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ON AN EXPANSION FORMULA FOR THE MULTIVARIABLE I - FUNCTION INVOLVING GENERALIZED LEGENDRE'S ASSOCIATED FUNCTION

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ABSTRACT

The authors have established a new expansion formula for multivariable I-function due to Prasad [5] in terms of products of the multivariable I-function and the generalized Legendre's associated function due to Meulenbeld [3]. Some special cases are given in the last.

Keywords:
Multivariable I-function, Generalized Legendre's associated function, Multivariable H-function.

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1. INTRODUCTION

The multivariable I-function introduced by Prasad [5] will be define and represent it in the following manner:

$$I\left[z_1,\dots,z_r\right]=I^{0,n_2:\dots:0,n_r:(m',n'):\dots:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:\dots:p_r,q_r:(p',q'):\dots:\left(p^{(r)},q^{(r)}\right)}$$

$$\left[z_1,\dots,z_r\left|\begin{matrix}(a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\dots:(\alpha_{rj},\alpha'_{rj},\dots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:\dots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\(b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\dots:(b_{rj},\beta'_{rj},\dots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'}:\dots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right.\right]$$

$$=\frac{1}{(2\pi w)^r}\int_{L_1}\dots\int_{L_r}\phi_1(s_1)\dots\phi_r(s_r)\psi(s_1,\dots,s_r)z_1^{s_1},\dots,z_r^{s_r}\,ds_1\dots ds_r \qquad (1.1)$$

Where

$w=\sqrt{(-1)}$

$$\phi_i(s_i)=\frac{\prod_{j=1}^{m^{(i)}}\Gamma\left(b_j^{(i)}-\beta_j^{(i)}s_i\right)\prod_{j=1}^{n^{(i)}}\Gamma\left(1-a_j^{(i)}+\alpha_j^{(i)}s_i\right)}{\prod_{j=m^{(i)}+1}^{q^{(i)}}\Gamma\left(1-b_j^{(i)}+\beta_j^{(i)}s_i\right)\prod_{j=n^{(i)}+1}^{p^{(i)}}\Gamma\left(a_j^{(i)}-\alpha_j^{(i)}s_i\right)}\ \forall i\in(1,2,...,r) \qquad (1.2)$$

$$\psi(s_1,...,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma\left(1-a_{2j}+\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i\right)\prod_{j=1}^{n_3}\Gamma\left(1-a_{3j}+\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i\right)}{\prod_{j=n_2+1}^{p_2}\Gamma\left(a_{2j}-\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i\right)\prod_{j=n_3+1}^{p_3}\Gamma\left(a_{3j}-\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i\right)}$$

$$\frac{...\prod_{j=1}^{n_r}\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)}{...\prod_{j=n_r+1}^{p_r}\Gamma\left(a_{rj}-\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)\prod_{j=1}^{q_2}\Gamma\left(1-b_{2j}-\sum_{i=1}^{2}\beta_{2j}^{(i)}s_i\right)...\prod_{j=1}^{q_r}\Gamma\left(1-b_{rj}-\sum_{i=1}^{r}\beta_{rj}^{(i)}s_i\right)} \qquad (1.3)$$

$\alpha_j^{(i)},\beta_j^{(i)},\alpha_{kj}^{(i)},\beta_{kj}^{(i)}(i=1,...,r)(k=1,...,r)$ are positive numbers,
$a_j^{(i)},b_j^{(i)}(i=1,...,r),a_{kj},b_{kj}(k=2,...,r)$ are complex numbers and here
$m^{(i)},n^{(i)},p^{(i)},q^{(i)}(i=1,...,r),n_k,p_k,q_k(k=2,...,r)$ are non-negative integers where
$0\le m^{(i)}\le q^{(i)},0\le n^{(i)}\le p^{(i)},q^k\ge 0,0\le n_k\le p_k$. Here (i) denotes the numbers of dashes. The contours L_i in the complex s_i-plane is of the Mellin-Barnes type which runs from $-w\infty$ to $+w\infty$ with indentations, if necessary, to ensure that all the poles of $\Gamma\left(b_j^{(i)}-\beta_j^{(i)}s_i\right)\left(j=1,...,m^{(i)}\right)$ are separated from those of $\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i\right)\left(j=1,...,n_r\right)$.

For further details and asymptotic expansion of the I-function one can refer by Prasad [5].

In what follows, the multivariable I-function defined byPrasad [5] will be represented in the contracted notation:

$$I_{p_2,q_2:...:p_r,q_r:(p',q'):...:\left(p^{(r)},q^{(r)}\right)}^{0,n_2:...:0,n_r:(m',n'):...:\left(m^{(r)},n^{(r)}\right)}\left[z_1,...,z_r\right]$$

Or simply by $I\left[z_1,...z_r\right]$.

According to the asymptotic expansion of the gamma function, the counter integral (1.1) is absolutely convergent provided that

$$\left|\arg z_i\right| < \frac{1}{2}\pi U_i, U_i > 0 \quad ; \quad i = 1, 2, \ldots, r \tag{1.4}$$

Where

$$U_i = \sum_{j=1}^{n^i} \alpha_j^{(i)} - \sum_{j=n^{(i)}+1}^{p^{(i)}} \alpha_j^{(i)} + \sum_{j=1}^{m^i} \beta_j^{(i)} - \sum_{j=m^{(i)}+1}^{q^{(i)}} \beta_j^{(i)}$$
$$+\left(\sum_{j=1}^{n_2} \alpha_{2j}^{(i)} - \sum_{j=n_2+1}^{p_2} \alpha_{2j}^{(i)}\right) + \left(\sum_{j=1}^{n_3} \alpha_{3j}^{(i)} - \sum_{j=n_3+1}^{p_3} \alpha_{3j}^{(i)}\right)$$
$$+ \ldots + \left(\sum_{j=1}^{n_r} \alpha_{rj}^{(i)} - \sum_{j=n_r+1}^{p_r} \alpha_{rj}^{(i)}\right)$$
$$-\left(\sum_{j=1}^{q_2} \beta_{2j}^{(i)} + \sum_{j=1}^{q_3} \beta_{3j}^{(i)} + \ldots + \sum_{j=1}^{q_r} \beta_{rj}^{(i)}\right) \tag{1.5}$$

The asymptotic expansion of the I-function has been discussed by Prasad [5]. His results run as follow:

$I[z_1, \ldots, z_r] = 0(|z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}), \max\{|z_1|, \ldots, |z_r|\} \to 0$

Where

$$\alpha_i = \min \mathrm{Re}(b_j^{(i)} / \beta_j^{(i)}),\ j = 1, \ldots, m^{(i)} \ ;\ i = 1, \ldots, r \tag{1.6}$$

And $I[z_1, \ldots, z_r] = 0(|z_1|^{\beta_1} \ldots |z_r|^{\beta_r}), \min\{|z_1|, \ldots, |z_r|\} \to \infty$

Where

$$\beta_i = \max \mathrm{Re}\left(\frac{a_j^{(i)} - 1}{\alpha_j^{(i)}}\right) \ ;\ j = 1, \ldots, n^{(i)} \ ,\ i = 1, \ldots, r \tag{1.7}$$

The details of the function can be found in the paper of Prasad [5].

In this paper we will evaluate an integral involving generalized associated Legendre's function and the multivariable I-function due to Prasad [5] and apply it in deriving an expansion for the multivariable I-function in series of products of associated Legendre's function and the multivariable I-function.

2. THE INTEGRAL

The integral to be evaluated is:

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$$\int_{-1}^{1}(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}P_{k-\frac{u-v}{2}}^{u,v}(x)$$

$$\times I\left[(1-x)^{\alpha_1}(1+x)^{\beta_1}z_1,\ldots,(1-x)^{\alpha_r}(1+x)^{\beta_r}z_r\right]dx$$

$$=2^{\rho-u+v+\sigma+1}\sum_{t=0}^{\infty}\frac{(-k)_t(v-u+k+1)_t}{\Gamma(1-u+t)\,t!}\,I_{p_2,q_2:\ldots:p_r+2,q_r+1:(p',q'):\ldots:\left(p^{(r)},q^{(r)}\right)}^{0,n_2:\ldots:0,n_r+2:(m',n'):\ldots:\left(m^{(r)},n^{(r)}\right)}$$

$$\left[\begin{matrix}2^{\alpha_1+\beta_1}z_1\\ \vdots\\ 2^{\alpha_r+\beta_r}z_r\end{matrix}\left|\begin{matrix}(a_{2j},\alpha_{2j}',\alpha_{2j}'')_{1,p_2}:\ldots:(-\sigma-v;\beta_1,\ldots,\beta_r),\\ \\ (b_{2j},\beta_{2j}',\beta_{2j}'')_{1,q_2}:\ldots:(b_{rj},\beta_{rj}',\ldots,\beta_{rj}^{(r)})_{1,q_r},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(u-\rho-t;\alpha_1,\ldots,\alpha_r),(\alpha_{rj},\alpha_{rj}',\ldots,\alpha_{rj}^{(r)})_{1,p_r}:(a_j',\alpha_j')_{1,p'}:\ldots:(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}}\\ (u-v-\rho-\sigma-t-1;\alpha_1+\beta_1,\ldots,\alpha_r+\beta_r),:(b_j',\beta_j')_{1,q'}:\ldots:(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}}\end{matrix}\right] \qquad (2.1)$$

The integral (2.1) is valid under the following set of conditions:

(i) $(\alpha_i,\beta_i)>0;\forall i\in\{1,2,\ldots,r\};k-\dfrac{u-v}{2}$ is a positive integer, k is a integer ≥ 0.

(ii) $\mathrm{Re}\left(\rho-u+\sum_{i=1}^{r}\alpha_i\dfrac{b_j^{(i)}}{\beta_j^{(i)}}\right)>-1;\mathrm{Re}\left(\sigma+v+\sum_{i=1}^{r}\beta_i\dfrac{b_j^{(i)}}{\beta_j^{(i)}}\right)>-1;(j=1,2,\ldots,m_i;i=1,2,\ldots,r)$

And the conditions given in (1.4) to (1.7) are also satisfied.

Proof: On expressing the multivariable I-function in the integrand as a multiple Mellin-Barnes type integral (1.1) and inverting the order of integrations, which is justified due to the absolute convergence of the integrals involved in the process, the value of the integral

$$=(2\pi w)^r\int_{L_1}\ldots\int_{L_r}\psi(s_1,\ldots,s_r)\sum_{i=1}^{r}\left\{\phi_i(s_i)z_i^{\xi_i}\right\}$$

$$\times\left\{\int_{-1}^{1}(1-x)^{\rho-\frac{u}{2}+\sum_{i=1}^{r}\alpha_i\xi_i}(1+x)^{\sigma+\frac{v}{2}+\sum_{i=1}^{r}\beta_i\xi_i}\right.$$

$$\left.P_{k-\frac{u-v}{2}}^{u,v}(x)dx\right\}d\xi_1\ldots d\xi_r$$

On evaluating the x-integral with the help of the integral ([4], p.343, eq. (38)):

$$\int_{-1}^{1}(1-x)^{\rho}(1+x)^{\sigma}P_{k-\frac{m-n}{2}}^{m,n}(x)dx$$

$$=\frac{2^{\rho+\sigma-\frac{m-n}{2}}\Gamma\left(\rho-\frac{m}{2}+1\right)\Gamma\left(\sigma+\frac{n}{2}+1\right)}{\Gamma(1-m)\Gamma\left(\rho+\sigma-\frac{m-n}{2}+2\right)}$$

$$\times{}_3F_2\left(-k,n-m+k+1,\rho-\frac{m}{2}+1;1-m,\rho-\sigma-\frac{m-n}{2}+2;1\right) \quad (2.2)$$

Provided that $\mathrm{Re}\left(\rho-\frac{m}{2}\right)>-1;\mathrm{Re}\left(\sigma+\frac{n}{2}\right)>-1$ and interpreting the result with the help of (1.1), the integral (2.1) is established.

3. EXPANSION THEOREM

Let the following conditions be established:

(i) $\beta_1,...,\beta_r>0;\alpha_1,...,\alpha_r\geq 0(or\,\beta_1,...,\beta_r\geq 0;\alpha_1,...,\alpha_r>0)$;

(ii) $m^{(i)},n^{(i)},p^{(i)},q^{(i)}(i=1,...,r),n_k,p_k,q_k(k=2,...,r)$ are non-negative integers where $0\leq m^{(i)}\leq q^{(i)},0\leq n^{(i)}\leq p^{(i)},q^k\geq 0,0\leq n_k\leq p_k$ and the conditions given by (1.4) to (1.7) are also satisfied.

(iii) $\mathrm{Re}(u)>-1,\mathrm{Re}(v)>-1,\mathrm{Re}\left(\rho-u+\sum_{i=1}^{r}\alpha_i\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)>-1;$

$$\mathrm{Re}\left(\sigma+v+\sum_{i=1}^{r}\beta_i\frac{b_j^{(i)}}{\beta_j^{(i)}}\right)>-1;(j=1,2,...,m_i;i=1,2,...,r).$$

Then the following expansion formula holds:

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}I\left[(1-x)^{\alpha_1}(1+x)^{\beta_1}z_1,...,(1-x)^{\alpha_r}(1+x)^{\beta_r}z_r\right]$$

$$=2^{\rho+\sigma}\sum_{N=0}^{\infty}\sum_{\mu=0}^{N}\frac{(2N-u+v+1)\Gamma(N-u+1)\Gamma(1+v-u+N+\mu)(-N)_\mu}{N!\mu!\Gamma(1+v+N)\Gamma(1-u+\mu)}$$

$$P^{u,v}_{N-\frac{u-v}{2}}(x)I^{0,n_2:...:0,n_r+2:(m',n'):...:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:...:p_r+2,q_r+1:(p',q'):...:\left(p^{(r)},q^{(r)}\right)}$$

$$\left[\begin{matrix}2^{\alpha_1+\beta_1}z_1\\ \vdots\\ 2^{\alpha_r+\beta_r}z_r\end{matrix}\left|\begin{matrix}(a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:...:(-\sigma-v;\beta_1,...,\beta_r),\\ \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:...:(b_{rj},\beta'_{rj},...,\beta^{(r)}_{rj})_{1,q_r},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(u-\rho-\mu;\alpha_1,...,\alpha_r),(\alpha_{rj},\alpha'_{rj},...,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:...:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\(u-v-\rho-\sigma-\mu-1;\alpha_1+\beta_1,...,\alpha_r+\beta_r),:(b'_j,\beta'_j)_{1,q'}:...:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right] \quad (3.1)$$

Proof: Let

$$f(x)=(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}I\left[(1-x)^{\alpha_1}(1+x)^{\beta_1}z_1,...,(1-x)^{\alpha_r}(1+x)^{\beta_r}z_r\right]$$

$$=\sum_{N=0}^{\infty}C_N P^{u,v}_{N-\frac{u-v}{2}}(x) \quad (3.2)$$

Equation (3.2) is valid since $f(x)$ is continuous and of bounded variation in the interval (-1,1).

Now, multiplying both the sides of (3.2) by $P^{u,v}_{N-\frac{u-v}{2}}(x)$ and integrating with respect to x from -1 to 1; evaluating the L.H.S. with the help of (2.1) and on the R.H.S. interchanging the order of summation, using ([2],p.176,eq. (75)) and then applying orthogonality property of the generalized Legendre's associated functions ([4],p.340,eq.(27)):

$$\int_{-1}^{1}P^{u,v}_{k-\frac{u-v}{2}}(x)P^{u,v}_{N-\frac{u-v}{2}}(x)dx$$

$$=\begin{cases}0; \text{ if } k\neq N\\ \dfrac{2^{u-v+1}k!\Gamma(k+v+1)}{(2k-u+v+1)\Gamma(k-u+1)\Gamma(k-u+v+1)}; \text{if } \quad k=N\end{cases} \quad (3.3)$$

Provided that $\text{Re}(u),1,\text{Re}(v)<1$; we obtain

$$C_k=\frac{2^{\rho+\sigma}(2k-u+v+1)\Gamma(k-u+1)}{k!\Gamma(k+v+1)}\sum_{\mu=0}^{k}\frac{(-k)_\mu\Gamma(k-u+v+\mu+1)}{\mu!\Gamma(k-u+\mu)}$$

$$I^{0,n_2:...:0,n_r+2:(m',n'):...:\left(m^{(r)},n^{(r)}\right)}_{p_2,q_2:...:p_r+2,q_r+1:(p',q'):...:\left(p^{(r)},q^{(r)}\right)}\left[\begin{matrix}2^{\alpha_1+\beta_1}z_1\\ \vdots\\ 2^{\alpha_r+\beta_r}z_r\end{matrix}\left|\begin{matrix}(a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:...:(-\sigma-v;\beta_1,...,\beta_r),\\ \\(b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:...:(b_{rj},\beta'_{rj},...,\beta^{(r)}_{rj})_{1,q_r},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(u-\rho-\mu;\alpha_1,...,\alpha_r),(\alpha_{rj},\alpha'_{rj},...,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:...:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\(u-v-\rho-\sigma-\mu-1;\alpha_1+\beta_1,...,\alpha_r+\beta_r),:(b'_j,\beta'_j)_{1,q'}:...:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right] \quad (3.4)$$

Now on substituting the values of C_k in (3.2), the result follows.

4. SPECIAL CASES

If in (2.1), we put $n_2=...=n_{r-1}=0=p_2=...=p_{r-1},q_2=...=q_{r-1}=0$, the multivariable I -function reduces to multivariable H -function and we get result given by Saxena and Ramawat [6]

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}} H\left[(1-x)^{\alpha_1}(1+x)^{\beta_1} z_1,\dots,(1-x)^{\alpha_r}(1+x)^{\beta_r} z_r\right]$$

$$=2^{\rho+\sigma}\sum_{N=0}^{\infty}\sum_{\mu=0}^{N}\frac{(2N-u+v+1)\Gamma(N-u+1)\Gamma(1+v-u+N+\mu)(-N)_\mu}{N!\mu!\Gamma(1+v+N)\Gamma(1-u+\mu)}$$

$$P^{u,v}_{N-\frac{u-v}{2}}(x) H^{0,n+2:(m',n');\dots;(m^{(r)},n^{(r)})}_{p+2,q+1:(p',q');\dots;(p^{(r)},q^{(r)})}$$

$$\left[\begin{matrix}2^{\alpha_1+\beta_1}z_1\\ \vdots\\ 2^{\alpha_r+\beta_r}z_r\end{matrix}\left|\begin{matrix}(-\sigma-v;\beta_1,\dots,\beta_r),\\ (b_j,\beta_j^{'},\dots,\beta_j^{(r)})_{1,q},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(u-\rho-\mu;\alpha_1,\dots,\alpha_r),(\alpha_j,\alpha_j^{'},\dots,\alpha_j^{(r)})_{1,p}:(a_j^{'},\alpha_j^{'})_{1,p'};\dots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}}\\ (u-v-\rho-\sigma-\mu-1;\alpha_1+\beta_1,\dots,\alpha_r+\beta_r),:(b_j^{'},\beta_j^{'})_{1,q'};\dots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}}\end{matrix}\right] \quad (4.1)$$

Provided all the conditions given with (3.1) and the conditions ([7], p.252-253, eq. (c.4), (c.5) and (c.6)) are satisfied.

For $n=0=p, q=0$, the multivariable H -function breaks up into a product of r H -function and consequently, (4.1) reduces to

$$(1-x)^{\rho-\frac{u}{2}}(1+x)^{\sigma+\frac{v}{2}}\prod_{i=1}^{r}\left\{H^{m_i,n_i}_{p_i,q_i}\left[(1-x)^{\alpha_i}(1+x)^{\beta_i}\left|\begin{matrix}(a_j^{(i)},\alpha_j^{(i)})_{1,p_i}\\ (b_j^{(i)},\beta_j^{(i)})_{1,q_i}\end{matrix}\right.\right]\right\}$$

$$=2^{\rho+\sigma}\sum_{N=0}^{\infty}\sum_{\mu=0}^{N}\frac{(2N-u+v+1)\Gamma(N-u+1)\Gamma(1+v-u+N+\mu)(-N)_\mu}{N!\mu!\Gamma(1+v+N)\Gamma(1-u+\mu)}$$

$$P^{u,v}_{N-\frac{u-v}{2}}(x) H^{0,2:(m',n');\dots;(m^{(r)},n^{(r)})}_{2,1:(p',q');\dots;(p^{(r)},q^{(r)})}\left[\begin{matrix}2^{\alpha_1+\beta_1}z_1\\ \vdots\\ 2^{\alpha_r+\beta_r}z_r\end{matrix}\left|\begin{matrix}(-\sigma-v;\beta_1,\dots,\beta_r),\\ -\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(u-\rho-\mu;\alpha_1,\dots,\alpha_r):(a_j^{'},\alpha_j^{'})_{1,p'};\dots;(a_j^{(r)},\alpha_j^{(r)})_{1,p^{(r)}}\\ (u-v-\rho-\sigma-\mu-1;\alpha_1+\beta_1,\dots,\alpha_r+\beta_r),:(b_j^{'},\beta_j^{'})_{1,q'};\dots;(b_j^{(r)},\beta_j^{(r)})_{1,q^{(r)}}\end{matrix}\right] \quad (4.2)$$

For $r=1$, (4.2) gives riseto the result due to Anandani [1].

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