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# FINDING STANDARD DEVIATION OF A FUZZY NUMBER

**Fokrul Alom Mazarbhuiya** *1
*1 College of Computer Science & IT, Albaha University, KSA

## ABSTRACT

***Two probability laws can be root of a possibility law. Considering two probability densities over two disjoint ranges, we can define the fuzzy standard deviation of a fuzzy variable with the help of the standard deviation two random variables in two disjoint spaces.***

**Keywords:**
*Probability density function, probability distribution, fuzzy measure, fuzzy mean, fuzzy variance, fuzzy standard deviation, fuzzy membership function, Dubois-Prade reference functions.*

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## 1. INTRODUCTION

*Zadeh* [1] proposed the concept of fuzziness into the realm of Mathematics. Accordingly, various authors have made study on the mathematics related to the fuzzy measure and the associated fuzzy expected value of a possibility distribution [[2], [3], [4], [5], [6], [7]]. In [8] author discussed the possibilistic means and variance of a fuzzy number. In [[9], [10], authors used variance of a fuzzy number for clustering of temporal and fuzzy temporal data. In [11], authors proposed a method of analysis of possibilistic portfolio that associates a probabilistic portfolio. Similar attempts were made in associating possibility and probability [[12], [13]]. In,[[14],[15]], author have tried to establish a link between possibility law and probability law using a concept discussed in the paper called set superimposition [16]. In [17], author tries to establish a link between fuzziness and randomness. In [18], authors propose the definition of expected value of fuzzy number using the set superimposition. In this article we have attempted define the variance and standard definition of a fuzzy number. The idea is based on the fact that a fuzzy variable can be defined in terms of two random variables in two disjoint space [14], The paper is organized as follows. In section 2, we review some definitions and notations used in the article. In section 3, we give the standard deviation of a fuzzy numbers. We conclude the paper with conclusions and lines for future work in section 4.

## 2. DEFINITION AND NOTATION

In this section we review some important definitions related to randomness and fuzziness.

Suppose $X$ be a continuous random variable in $[a,b]$ with probability density function $f(x)$ and probability distribution function $F(x)$. Then, we can write

$$\Pr ob(a \le X \le b) = \int_a^b f(y)dy = F(b) - F(a)$$

The expectation of $X$ is given by

$$E(X) = \int_{-\infty}^{\infty} xdF(x) \qquad ..... \qquad (1)$$

where the integral is absolutely convergent.

Let $E$ be a set and $x \in X$ then the fuzzy subset $A$ of $E$ is defined as

$$A = \{x, \chi_A(x); x \in E\}$$

where $\chi_A(x) \in [0,1]$ is the fuzzy membership function of the fuzzy set $A$ for an ordinary set, $\chi_A(x) = 0$ or 1.

A fuzzy set $A$ is said to be normal if $\chi_A(x) = 1$ for at least one $x \in E$.

An $\alpha$ -cut $A_\alpha$ for a fuzzy set $A$ is an ordinary set of elements such that $\chi_A(x) \ge \alpha$ for $0 \le \alpha \le 1$, i.e. $A_\alpha = \{x \in E, \chi_A(x) \ge \alpha\}$.

The membership function of a fuzzy set is known as a possibility distribution [19]. A fuzzy number is usually denoted by a triad $[a,b,c]$ such that $\chi_A(a) = 0 = \chi_A(c)$ and $\chi_A(b) = 1$. $\chi_A(x)$, for $x \in [a,b]$ is the left reference function and for $x \in [b,c]$ is the right reference function. The left reference function is right continuous, monotone and non-decreasing., while the right reference function is left continuous, monotone and non-increasing. The above definition of a fuzzy number is known as an L-R fuzzy number.

### *2.1.FUZZY MEASURE*

*Kandel* [[5], [20]] has defined a fuzzy measure as follows: Let $B$ be a Borel field ($\sigma$ -algebra) of subset of the real line $\Omega$. A set function $\mu(.)$ defined on $B$ is called fuzzy measure if it has the following properties:

(1) $\mu(\phi) = 0$ ($\phi$ is the empty set )

(2) $\mu(\Omega) = 1$

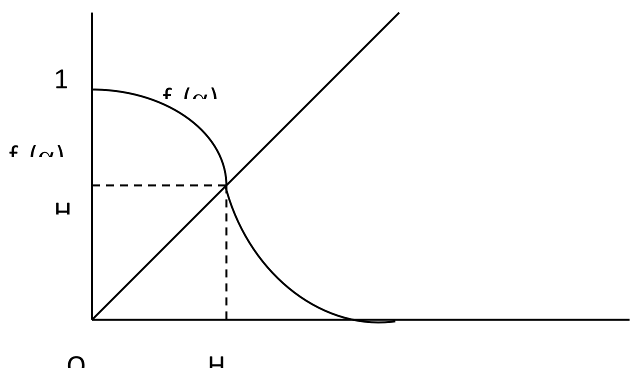
(3) If $\mathrm{A}, \mathrm{B} \in B$ with $\mathrm{A} \subset \mathrm{B}$, then $\mu(\mathrm{A}) \le \mu(\mathrm{B})$.

(4) If $\{A_i; 1 \le i \le \infty\}$ is a monotonic sequence, then $\lim_{i\to\infty}[\mu(A_i)] = \mu[\lim_{i\to\infty}(A_i)]$ Clearly $\phi, \Omega \in B$. Also, if $A_i \in B$,

then $\lim_{i\to\infty}(A_i) \in B$. $(\Omega, B, \mu)$ is called a fuzzy measure space. $\mu(.)$ is the fuzzy measure of $(\Omega, B)$.

Let $\chi_A : \Omega \to [0,1]$ and $A_\alpha = \{x; \chi_A(x) \ge \alpha\}$. The function $\chi_A$ is called a $B$- measurable function, if $A_\alpha \in B$ for all $\alpha \in [0,1]$. In their notations, fuzzy expected value is defined as follows: Let $\chi_A$ be a $B$- measurable function such that $\chi_A \in [0,1]$. The fuzzy expected value (FEV) of $\chi_A$ over a set $A$ with respect to the measure $\mu(.)$ is defined as $\sup_{\alpha\in[0,1]}\{\min[\alpha.\mu(A_\alpha)]\}$

Now $\mu\{x; \chi_A(x.) \ge \alpha\} = f_A(\alpha)$ is a function of the threshold $\alpha$. The calculation of FEV( $\chi_A$) then consists of finding the intersection of the curves of $\alpha = f_A(\alpha)$. The intersection of the curves will be at a value $\alpha = H$ so that FEV( $\chi_A$) = $H \in [0,1]$ as in the diagram.




## *2.2.EXPECTED VALUE OF FUZZY NUMBER*

*Kandel's* definition of a fuzzy expected value is based on the definition of the fuzzy measure. However, the fuzzy measure is not a measure in traditional sense because of it is non-additive nature.

In [14], author has shown that if the possibility distribution in $[a,b,c]$, is expressed as a probability distribution function in $[a,b]$ and a complementary probability distribution function in $[b,c]$, the underlying mathematics can be seen to be governed by the product measure on $[a,b]$ and $[b,c]$. Using the idea of [14], in [18], authors propose to define the expected of fuzzy number as follows.

Let $X$ be a fuzzy variable in the fuzzy set $A=[a,b,c]$. We divide $A$ into two intervals $A_1=[a,b]$ and $A_2=[b,c]$ such that $A_1 \cup A_2 = A$ and $A_1 \cap A_2 = \phi$. Let $X$ be a random variable on $A_1$. Then from (1), the mean of $X$ would be

$$E_1(X)=\int_a^b xf(x)dx \tag{2}$$

where $f(x)$ is the concerned probability density function defined on $[a,b]$. Let the mean of the random variable an $A_2$ be

$$E_2(X)=\int_b^c xg(x)dx \tag{3}$$

where $g(x)$ is the concerned probability density function defined on $[b,c]$.

Using (2) and (3), we get the possibilistic mean of $X \in [a,b,c]$ as

$$M=[\int_a^b xf(x)dx, b, \int_b^c xg(x)dx]$$

$$=\{x, \chi_M(x) \in [r,1], x \in E\} \tag{4}$$

where $r=\min\{\chi_M(\int_a^b xf(x)dx), \chi_M(\int_b^c xg(x)dx)\}$

The equation (4) is the required result that shows that poissibilistic mean of a fuzzy variable is again a fuzzy set.

## 3. STANDARD DEVIATION OF FUZZY NUMBER

To find the standard deviation of a fuzzy number let us proceed as follows.
The expected value of $X^2$ where $X \in$ [a, b] is

$$E_1(X^2)=\int_a^b x^2 f(x)dx \tag{5}$$

Similarly the expected value of $Y^2$ where $Y \in$ [b, c] is

$$E_2(y^2)=\int_b^c y^2 g(x)dx \tag{6}$$

where f(x) and g(x) are the probability density functions on [a, b] and [b, c] respectively
Using the definition variance of random variable, we have

$V_1(X) = E(X^2)- [E(X)]^2$

Using (2) and (5), we get

$$V_1(X) = \int_a^b x^2 f(x)dx - [\int_a^b xf(x)dx]^2 \quad (7)$$

Similarly, using (3) and (6), we get

$$V_2(Y) = \int_a^b y^2 f(y)dy - [\int_a^b yf(y)dy]^2 \quad (8)$$

Thus from (7) and (8) the standard deviation of fuzzy number [a, b, c] is defined as

$$\sigma_{[X, Y]} = [\sqrt{V_1(X)}, \sqrt{V_2(Y)}]$$

$$= \sqrt{\int_a^b x^2 f(x)dx, -[\int_a^b xf(x)dx]^2}, \sqrt{\int_a^b x^2 f(x)dx, -[\int_a^b xf(x)dx]^2} \quad (9)$$

To illustrate the result (9), we take $A = [a, b, c]$, a triangular number such that $\chi_A(a) = 0 = \chi_A(c)$ and $\chi_A(b) = 1$. The probability distribution function is given by

$$F(x) = \begin{cases} 0 & x \le a \\ \frac{(x-a)}{(b-a)} & a \prec x \prec b \\ 1 & x \ge b \end{cases} \quad (10)$$

where $f(x) = \frac{1}{(b-a)}$ (11)

is the probability density function in a ≤ x ≤ b.

The complementary probability distribution or the survival function is given by

$$G(x) = \begin{cases} 1 & x \le b \\ 1 - \frac{(x-b)}{(c-b)} & b \prec x \prec c \\ 0 & x \ge c \end{cases} \quad (12)$$

where $F(x) = 1 - G(x)$ and the probability density function in $b \le x \le c$ is

$$g(x) = \frac{1}{(c-b)} \quad (13)$$

Therefore, the expected value of a uniform random variable X on [a, b] is

$$E_1(x) = \frac{(a+b)}{2} \tag{14}$$

and, similarly, the expected value of another uniform random variable X on [b, c] is

$$E_2(x) = \frac{(b+c)}{2} \tag{15}$$

$$E_1(X^2) = (a^2 + b^2 + ab)/3 \tag{16}$$

$$V_1(X) = (a^2 + b^2 + ab)/3 - (a+b)^2/4$$

$$= (a-b)^2/12 \tag{17}$$

Similarly,

$$V_2(Y) = (b-c)^2/12 \tag{18}$$

From (17) and (18), the standard deviation is

$$\sigma_{[X,\,Y]} = [\,(a-b)/\sqrt{12}\,,\,(b-c)/\sqrt{12}\,] \tag{19}$$

## 4. CONCLUSION

The very definition of a fuzzy expected value as given by *Kandel* is based on the understanding that the so called fuzzy measure is not really a measure in the strict sense. The possibility distribution function is viewed as two reference function. Using left reference function as probability distribution function and right reference function survival function, in this article we redefine the standard deviation of a fuzzy number.

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