1} + \sum_{n\geq 0}\frac{1}{n+1}t^{n+1}\left(\sum_{k=0}^{n} b_k u_{n-k}\right) + \sum_{n\geq 0}\frac{1}{n+1}t^{n+1}\left(\sum_{k=0}^{n} c_k A_{n-k}\right)$$

$$= u(0) + \sum_{n\geq 1}\frac{1}{n}\left(a_{n-1} + \sum_{k=0}^{n-1} b_k u_{n-1-k} + \sum_{k=0}^{n-1} c_k A_{n-1-k}\right)t^n$$

and the solution given by:

$$u_0 = u(0)$$

$$u_n = \left(\frac{1}{n}\right)\left(a_{n-1} + \sum_{k=0}^{n-1} b_k u_{n-1-k} + \sum_{k=0}^{n-1} c_k A_{n-1-k}\right) \quad (15)$$

## 4. APPLICATION EXAMPLES

### *4.1. EXAMPLE*1

We consider the following problem (Riccati equation):

$$u' = -t + (2t - 1)u + (1 - t)u^2 \quad (16)$$
$$u(0) = 2$$

with the particular solution $u_p = 1$

### *4.1.1. DIRECT RESOLUTION*

We have the following equation according to (7), (9) and (10) with the transformation $z = 1/u$:

$$z' + z = t - 1 \quad (17)$$

it is a first-order non-homogeneous equation, and the solution given by:

$$z(t) = t - 2 + ke^{-t} \quad (18)$$

and the solution of the problem (16) with the initial condition given by:

$$u(t) = 1 + \frac{1}{t - 2 + 3e^{-t}} \quad (19)$$

the D, L of the solution function is: We have:

$$e^t = \sum_{n\geq 0} \frac{(-t)^n}{n!} = 1 - t + \left(\frac{1}{2}\right).t^2 - \left(\frac{1}{6}\right).t^3 + \left(\frac{1}{24}\right)t^4 - \cdots$$

and

$$t - 2 + 3e^{-t} = 1 - 2t + \left(\frac{3}{2}\right).t^2 - \left(\frac{1}{2}\right).t^3 + \left(\frac{1}{8}\right).t^4 + o(t^4)$$

Dividing the polynomial ($f_1(t) = 1$) by the polynomial ($f_2(t) = t - 2 + 3e^{-t}$), we obtain:

$$\frac{1}{t - 2 + 3e^{-t}} = 1 + 2t + \left(\frac{5}{2}\right).t^2 + \left(\frac{5}{2}\right).t^3 + \left(\frac{17}{8}\right).t^4 + o(t^4)$$

So the order of D,L. 4 in the neighborhood of 0 of the solution function u(t) is:

$$u(t) = 1 + 2t + \left(\frac{5}{2}\right).t^2 + \left(\frac{5}{2}\right).t^3 + \left(\frac{17}{8}\right).t^4 + o(t^4) \quad (20)$$

### *4.1.2. RESOLUTION BY THE METHOD ADOMIAN*

Consider the Riccati equation:

$$u' = -t + (2t - 1)u + (1 - t)u^2 \quad (21)$$

Applying the method of Adomian with $F = L + R + N$ as:

$$L = D^1 = \frac{d}{dt}$$
$$Ru = (2t - 1)u$$
$$Nu = (1 - t)u^2$$

We have

$$L = D^1 = \frac{d}{dt} \Longrightarrow L^{-1} = \int_0^t .\,ds$$

Is applied $L^{-1}$ in equation (21) we obtain:

$$L^{-1}u' = L^{-1}(-t) + L^{-1}Ru + L^{-1}Nu$$

$$u(t) - u(0) = L^{-1}(-t) + L^{-1}((2t-1)u(t)) + L^{-1}((1-t)u^2)$$

$$\sum_{n\geq 0} u_n t^n = u(0) - \frac{1}{2}t + \sum_{n\geq 0} \frac{2u_n}{n+2} t^{n+2} - \sum_{n\geq 0} \frac{u_n}{n+1} t^{n+1} + \sum_{n\geq 0} \frac{A_n}{n+1} t^{n+1} - \sum_{n\geq 0} \frac{A_n}{n+2} t^{n+2}$$

$$\sum_{n\geq 0} u_n t^n = u(0) - \frac{1}{2}t + \sum_{n\geq 2} \frac{2u_{n-2}}{n} t^n - \sum_{n\geq 1} \frac{u_{n-1}}{n} t^n + \sum_{n\geq 1} \frac{A_{n-1}}{n} t^n - \sum_{n\geq 2} \frac{A_{n-2}}{n} t^n$$

$$\sum_{n\geq 0} u_n t^n = u(0) - \frac{1}{2}t + \sum_{n\geq 1} \frac{1}{n}(A_{n-1} - u_{n-1}) t^n + \sum_{n\geq 2} \frac{1}{n}(2u_{n-1} - A_{n-1}) t^n$$

So the coeffcients of the series solution Adomian is given by:

$$\begin{cases} u_0 = u(0) = 2 \\ u_1 = A_0 - u_0 \\ u_2 = \frac{1}{2}(A_1 - u_1 + 2u_0 - A_0) - \frac{1}{2} \\ . \\ . \\ u_n = \frac{1}{2}(A_{n-1} - u_{n-1} + 2u_{n-2} - u_{-2}), \forall n \geq 3. \end{cases}$$

Or $A_i$ are polynomials of Adomian of $u^2$ function [4,5]

$$\begin{cases} A_0 = u_0^2 \\ A_1 = 2u_0u_1 \\ A_2 = u_1^2 + 2u_0u_2 \\ A_3 = 2u_1u_2 + 2u_0u_3 \\ A_4 = u_2^2 + 2u_1u_3 + 2u_0u_4 \\ A_5 = 2u_2u_3 + 2u_1u_4 + 2u_0u_5 \\ . \\ . \end{cases}$$

So:

$$\begin{cases} u_0 = 2 \\ u_1 = 2 \\ u_2 = \frac{5}{2} \\ u_3 = \frac{5}{2} \\ u_4 = \frac{17}{8} \\ . \\ . \end{cases}$$

Finally the solution series is:

$$u(t) = 2 + 2t + \frac{5}{2}t^2 + \frac{5}{2}t^3 + \frac{17}{8}t^4 + \cdots. \qquad (23)$$

And we note that the results (19) and (22) are equal.

***4.2.EXAMPLE2***

We consider the following problem (Riccati equation):

$$\begin{cases} (1+t^2)u' = u^2 - 1 \\ u(0) = 2 \end{cases} \qquad (24)$$

with the particular solution: $u_p = 1$.

***4.2.1. DIRECT RESOLUTION***

We have the following equation according to (7), (9) and (10) with the transformation $z = \frac{1}{u}$:

$$z' = \frac{2}{1+t^2} z = \frac{-1}{1+t^2} \qquad (25)$$

it is a first-order non-homogeneous equation, and the solution given by:

$$z(t) = -\frac{1}{2} + ke^{2\arctan t} \qquad (26)$$

and the solution of the problem (16) with the initial condition given by:

$$u(t) = 1 + \frac{1}{-\frac{1}{2} + 3e^{2\arctan t}} = 1 + \frac{2}{3e^{2\arctan t} - 1} \qquad (27)$$

the D, L of the solution function is we have:

$$\arctan t = t - \frac{1}{3}t^3 + o(t^4)$$

So:

$$e^{2\arctan t} = e^{2t - \frac{2}{3}t^3 + o(t^4)}$$

if we pose: $v = 2t - \frac{2}{3}t^3 + o(t^4)$

Then:

$$e^v = 1 + v + \frac{1}{2}v^2 + o(v^2)$$

from where:

$$e^v = 1 + (2t - \frac{2}{3}t^3) + \frac{1}{2}(2t - \frac{2}{3}t^3)^2 + o(t^4)$$

$$= 1 - 2t + 2t^2 - \frac{2}{3}t^3 + \frac{4}{3}t^4 + o(t^4)$$

So we have:

$$3e^v - 1 = 2 - 6t + 6t^2 - 2t^3 - 4t^4 + o(t^4) \quad (28)$$

Dividing the polynomial $(f_1(t) = 2)$ by the polynomial $(f_2(t) = 3e^{2\,arctan\,t} - 1)$, we obtain:

$$\frac{2}{3e^{2\arctan t} - 1} = 1 + 3t + 6t^2 + 10t^3 + 17t^4 + o(t^4)$$

So the order of D, L 4 to neighborhood of the origin of the solution function $u(t)$ is:

$$u(t) = 2 + 3t + 6t^2 + 10t^3 + 17t^4 + o(t^4) \quad (29)$$

#### *4.2.2. RESOLUTION BY THE METHOD ADOMIAN*

Consider the Riccati equation:

$$u' = \frac{-1}{1+t^2} + \frac{1}{1+t^2}u^2 \quad (30)$$

Applying the method of Adomian with $F = L + R + N$ as:

$$\begin{cases} L = D^1 = \dfrac{d}{dt} \\ Nu = \dfrac{1}{1+t^2}u^2 \end{cases}$$

We have :

$$L = \frac{d}{dt} \Longrightarrow L^{-1} = \int_0^t .\,ds$$

Applying the operator $L^{-1}$ in equation (30) we obtain:

$$L^{-1}u' = L^{-1}\left(\frac{-t}{1+t^2}\right) + L^{-1}Nu$$

$$\Leftrightarrow u(t)-u(0)=L^{-1}\left(\frac{-t}{1+t^2}\right)+L^{-1}\left(\frac{1}{1+t^2}u^2(t)\right) \qquad (31)$$

$$\sum_{n\geq 0} u_n t^n = u(0)+L^{-1}\sum_{n\geq 0}(-1)^{n+1}t^{2n}+L^{-1}\sum_{n\geq 0}(-1)^n t^{2n}.\sum_{n\geq 0} A_n t^n$$

$$\sum_{n\geq 0} u_n t^n = u(0)+L^{-1}\sum_{n\geq 0}(-1)^{n+1}t^{2n}+L^{-1}\sum_{n\geq 0}(-1)^n t^{2n}.\left(\sum_{n\geq 0} A_{2n}t^{2n}+\sum_{n\geq 0} A_{2n+1}t^{2n+1}\right)$$

$$\sum_{n\geq 0} u_n t^n = u(0)+\sum_{n\geq 0}\frac{(-1)^{n+1}}{2n+1}t^{2n+1}+\sum_{n\geq 0}\frac{(-1)^n}{2n+1}t^{2n+1}.\left(\sum_{k=0}^{n}(-1)^k A_{2(n-k)}\right)$$
$$+\sum_{n\geq 0}\frac{(-1)^n}{2n+2}t^{2n+2}.\left(\sum_{k=0}^{n}(-1)^k A_{2(n-k)-1}\right)$$

So the coefficients of the series solution Adomian is given by:

$$u_0 = u(0) = 2$$

$$u_{2n+1} = \frac{1}{2n+1}\left((-1)^{n+1}+\sum_{k=0}^{n}(-1)^k A_{2(n-k)}\right), \forall n \geq 0$$

$$u_{2n+1} = \frac{1}{2n+2}\left(\sum_{k=0}^{n}(-1)^k A_{2(n-k)-1}\right), \forall n \geq 0$$

Or $Ai$ are polynomials of Adomian of the function $u^2$ [4,5].
Then:

$$\begin{cases} u_0 = 2 \\ u_1 = (-1)^1+(-1)^0 A_0 = -1+4 = 3 \\ u_2 = \frac{1}{2}A_1 = \frac{1}{2}.12 = 6 \\ u_3 = \frac{1}{3}((-1)^2+A_2+(-1)^1 A_0) = \frac{1}{3}(1+33-4) = 10 \\ u_4 = \frac{1}{4}(A_3+(-1)A_1) = \frac{1}{4}(80-12) = 17 \end{cases}$$

Finally the solution series is:

$$u(t) = 2+3t+6t^2+10t^3+17t^4+\cdots \qquad (32)$$

And we note that the results (29) and (32) are equal.

## 5. CONCLUSION

Despite generally, there is no Resolution of Riccati equation, but the method of decomposition Adomian always given an approximate solution in the form of a convergent series.

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