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A STUDY ON POLYNOMIAL TERNARY SEMI RING

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ABSTRACT

In this paper we discussed about the polynomial ternary semiring and characterized them.

Mathematical Subject Classification : 16Y30, 16Y99

Keywords:
Polynomial Ternary Semiring, Commutative polynomial ternary semiring, Quasi Commutative polynomial ternary Semiring, Normal, pseudo Commutative.

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1. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like. The theory of ternary algebraic systems was introduced by D. H. Lehmer [2]. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary groups. D. Madhusudhana Rao[3] characterized the primary ideals in ternary semigroups. About T. K. Dutta and S. Kar [1] introduced and studied some properties of ternary semirings which is a generalization of ternary rings. Our main purpose in this paper is to introduce the notion of some special class of ternary semirings.

2. PRELIMINARIES

Definition 2.1 : A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a ***ternary semiring*** if T is an additive commutative semigroup satisfying the following conditions :
i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,
ii) $[(a + b)cd] = [acd] + [bcd]$,
iii) $[a(b + c)d] = [abd] + [acd]$,
iv) $[ab(c + d)] = [abc] + [abd]$ for all $a; b; c; d; e \in$ T.

Throughout T will denote a ternary semiring unless otherwise stated.
Note 2.2 : For the convenience we write $x_1x_2x_3$ instead of $\left[x_1x_2x_3\right]$
Note 2.3 : Let T be a ternary semiring. If A,B and C are three subsets of T , we shall denote the set ABC = $\left\{\Sigma abc : a \in A, b \in B, c \in C\right\}$.
Note 2.4 : Let T be a ternary semiring. If A,B are two subsets of T , we shall denote the set A + B = $\left\{a+b : a \in A, b \in B\right\}$.
Note 2.5 : Any semiring can be reduced to a ternary semiring.
Example 2.6 : Let T be an semigroup of all $m \times n$ matrices over the set of all non negative rational numbers. Then T is a ternary semiring with matrix multiplication as the ternary operation.

3. MAIN RESULTS

Definition 3.1: Let T, be a Ternary Semi ring. The set P[x] of all polynomials over a Ternary semi ring, T is said to be ***Polynomial Ternary Semiring*(denoted by** $P_T[x]$) with respect to addition + and Ternary multiplication [ ] of the polynomials provided
$[(fgh)ij] = [f(ghi)j] = [fg(hij)]$
$[(f + g)\, hi] = [fhi] + [ghi]$
$[f(g + h)i] = [fgi] + [fhi]$
$[fg(h + i)] = [fgh] + [fgi]$
$(f + g) \in P[x],\ \forall f, g, h, i, j \in P[x]$.
Note 3.2: Let $P_T\left[x\right]$ be the polynomial ternary semiring, then the operations addition and multiplication on P[x] is defined as follows:.
For any $f, g, h \in P[x]$ where,
$f = f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$
$g = g(x) = b_0 + b_1x + bx^2 + \cdots + bx^n$
$h = h(x) = c_0 + cx + cx^2 + \cdots + cx^n$
Then, $f + g = e_0 + ex + e_2x^2 + \cdots + e_nx^n$, $e_0 = a_0 + b_0,\ e_1 = a_1 + b_1,\ e_2 = a_2 + b_2, + \cdots$
Also $f.g.h$ = f(x). g(x). h(x) = $d_0 + d_1x + d_2x^2 + \cdots + d_nx^n$, where $d_n = \sum_{i+j+k}(a_ib_jc_k)$
And also $d_1 = d_3 = \ldots + d_{2n-1} \neq 0$, $d_2 = d_4 = \ldots d_{2n} = 0$.
Example 3.3: The set of all polynomials over real numbers forms a Ternary polynomial semi ring $P_T[x]$ by using polynomial addition and Ternary multiplication.
Example 3.4: The set P[x] consisting of a single polynomial o[x] with binary operation defined 0+0 = 0 and Ternary multiplication defined by 0.0.0 = 0 is a Ternary polynomial semi ring, $P_T[x]$ this ring is called the Null polynomial ternary semi ring.

Definition 3.5: A ternary polynomial semi ring, $P_T[x]$ is said to be a ***Commutative polynomial Ternary semi ring*** if $[fgh] = [ghf] = [hfg] = [fhg] = [ghf] = [hgf]$, $\forall f, g, h \in P_T[x]$. Where,
$f = f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$
$g = g(x) = b_0 + b_1x + bx^2 + \cdots + bx^n$
$h = h(x) = c_0 + cx + cx^2 + \cdots + cx^n$
Then $\sum_{l+m+k=n}(a_lb_mc_k)$ =$\sum_{l+m+k=n}(b_mc_kl)$= $\sum_{l+m+k=n}(c_k\,a_l\,b_m)$ and also $\sum_{l+m+k=n}(a_lb_mc_k)$, $\sum_{l+m+k=n}(b_lc_ma_k)$, $\sum_{l+m+k=n}(c_l\,a_m\,b_k)$.
Example 3.6: A polynomial ternary semi ring $P_T[x]$ over the set of Complex numbers, C is a Commutative Ternary polynomial semi ring.

Definition 3.7: A polynomial ternary semi ring, $P_T[x]$ is said to be a ***Quasi Commutative polynomial ternary semi ring*** if $[fgh] = [g^nfh] = [ghf] = [h^ngf] = [hfg] = [f^nhg]$, $\forall f, g, h \in P_T[x]$ and n must be an odd natural number.
Example 3.8 : A polynomial ternary semi ring $P_T[x]$ over the set of Complex numbers, C is a Quasi Commutative Ternary polynomial semi ring.

Theorem 3.9: A polynomial ternary semi ring $P_T[x]$ is commutative iff it is quasi commutative Ternary polynomial semi ring.
Proof: Let $P_T[x]$ be a Commutative Ternary polynomial semi ring.
We have to show that $P_T[x]$ be a quasi-commutative Ternary polynomial semi ring.
for any f, g, h $\in P_T[x]$, where

$$f = f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$
$$g = g(x) = b_0 + b_1x + bx^2 + \cdots + bx^n$$
$$h = h(x) = c_0 + cx + cx^2 + \cdots + cx^n.$$

since $P_T[x]$ be a Commutative polynomial ternary semi ring by def.
$[fgh] = [ghf] = [hfg] = [fhg] = [ghf] = [hgf]$, $\forall f, g, h \in P_T[x]$
we can also observe, $[fgh] = [g^1hf] = [hfg] = [f^1hg] = [ghf] = [h^1gf]$
$[fgh] = [g^nhf] = [hfg] = [f^nhg] = [ghf] = [h^ngf]$ this true for every odd natural number, n.
Therefore, $P_T[x]$ is a quasi commutative Ternary polynomial semi ring.
Conversely suppose that $P_T[x]$ be a quasi commutative Ternary polynomial semi ring.
Now we have to verify $P_T[x]$ be a Commutative Ternary polynomial semi ring.
by the definition of quasi commutative $[fgh] = [ghf] = [hfg] = [f^nhg] = [g^nhf] = [i]$, for some odd natural n and $f, g, h \in P_T[x]$.
$[fgh] = [g^1hf] = [hfg] = [f^1hg] = [ghf] = [h^1gf]$, $\forall f, g, h \in P_T[x]$.
Therefore $P_T[x]$ be a commutative x polynomial ternary semi ring.

Definition 3.10: A polynomial ternary semi ring, $P_T[x]$ is said to be a ***normal polynomial ternary semi ring*** if $fg.P_T[x] = P_T[x].fg$, $\forall f, g \in P_T[x]$.
Example 3.11: A polynomial ternary semi ring, $P_T[x]$ over set of Complex numbers, C is a normal polynomial ternary semi ring.

Theorem 3.12. A polynomial ternary semi ring $P_T[x]$ is quasi commutative iff it is normal polynomial ternary semi ring.
Proof: Let $P_T[x]$ be a Commutative polynomial ternary semi ring.
by definition of quasi commutative for any $f, g, h \in P_T[x]$,
$[fgh] = [ghf] = [hfg] = [f^nhg] = [g^nhf] = [h^ngf]$, for some odd natural n.
let, $k \in fg.P_T[x] \Rightarrow k = [fgh]$, for some $h \in P_T[x]$.
since $P_T[x]$ is quasi then, $k = [fgh] = [h^ngf] = [hfg]$, where $h, f, g \in P_T[x]. fg$
therefore, $k \in fg.P_T[x] \Rightarrow k \in P_T[x]$. fg
therefore $fg.P_T[x]$ is contained in $P_T[x]. fg$
similarly we can show that $P_T[x].fg$ is contained in $fg.P_T[x]$.
Therefore $fg.P_T[x] = P_T[x].fg$, $\forall f, g \in P_T[x]$.
Conversely suppose that $P_T[x]$ is a normal Ternary polynomial semi ring.
We have to show $P_T[x]$ is quasi Ternary polynomial semi ring.
by the definition of normal for any $f, g, h \in P_T[x]$, then $fg.P_T[x] = P_T[x].fg$
$\Rightarrow f.g.h = h.f.g$ for any $h \in P_T[x] \Rightarrow f.g.h^n = h^n.f.g$ for any $h \in P_T[x]$, when n=1

⇒ This is true for any odd natural number *n*.
Therefore $P_T[x]$ is quasi polynomial ternary semi ring. Hence the theorem is proved.

Corollary 3.13: **Every commutative polynomial ternary semi ring is quasi commutative Ternary polynomial ring.**
Corollary 3.14: **Every quasi commutative polynomial ternary semi ring is normal commutative polynomial ternary semi ring.**

Definition 3.15: A polynomial ternary semi ring, $P_T[x]$ is said to be **left pseudo commutative polynomial ternary semi ring** if $[(fgh)ij] = [(ghf)ij] = [(hfg)ij] = [(fhg)ij] = [(gfh)ij] = [(hgf)ij]$, $\forall f, g, h, i, j \in P_T[x]$.
Example 3.16: A polynomial ternary semi ring, $P_T[x]$ over set of Complex numbers, C is left pseudo commutative Ternary semi ring.

Theorem 3.17: If $P_T[x]$ is a commutative polynomial ternary semi ring then $P_T[x]$ is left pseudo commutative polynomial ternary semi ring.
Proof: Let $P_T[x]$ is a commutative polynomial ternary semi ring.
By definition for any $f, g, h \in P_T[x] \Rightarrow [fgh\ ] = [ghf] = [hfg] = [fhg] = [ghf] = [hgf]$
⇒ for any $i, j \in P_T[x]$, $[(fgh)ij] = [(ghf)ij] = [(hfg)ij] = [(fhg)ij] = [(gfh)ij] = [(hgf)ij]$.
Therefore $P_T[x]$ is left pseudo commutative Ternary semi ring.
Note 3.18: **The converse of the above theorem is need not be true.**

Definition 3.19 : A polynomial ternary semi ring, $P_T[x]$ is said to be **lateral pseudo commutative polynomial ternary semi ring** if $[(f(ghi)j] = [f(hig)j] = [f(igh)j] = [(f(gih)j] = [f(h\ gi)j] = [f(ihg)j]$, $\forall f,, g, h, I, j \in P_T[x]$.

Definition 3.20: A polynomial ternary semi ring, $P_T[x]$ is said to be **right pseudo commutative polynomial ternary semi ring** if $[fg(hij)] = [fg(ijh)] = [fg(jhi)] = [fg(hji)] = [fg(ihj)] = [fg(jih)]$, $\forall f, g, h, I, j \in P_T[x]$.

Theorem 3.21. **If $P_T[x]$ is a commutative polynomial ternary semi ring then it is lateral pseudo commutative polynomial ternary semi ring**.
Proof: by the definition of commutative polynomial ternary semi ring, for any $f, g, h \in P_T[x]$
$[fgh] = [ghf] = [hfg] = [fhg] = [ghf] = [hgf]$.
For any $i, j \in P_T[x]$, $\Rightarrow [i(fgh)j] = [i(ghf)j] = [i(hfg)j] = [i(fhg)j] = [i(gfh)j] = [i(hgf)j]$
⇒ $P_T[x]$ is lateral pseudo commutative polynomial ternary semi ring.

Corollary 3.22: **If P_T[x] is a commutative Ternary polynomial semi ring then it is left and right pseudo commutative Ternary semi ring.**
Corollary 3.23: **The converse of the above theorem is need not be true.**

Definition 3.24: A polynomial ternary semi ring, $P_T[x]$ is said to be **pseudo commutative polynomial ternary semi ring** if it is left and right pseudo commutative polynomial ternary semi ring.

Theorem 3.25: **Every Ternary polynomial semi ring, $P_T[x]$ is pseudo commutative Ternary semi ring.**
Proof: By theorem 3.17, theorem 3.21, and corollary 3.22, a polynomial ternary semi ring, $P_T[x]$ is left, right and lateral pseudo polynomial ternary semi ring. Hence the theorem.

Definition 3.26: if $P_T[x]$, is a polynomial ternary semi ring, and $S_T[x]$ is a non-empty sub set of $P_T[x]$ then $S_T[x]$ is said to be a **polynomial sub ternary semi ring** if $S_T[x]$ is a polynomial ring under the addition and Ternary multiplication defined on $P_T[x]$.
Example 3.27: $S_Z[x]$, the set of polynomials over the set of integers is a polynomial sub ternary semi ring of $P_R[x]$, the polynomial sub ternary semi ring over the set of real numbers.

Theorem 3.28: The non-empty intersection of two polynomial sub ternary semi ring is again a polynomial ternary sub semi ring.
Proof: Let $S_P[x]$ and $S_Q[x]$ be two polynomial sub ternary semi ring of $P_T[x]$, where P and Q are ternary sub semiring of T.
For any $f, g, h \in S_P[x] \cap S_Q[x] \Rightarrow f, g, h \in S_P[x]$ and $f, g, h \in S_Q[x]$.
Since $S_P[x]$ and $S_Q[x]$ are polynomial sub ternary semi rings then
$f + g \in S_P[x]$ and $f + g \in S_Q[x] \Rightarrow f + g \in S_P[x] \cap S_Q[x]$
and also by ternary multiplication, $f.g.h \in S_P[x]$ and $f.g.h \in S_Q[x] \Rightarrow f.g.h \in S_P[x] \cap S_Q[x]$
Therefore $S_P[x] \cap S_Q[x]$ is a polynomial sub ternary semi ring.

Definition 3.29: Let $S_T[x]$ is a non-empty sub set of a polynomial ternary semi ring of $P_T[x]$. The smallest polynomial ternary sub semi ring of $P_T[x]$ containing $S_T[x]$ is called a **polynomial sub ternary semiring of $P_T[x]$ generated by $S_T[x]$**. It is denoted by $< S_T[x] >$.
Theorem 3.30: **$P_T[x]$ is a polynomial ternary semi ring and $S_T[x]$ is a polynomial sub ternary semi ring of $P_T[x]$ then $< S_T[x] > = \{f_1.f_2.f_3.---.f_n \ / \ f_1, f_2, f_3, \dots f_n \ \in S_T[x]\}$.**
Proof: Let $< K_T[x] > = \{f_1.f_2.f_3.---f_n \ / \ f_1, f_2, f_3, \dots f_n \ \in S_T[x]\}$.
Let $r, s, t \in K_T[x]$, where

$$r = r(x) = r_1.r_2.r_3.---.r_n$$
$$s = s(x) = s_1.s_2.s_3.---.s_n$$
$$t = t(x) = t_1.t_2.t_3.---.t_n \ .$$

where $r_1.r_2.r_3.---.r_n$, $s_1.s_2.s_3.---.s_n$, $t_1.t_2.t_3.---.t_n \in S_T[x]$
now $r.s.t = (r_1.r_2.r_3.---.r_n)(s_1.s_2.s_3.---.s_n)(t_1.t_2.t_3.---.t_n) \in K_T[x]$.
$\Rightarrow r.s.t \in K_T[x] \Rightarrow K_T[x]$ is a Ternary sub polynomial semi ring of $P_T[x]$.
Let $M_T[x]$ be a polynomial sub ternary semiring of $P_T[x]$ which containing $S_T[x]$.
let $r \in K_T[x] \Rightarrow r = r_1.r_2.r_3.---.r_n$, where $r_1, r_2, r_3, ---, r_n \in S_T[x]$.
since $r_1, r_2, r_3, ---, r_n \in S_T[x]$ and $S_T[x]$ is contained in $M_T[x]$,
then $r_1, r_2, r_3, ---, r_n \in M_T[x] \Rightarrow M_T[x]$ is a polynomial sub ternary semiring.
Therefore $K_T[x]$ is the smallest polynomial sub ternary semiring of $P_T[x]$ containing $S_T[x]$.
Hence $P_T[x]$ is a polynomial ternary semiring generated by a polynomial sub ternary semiring $S_T[x]$.

4. CONCLUSION

In this paper mainly we studied about Polynomial ternary semirings.

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5. ACKNOWLEDGMENTS

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