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TREE RELATED EXTENDED MEAN CORDIAL GRAPHS

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ABSTRACT

*Let $G = (V, E)$ be a graph with p vertices and q edges. A *Extended Mean Cordial Labeling* of a Graph G with vertex set V is a bijection from V to $\{0, 1, 2\}$ such that each edge uv is assigned the label $(\lceil f(u) + f(v) \rceil)/2$ where $\lceil x \rceil$ is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by almost 1. The graph that admits an *Extended Mean Cordial Labeling* is called *Extended Mean Cordial Graph*. In this paper, we proved that tree related graphs Hdn , $K_{1,n}$, Tg_n , $\langle K_{1,n}:n \rangle$ are *Extended Mean Cordial Graphs*.*

Keywords:

Extended Mean Cordial Graph, Extended Mean Cordial Labeling.

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1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e = \{u, v\}$ of vertices in E is called edges or a line of G . In this paper, we proved that tree related graphs Hdn , $K_{1,n}$, Tg_n , $\langle K_{1,n}:n \rangle$ are *Extended mean cordial graphs*. For graph theory terminology, we follow [2].

2. PRELIMINARIES

Let $G = (V, E)$ be a graph with p vertices and q edges. A *Extended Mean Cordial Labeling* of a Graph G with vertex set V is a bijection from V to $\{0, 1, 2\}$ such that each edge uv is assigned the label $(\lceil f(u) + f(v) \rceil)/2$ where $\lceil x \rceil$ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits an Extended Mean Cordial Labeling is called Extended Mean Cordial Graph. In this paper, we proved that tree related graphs H_{dn} , $K_{1,n}$, Tg_n , $\langle K_{1,n}:n \rangle$ are Extended Mean Cordial Graphs.

Definition: 2.1

A graph obtained from a path P_n by attaching a pendent edges to every internal vertices of the Path. It is called Hurdle graph with $n-2$ hurdles and is denoted by H_{dn} .

Definition: 2.2

A bipartite graph is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every edge of G has one end in V_1 and the other end in V_2 ; (V_1, V_2) is called a bipartition of G . If further, every vertex of V_1 is joined to all the vertices of V_2 , then G is called a complete bipartite graph. The complete bipartite graph with bipartition (V_1, V_2) such that $|V_1|=m$ and $|V_2|=n$ is denoted by $K_{m,n}$. A complete bipartite graph $K_{1,n}$ is called a star

Definition: 2.3

Subdivided star $\langle K_{1,n}:n \rangle$ is a graph obtained as one point union of n paths of path-length 2

Definition: 2.4

A graph obtained from a path by attaching exactly two pendent edges to each internal vertex of a path is called a twig and is denoted by Tg_n , $n \geq 1$

3. MAIN RESULTS**Theorem 3.1**

Graph H_{dn} is a Extended Mean Cordial Graph.

Proof:

Let $V(H_{dn}) = \{[u_i: 1 \leq i \leq n], [v_i: 1 \leq i \leq n-2]\}$

Let $E(H_{dn}) = \{[(u_i u_{i+1}): 1 \leq i \leq n-1] \cup [(v_i u_{i+1}): 1 \leq i \leq n-2]\}$

Define $f: V(G) \rightarrow \{0, 1, 2\}$

The Vertex labeling are

$$f(u_i) = 1 \quad 1 \leq i \leq n$$

$$f(v_i) = 0 \quad 1 \leq i \leq n-2$$

The induced edge labeling are,

$$f(u_i u_{i+1}) = 1 \quad 1 \leq i \leq n-1$$

$$f(v_i u_{i+1}) = 0 \quad 1 \leq i \leq n-2$$

Here, $e_f(1) = e_f(0) + 1$

Hence, H_{dn} satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, H_{dn} is a Extended Mean Cordial Graph.

For example, H_{d4} is a Extended Mean Cordial Graph as shown in the figure 3.2

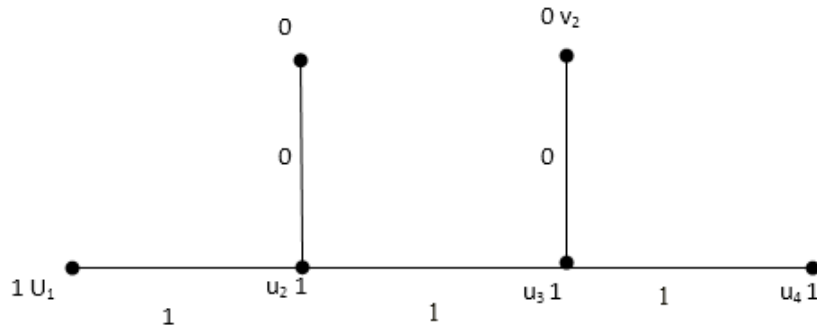


Figure 3.2:

Theorem 3.3

$k_{1,n}$ is a Extended Mean Cordial Graph.

Proof:

Let $V(k_{1,n}) = \{[u_i v_i : 1 \leq i \leq n]\}$

Let $E(k_{1,n}) = \{[(u_i v_i) : 1 \leq i \leq n]\}$

Define $f: (k_{1,n}) \rightarrow \{0,1,2\}$

The vertex labeling are,

$$f(u) = 1, 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases}, 1 \leq i \leq n$$

The edge labeling are,

$$f^*(u_i v_i) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ 0 & i \equiv 0 \pmod 2 \end{cases}, 1 \leq i \leq n$$

Here, $ef(1) = ef(0) + 1$

Hence $k_{1,n}$ is Satisfies the condition $|ef(0) - ef(1)| \leq 1$

Therefore, $k_{1,n}$ is a Extended Mean Cordial Graph.

For example, $k_{1,n}$ is a Extended Mean Cordial Graph as shown in the figure 3.4

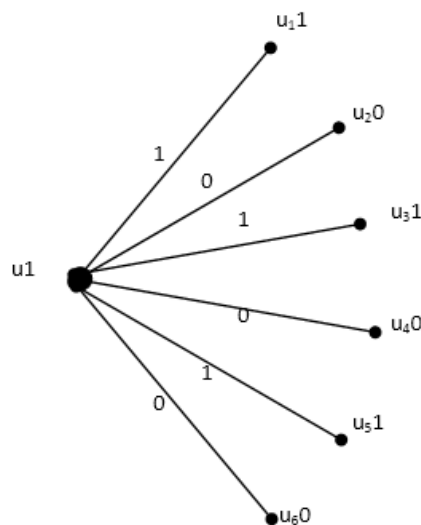


Figure 3.4:

Theorem: 3.5

Tg_n is a Extended Mean Cordial Graph

Proof:

Let $V(Tg_n) = \{[u_i : 1 \leq i \leq n] \cup [v_i : 1 \leq i \leq n-2] \cup [w_i : 1 \leq i \leq n-2]\}$

Let $E(Tg_n) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(v_i u_{i+1}) : 1 \leq i \leq n-2] \cup [w_i u_{i+1}) : 1 \leq i \leq n-2]\}$

Define $f: V(Tg_n) \rightarrow \{0,1,2\}$

The vertex labeling are,

$$f(u_i) = 1, \quad 1 \leq i \leq n$$

$$f(v_i) = 1, \quad 1 \leq i \leq n-2$$

$$f(w_i) = \begin{cases} 0 & i \equiv 1, 2 \pmod{4} \\ 1 & i \equiv 0, 3 \pmod{4} \end{cases}, 1 \leq i \leq n-2$$

The edge labeling are,

$$f^*(u_i u_{i+1}) = 1, \quad 1 \leq i \leq n$$

$$f^*(v_i u_{i+1}) = 0, \quad 1 \leq i \leq n-2$$

$$f^*(w_i u_{i+1}) = \begin{cases} 1 & i \equiv 3, 0 \pmod{4} \\ 0 & i \equiv 1, 2 \pmod{4} \end{cases}, 1 \leq i \leq n-2$$

Here, $ef(0) = ef(1) \quad n \equiv 1 \pmod{2}$

$ef(0) = ef(1) + 1 \quad n \equiv 0 \pmod{2}$

Hence Tg_n is Satisfies the condition $|ef(0) - ef(1)| \leq 1$

Therefore, Tg_n is a Extended Mean Cordial Graph.

For example, Tg_5 is a Extended Mean Cordial Graph as shown in the figure 3.6

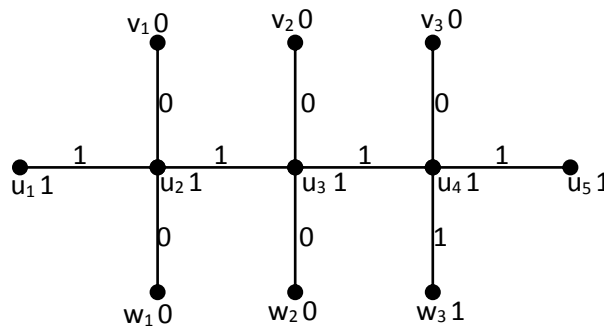


Figure 3.6:

Theorem : 3.7

Graph $\langle K_{1,n}: n \rangle$ be a Extended Mean Cordial Graph.

Proof:

Let $v(K_{1,n}: n) = \{[u, u_i, v_i : 1 \leq i \leq n]\}$

Let $E(K_{1,n}: n) = \{[(uu_i) \cup (u_i v_i) : 1 \leq i \leq n]\}$

Define $f: v(K_{1,n}: n) \rightarrow \{0,1,2\}$

The vertex labeling are,

$$f(u) = 1, \quad 1 \leq i \leq n$$

$$f(u_i) = 1, \quad 1 \leq i \leq n$$

$$f(v_i) = 0, \quad 1 \leq i \leq n$$

The edge labeling are,

$$f^*[(uu_i)] = 1, \quad 1 \leq i \leq n$$

$$f^*[(u_i v_i)] = 0, \quad 1 \leq i \leq n$$

Here , $ef(1) = ef(0)$

Hence $\langle K_{1,n}; n \rangle$ is Satisfies the condition $|ef(0) - ef(1)| \leq 1$

Therefore , $\langle K_{1,n}; n \rangle$ is a Extended Mean Cordial Graph.

For example, $\langle K_{1,4}; n \rangle$ is a Extended Mean Cordial Graph as shown in the figure 3.8

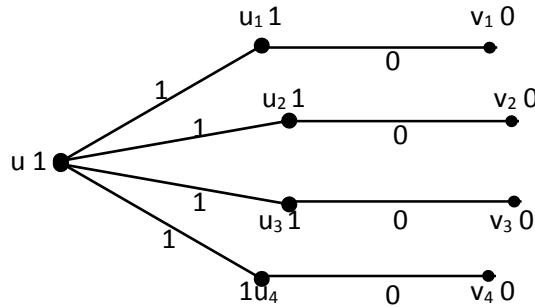


Figure 3.8:

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