

# NUMERICAL ANALYSIS OF FLEXURAL STRENGTH AND DUCTILITY OF RU-NC COMPOSITE CIRCULAR COLUMNS

Zhizhou Bai <sup>1</sup>✉, John Chen <sup>2</sup>

<sup>1</sup> Assistant Professor, College of Civil Engineering, Tongji University, Shanghai, China & Honorary Research Associate, Department of Civil Engineering, The University of Hong Kong, Hong Kong, China

<sup>2</sup> Associate, Binnies Hong Kong Limited, Hong Kong, China & Honorary Research Associate, Department of Civil Engineering, The University of Hong Kong, Hong Kong, China



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Corresponding Author

Zhizhou Bai, [zzbai@tongji.edu.cn](mailto:zzbai@tongji.edu.cn)

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## ABSTRACT

To theoretically investigate the flexural strength and ductility of a novel composite column structure comprising a Reinforced Ultra-High Performance Concrete (RU) shell and a core Normal Concrete (NC), this study proposes a comprehensive full-range moment-curvature analysis framework. Based on the derived moment-curvature relationships, a comparative analysis is conducted between RU-NC composite circular columns and conventional NC circular columns. The results demonstrate that replacing the NC outer layer with a thin-walled UHPC shell leads to significant enhancements in both flexural strength and ductility performance. Key findings reveal that the flexural strength and ductility of both traditional NC and proposed composite RU-NC circular columns exhibit strong dependence on the applied compressive axial load level. Furthermore, these mechanical properties show a pronounced correlation with the compressive strength of UHPC. Specifically, both flexural strength and ductility are further influenced by the compressive strength of UHPC. As the UHPC compressive strength increases, these mechanical properties exhibit a marked improvement.

**Keywords:** Ultra-High-Performance Concrete, Full-Range Moment-Curvature Curve, Flexural Strength and Ductility

## 1. INTRODUCTION

Ultra-high Performance Concrete (UHPC), as an ultra-strong and durable cement-based material, is effectively capable to protect structures from harsh environmental erosion and extend their service life [Larrard and Sedran \(1994\)](#), [Richard and Cheyrezy \(1995\)](#). In comparison to traditional Normal Concrete (NC), UHPC demonstrates superior tensile properties, resulting in significantly improved cracking resistance and enhanced corrosion prevention capabilities.

Many studies on the material properties of UHPC have been performed. [Abrishambaf et al. \(2017\)](#) investigated the effect of steel fiber on the axial tensile properties of UHPC and developed a novel method for calculating the axial tensile strength of UHPC. [Shaikh et al. \(2020\)](#) found that when the water-to-binder ratio (W/B) is below 0.2, the optimal steel fiber content in UHPC ranges from 2% to 3%. [Wille et al. \(2011\)](#) successfully prepared UHPC with good fluidity and excellent performance by using low-heat cement and ultra-fine silica fume without the need for thermal curing.

UHPC is characterized by its significantly reduced porosity and highly compact microstructure, ensuring exceptional resistance to permeability due to its low W/B [Abbas et al. \(2015\)](#). A reduction of one order of magnitude in chloride ion diffusion coefficient is observed in UHPC when contrasted with NC. According to [Roux et al. \(1996\)](#), the chloride ion diffusion coefficients at a testing age of 1 year measured for C30, C80, and UHPC were  $1.1 \times 10^{-12}$ ,  $6.0 \times 10^{-12}$ , and  $0.2 \times 10^{-13}$  m<sup>2</sup>/s, respectively. The diffusion coefficient has also been found to depend on factors such as W/B, curing conditions, solution concentration, steel fiber volume, and testing age and ranges from  $0.2 \times 10^{-13}$  to  $0.41 \times 10^{-12}$  m<sup>2</sup>/s in various studies [Kravanja et al. \(2024\)](#).

In recent years, UHPC has been increasingly applied in the field of bridge engineering. The Sherbrooke Footbridge, erected in Sherbrooke, Canada, in 1997, stands as the world's first engineering marvel to be built using UHPC [Blais and Couture \(1999\)](#). In 2014, China's Hebei Province constructed the nation's first road bridge utilizing prestressed UHPC box girders [Chen et al. \(2021\)](#). The application of UHPC in bridge superstructures has become widespread across the globe now [Graybeal et al. \(2020\)](#).

Several advanced techniques utilizing UHPC have also been proposed for seismic retrofitting of existing columns and have been successfully implemented in a number of real-world applications. [Beschi et al. \(2011\)](#), [Doiron \(2016\)](#). In addition to the application of UHPC in strengthening existing columns, various scholars have also proposed the concept of utilizing UHPC shells during the design phase. This involves using the UHPC shell as both the formwork for the core NC and the permanent load-bearing structure, thereby forming a UHPC-NC composite column [Lin et al. \(2017\)](#).

[Lin et al. \(2017\)](#) proposed a 100 mm thick UHPC shell scheme without reinforcement, while the core NC was reinforced. This composite circular column is referred to as the U-RC composite column in their work. [Shan et al. \(2021\)](#) conducted experimental research on the hysteretic loading performance of U-RC composite circular columns with UHPC shells provided with spiral hoops, discovering that they may exhibit superior seismic performance compared to traditional NC columns.

The use of UHPC shells during the design phase enhances durability of columns in harsh environments and significantly reduces steel formwork requirements, eliminates the need for formwork removal, offering potential cost-saving benefits. [Huang et al. \(2022\)](#) proposed an improved scheme to translate the reinforcement cage from the core concrete to UHPC and is referred to as the RU-NC composite column. In contrast to the U-RC composite columns, the RU-NC column features a reinforced UHPC shell, while its core NC remains unreinforced. The experimental study on RU-NC composite circular columns indicates that it may even enhance the axial compression performance.

When reinforcing UHPC shells, the combination of steel reinforcement and UHPC can create a complementary effect, theoretically enhancing bending, compression, and shear resistance, as well as further improving crack resistance,

thereby offering greater advantages. Meanwhile, the circular shape is a very commonly used cross-sectional form for piers and columns. In response to this, this study develops a full-range moment-curvature analysis method for RU-NC composite columns. And the method was applied to analyze and assess the flexural strength and ductility of RU-NC composite circular columns. The results are compared with those of traditional NC circular columns, yielding some beneficial conclusions that can serve as a reference for related research and application for RU-NC composite circular columns.

## 2. CONSTITUTIVE MODELS OF MATERIALS

### 2.1. NC

The compressive behavior of NC follows the constitutive model by [Attard and Setunge \(1996\)](#). This model requires four primary parameters: (a) peak stress  $f_{co}$ , (b) its associated strain  $\varepsilon_{co}$ , and (c) two shape coefficients  $A$  and  $B$ . The model is expressed as

$$\frac{\sigma_c}{f_{co}} = \frac{Ax+Bx^2}{1+(A-2)x+(B+1)x^2}, \quad x = \frac{\varepsilon_c}{\varepsilon_{co}} \quad (1)$$

The parameters  $A$  and  $B$  are determined by

$$\begin{cases} A = \frac{E_c \varepsilon_{co}}{f_{co}}, B = \frac{(A-1)^2}{0.55} - 1 & x \leq 1 \\ A = \frac{f_{ci}(\varepsilon_{ci} - \varepsilon_{co})^2}{\varepsilon_{co} \varepsilon_{ci} (f_{co} - f_{ci})}, B = 0 & x > 1 \end{cases} \quad (2)$$

The parameters  $E_c$ ,  $\varepsilon_{co}$ ,  $f_{ci}$  and  $\varepsilon_{ci}$  are determined as

$$E_c = 4370(f_{co})^{0.52} \quad (3a)$$

$$\varepsilon_{co} = 4.11(f_{co})^{0.75} / E_c \quad (3b)$$

$$\frac{f_{ci}}{f_{co}} = 1.41 - 0.17 \ln(f_{co}) \quad (3c)$$

$$\frac{\varepsilon_{ci}}{\varepsilon_{co}} = 2.50 - 0.30 \ln(f_{co}) \quad (3d)$$

### 2.2. UHPC

The compressive behavior of UHPC follows the constitutive model by [Yang and Fang \(2009\)](#), where the stress  $\sigma_c$  is expressed as a function of normalized strain  $x = \frac{\varepsilon_c}{\varepsilon_{co}}$ . This model requires three key inputs: (a) peak stress  $f_{co}$ , (b) its associated strain  $\varepsilon_{co}$ , and (c) shape coefficient  $A$ . The model is expressed as

$$\frac{\sigma_{co}}{f_{co}} = \begin{cases} \frac{Ax-x^2}{1+(A-2)x} & x \leq 1 \\ \frac{x}{2(x-1)^2+x} & x > 1 \end{cases} \quad (4)$$

The parameters  $A$ ,  $E_c$  (in MPa) and  $\varepsilon_{co}$  are determined as

$$A = \frac{E_c \varepsilon_{co}}{f_{co}} \quad (5a)$$

$$E_c = \frac{10^3}{0.0172 + 0.8364 f_{co}} \quad (5b)$$

$$\varepsilon_{co} = (6.7264 f_{co} + 2460.9) 10^{-6} \quad (5c)$$

Figure 1 shows one typical stress–strain curve of NC with  $f_{co} = 30$  MPa, along with three typical stress–strain curves of UHPC with  $f_{co} = 100$ , 120 and 140 MPa respectively used in this study.

Figure 1

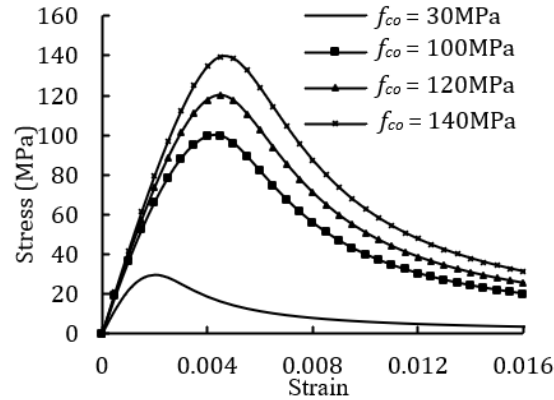


Figure 1 Typical Stress–Strain Curves of NC and UHPC

### 2.3. STEEL REINFORCEMENT

An idealized bilinear stress-strain model is applied to the steel reinforcement, where the stress  $\sigma_s$  relates to the strain  $\varepsilon_s$  as

$$\begin{cases} \sigma_s = E_s \varepsilon_s & \varepsilon_s \leq f_y / E_s \\ \sigma_s = f_y & \varepsilon_s > f_y / E_s \end{cases} \quad (6)$$

where  $E_s$  characterizes the elastic stiffness and  $f_y$  defines the yield plateau onset.

### 3. METHODS OF ANALYSIS

The analytical model begins with the application of a concentric axial load  $P$  (compression positive) prior to flexural loading. Consistent with conventional RC design practice and those findings by [Bai and Au \(2009\)](#), the tensile contributions of both NC and UHPC are excluded. The plane-section hypothesis enforces strain compatibility between steel and adjacent concrete. As can be seen in [Figure 2](#), for a cross-section subjected to curvature  $\phi$ , the longitudinal strain  $\varepsilon$  at

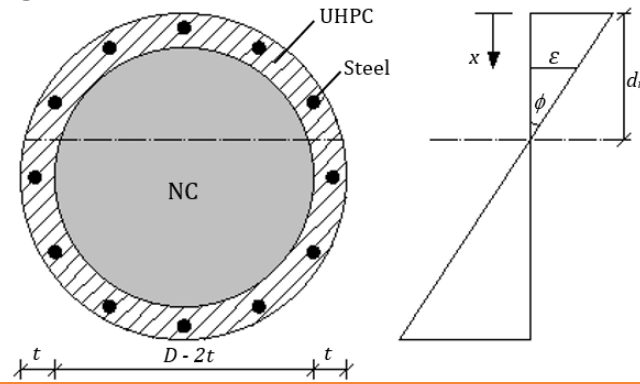
coordinate  $x$  relates to the neutral axis position  $d_n$  through the kinematic relationship as

$$\varepsilon = \phi(d_n - x) \quad (7)$$

Based on the established constitutive laws of the materials, the stress distributions in both concrete and steel reinforcement can be quantitatively determined. These internal stresses developed in the cross-section must collectively satisfy the fundamental equilibrium conditions, ensuring that their resultant forces equilibrate the external axial load and flexural moment about the geometric centroid.

To establish the moment-curvature response of a section, the neutral axis position must first be solved iteratively under deformation compatibility. For a prescribed curvature  $\phi$ , an initial estimate of the neutral axis depth  $d_n$  is typically non-equilibrium, leading to an unbalanced longitudinal force. Due to the nonlinear coupling between  $d_n$  and sectional forces, numerical iteration becomes essential to achieve force equilibrium. By progressively increasing  $\phi$  from zero until the moment capacity degrades substantially, the full-range moment-curvature relationship, including strain-softening regimes, can be traced.

**Figure 2**



**Figure 2** An RU-NC Composite Circular Column Section

## 4. OUTCOMES OF NUMERICAL ANALYSIS

### 4.1. CIRCULAR SECTIONS

The circular sections analysed are with a typical diameter  $D = 1000$  mm, with a UHPFRC shell thickness  $t = 100$  mm. The steel reinforcement, with a cover  $c = 50$  mm, has an elastic modulus  $E_s = 200$  GPa and yields at  $f_y = 400$  MPa. The  $f_{nc}$ , namely the in situ compressive strength of NC, is set at 30 MPa, while  $f_{uc}$ , namely that of UHPFRC, is considered for three cases: 100 MPa, 120 MPa and 140 MPa. Bai (2006) investigated size effects through a proposed normalization approach employing dimensionless parameters, enabling the extension of findings to geometrically scaled models with shared material and reinforcement properties. The current study specifically excludes discussion of confinement mechanisms. The reinforcement ratio  $\rho_s = \frac{A_s}{\pi D^2}$ , in which  $A_s$  is the total area of steel reinforcement, is considered for two cases: 1% and 2%. It is assumed that the steel reinforcement is uniformly and densely arranged along the same circumferential radius. The axial

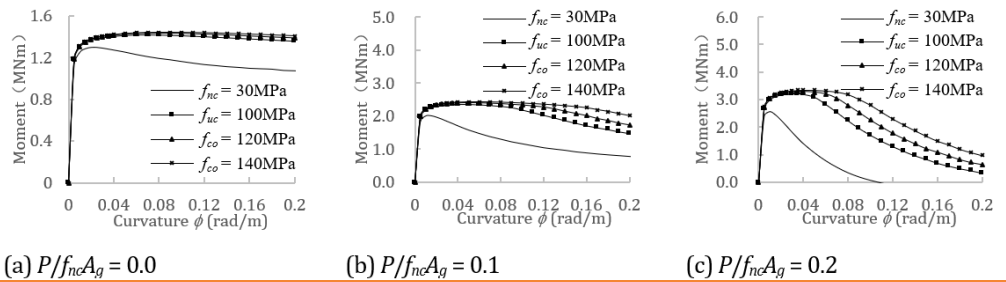
load ratio is defined as  $P/f_{nc}A_g$ , in which  $P$  is the resultant axial force and  $A_g$  is the gross cross-sectional area of the composite section.

#### 4.2. FULL-RANGE MOMENT-CURVATURE CURVES

As shown in Figure 3 with reinforcement ratio  $\rho_s = 1\%$ , the typical full-range moment-curvature response of RU-NC circular columns varies significantly with axial load levels. Under low axial load ratios, the post-peak stage exhibits a prolonged plateau, whereas higher axial loads lead to sharper peaks and reduced ductility. The axial force ratio predominantly influences the overall shape of the curves for both NC and RU-NC sections.

On the other hand, for a given axial force ratio, Figure 3 demonstrates the effect of replacing NC by UHPC on the full-range moment curvature response. It can be seen that UHPC not only helps to enhance peak resisting moment but also improve ductility, which is related with  $f_{nc}$ . Both flexural strength and ductility increase with  $f_{nc}$  increases.

**Figure 3**

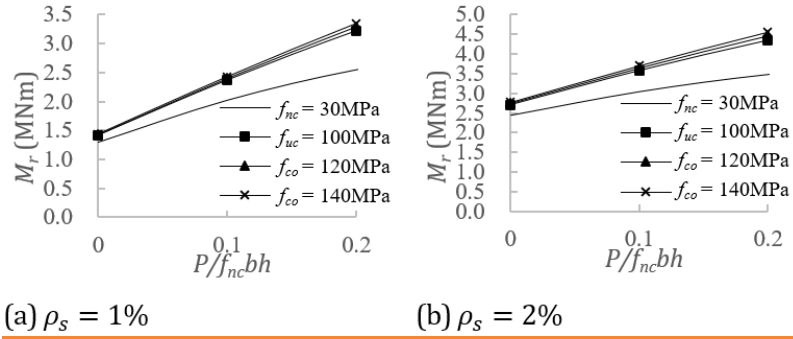


**Figure 3** Typical Full-Range Moment-Curvature Curves with  $\rho_s = 1\%$

#### 4.3. FLEXURAL STRENGTH AND DUCTILITY

Figure 4 illustrates the correlation between flexural strength  $M_r$  and axial load ratio  $P/f_{nc}A_g$ . It can be seen that the flexural strength  $M_r$  tends to increase with the compressive axial load. Obviously, the axial load ratio is an important parameter that determines not only the shape of moment-curvature curve but also the flexural strength. On the other hand, for a given axial force ratio, Figure 4 also demonstrates the effect of replacing NC by UHPC on the flexural strength.

It is clear that the flexural strength increases significantly when UHPC is adopted. The flexural strength ratio of an RU-NC circular column section to an NC circular column section for  $f_{uc}$  100MPa, 120 MPa and 140 MPa, respectively, is 1.099, 1.185 and 1.289 with axial load ratio  $P/f_{nc}A_g = 0.1$  and  $\rho_s = 1\%$ . Slightly higher flexural strength ratios can be obtained when the axial load ratio  $P/f_{nc}A_g$  or the reinforcement ratio  $\rho_s$  increases.

**Figure 4****Figure 4** Flexural Strength of RU-NC Circular Columns

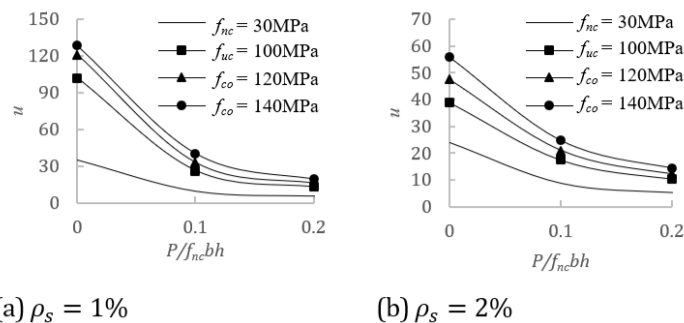
The flexural ductility factor  $\mu$  can be evaluated using ultimate curvature  $\phi_u$  and yield curvature  $\phi_y$  based on the full-range moment-curvature curve as

$$\mu = \phi_u / \phi_y \quad (8)$$

The ultimate curvature  $\phi_u$  is the section curvature corresponding to a resisting moment decayed to 85% of the peak moment. The yield curvature  $\phi_y$  is defined as the curvature at the idealized yield point of an equivalent elastic-perfectly plastic system, where the elastic stiffness matches the secant stiffness at 75% of the peak moment, and the yield moment equals the peak moment.

As shown in Figure 5, the correlation between flexural ductility  $\mu$  and axial load ratio  $P/f_{nc}Ag$  is presented. It can be seen that when the compressive axial load increases, the flexural ductility  $\mu$  generally reduces. Figure 5 also demonstrates the effect of replacing NC by UHPC. Notably, the use of UHPC significantly enhances flexural ductility.

The flexural ductility ratio of an RU-NC circular section to an NC circular section for  $f_{uc}$  100MPa, 120 MPa and 140 MPa, respectively, is 2.736, 3.454 and 4.243 with axial load ratio  $P/f_{nc}Ag = 0.1$  and  $\rho_s = 1\%$ . While that with axial load ratio  $P/f_{nc}Ag = 0.2$  and  $\rho_s = 1\%$ , respectively, is 2.337, 2.916 and 3.532. The flexural ductility ratio is relatively sensitive and decreases with axial load ratio  $P/f_{nc}Ag$ . Such value with axial load ratio  $P/f_{nc}Ag = 0.1$  and  $\rho_s = 2\%$ , respectively, is 2.001, 2.406 and 2.839. It seems that the flexural ductility ratio is also relatively sensitive and decreases with reinforcement ratio  $\rho_s$ .

**Figure 5****Figure 5** Flexural Ductility of RU-NC Circular Columns



## 5. CONCLUSION

Due to its exceptional performance, UHPC has witnessed increasingly widespread applications in recent years. UHPC is relatively expensive in material cost, resulting in significantly higher construction expenses for pure UHPC structures. Consequently, developing UHPC composite structures has emerged as a viable alternative solution.

This study investigates a novel UHPC composite circular column structure featuring reinforced UHPC thin-walled shells encasing NC cores, which is designed to achieve long service life in harsh environments. To evaluate the flexural performance of this innovative RU-NC composite circular column, a comprehensive full-range moment-curvature analysis procedure was developed and implemented for theoretical investigation.

Key findings demonstrate that the proposed RU-NC configuration not only enhances structural strength directly but also significantly improves ductility performance. These results substantiate the system's guaranteed seismic resistance capacity, suggesting its effectiveness for earthquake-resistance.

## CONFLICT OF INTERESTS

None.

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