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THE USE OF GOWIN'S V AND POLYA'S METHOD FOR SOLVING PROBLEMS IN COMPLEX ANALYSIS

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ABSTRACT

In this research article on the use of Gowin's V and Polya's problem-solving method, these strategies are presented as tools for solving problems in complex analysis. The study presents these strategies as direct help for students to understand the methodology of solving complex analysis problems. Similarly, each of these methods is also a teaching strategy to break down and simplify the abstract concepts of complex analysis. Both are pedagogical strategies well-utilized in simpler mathematics, but in this work, it is recommended that they be used for more advanced mathematics.

Keywords: Complex Analysis, Gowin's V Heuristic, Polya's Problem-Solving Method, Mathematical Strategies

1. INTRODUCTION

In the vast field of advanced mathematics, complex analysis represents a fascinating discipline that combines profound theoretical concepts with sophisticated analytical tools. Within this context, Gowin's V and Polya's method emerge as fundamental instruments for addressing problems of significant complexity, offering methodical strategies that transform seemingly insurmountable mathematical challenges into systematic and manageable processes.

Gowin's V, developed as an innovative epistemological tool, provides a conceptual framework that allows researchers and students to navigate the inherent complexity of mathematical problems. This methodology not only facilitates understanding of the cognitive processes involved in problem-solving but also offers a clear structure for decomposing and analyzing complex mathematical phenomena from multiple perspectives.

Polya's method, coined by Hungarian mathematician George Polya, has become a fundamental reference for systematic mathematical problem-solving. Its four fundamental stages - understanding the problem, designing a plan, executing the plan, and reviewing the solution - represent a universal guide that transcends the boundaries of different mathematical branches, becoming an indispensable tool for students, researchers, and professionals.

When these two approaches are integrated into the context of complex analysis, an extraordinary methodological synergy is produced. Gowin's V allows for a deep epistemological exploration of underlying concepts, while Polya's method provides an algorithmic structure for addressing specific problems. This combination enables a more holistic and rigorous approach to the most complex mathematical challenges.

The objective of this research is precisely to examine how Gowin's V and Polya's method can be applied complementarily in complex analysis, revealing not only the technical problem-solving strategies but also the cognitive and conceptual processes underlying advanced mathematical understanding. Through this study, we seek to provide an integrated perspective that enriches the understanding of these methods and their potential in solving complex mathematical problems.

2. PROBLEM STATEMENT

One of the mathematical branches that causes the greatest difficulty for students is complex analysis. One of the fundamental challenges lies in the profound understanding of complex variable functions and their behavior in the complex plane. The theory of analytic functions presents a unique mathematical richness that transcends the simplicity of real numbers, allowing exploration of phenomena and properties not observable in the real domain. Singularities, Cauchy's theorems, and conformal transformations represent conceptual pillars that reveal the intrinsic complexity of these functions, generating multiple questions about their nature and scope.

In this context, the problem statement focuses on unraveling the mechanisms governing the continuity, differentiability, and analyticity of complex functions, exploring how the Cauchy-Riemann conditions establish fundamental criteria for determining the regularity of such functions. The research seeks to understand how transformations in the complex plane can represent mappings that preserve angles and local structures, revealing a dynamic geometry that connects seemingly distant mathematical concepts, from function theory to applications in theoretical physics, quantum mechanics, and approximation theory.

González-Martínez (2019) in a study conducted at the University of Barcelona analyzed students' difficulties in interpreting abstract concepts of complex analysis. The study revealed that students have greater complexity in visualizing transformations in the complex plane, suggesting the need to implement computational tools and graphical representations that facilitate the geometric understanding of functions. Similarly, Chen & Wu (2021) developed in "Interactive Visualization Techniques for Complex Analysis" an innovative model using computational learning tools to simplify the understanding of advanced mathematical concepts. The research implemented interactive simulations that allowed students to manipulate complex functions in real-time, significantly improving the understanding of singularities, conformal mappings, and analytic behaviors.

Likewise, Rodríguez-Pérez (2018) in his doctoral thesis "Epistemological Obstacles in Complex Analysis" identified the main cognitive barriers faced by students. The qualitative study revealed that mathematical abstraction, lack of connection with geometric representations, and algebraic complexity are the main impediments to a deep understanding of complex functions. Similarly, Kimura & Nakamura (2020) presented in "New Pedagogical Approaches for Complex Analysis" a research that proposes teaching methodologies based on active learning and problem-solving. The methodology integrates digital tools, mathematical simulations, and multi-representational approaches to facilitate the understanding of abstract concepts.

Santos-Lima, F. (2017) in his research "Mental Processes in Understanding Complex Functions" explored the cognitive mechanisms involved in learning complex analysis. The neuroeducational study used neuroimaging techniques to understand how students process abstract mathematical information, revealing brain activation patterns associated with the understanding of complex transformations.

3. RESEARCH OBJECTIVE

The objective of the research is to present two pedagogical methods, namely Gowin's V and Polya's problem-solving method, as a pedagogical technique for solving complex analysis problems.

3.1. WHAT IS COMPLEX ANALYSIS

Complex analysis (also called the theory of functions of a complex variable, or infrequently Complex Calculus) is the branch of mathematics that partly investigates holomorphic functions, also called analytic functions. A function is holomorphic in an open region of the complex plane if it is defined in this region, takes complex values, and is differentiable at each point of this open region with continuous derivatives.

The fact that a complex function is differentiable in the complex sense (that it is holomorphic) has much stronger consequences than the usual differentiability in reals. For example, every holomorphic function can be represented as a power series in some open disk where the series converges to the function. If the power series converges over the entire complex plane, the function is said to be entire. A definition related to holomorphic function is analytic function: a complex function over the complexes that can be represented as a power series.

A fundamental result of complex analysis is that every holomorphic function also meets the definition of an analytic function, a situation different from what occurs with functions that only admit real values, since in the real case there are differentiable functions at a point that are not analytic at that point; this situation constitutes a crucial difference between differentiable functions with real values and differentiable functions with complex values. In particular, holomorphic functions are infinitely differentiable (since analytic functions are), a fact that is markedly different from what occurs in differentiable real functions. Most elementary functions such as polynomials, the exponential function, and trigonometric functions are holomorphic.

Complex analysis is one of the classic branches of mathematics that has its roots beyond the 19th century. The notable names in its development are Euler, Gauss, Riemann, Cauchy, Weierstrass, and many more in the 20th century. Traditionally, complex analysis, particularly the theory of conformal mappings, has many applications in engineering but is also widely used in analytic number theory. In modern times, it became popular thanks to the push of complex dynamics and fractal drawings, produced by the iteration of holomorphic functions, of which the most popular is the Mandelbrot set. Other important applications of complex analysis are in string theory, a quantum field theory that is conformal-invariant.

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4. WHAT IS THE GOWIN V METHOD

The Gowin V is a methodological tool developed by David Gowin and Bob Novak that represents an epistemic diagram for understanding and analyzing scientific knowledge construction. Visualized as a V, this tool conceptually divides two fundamental dimensions of the research process: the left side represents conceptual elements such as theories, principles, and theoretical frameworks, while the right side shows methodological aspects like records, transformations, and data collection methods. In its essence, the Gowin V seeks to reveal the interconnection between theoretical knowledge and research practice, allowing researchers and students to understand how scientific knowledge is generated and structured. Its value lies in its ability to break down complex processes, show the dialectical relationship between concepts and procedures, and facilitate a deeper understanding of how knowledge is built and validated in different fields of knowledge. As a didactic and analytical tool, the Gowin V promotes critical thinking, helps plan research, and serves as a conceptual map that guides both the generation and interpretation of scientific knowledge.

For his part, Campos-Silva, (2017) points out that the heuristic V is configured as a conceptual map that allows visualizing the epistemological structure of research, connecting theoretical and methodological elements which help students better understand complex analysis concepts. On the other hand, this methodology is greatly used in solving verbal problems together with Polya's method. Furthermore, Hernández-Mosqueda (2016) adds that the Gowin V transcends its initial role as a didactic tool, consolidating itself as a rigorous analysis method in qualitative and quantitative research by combining the theoretical and quantitative parts. Often in social sciences and education. In turn, researchers Triana-Vargas & Castaño-Vélez (2022) propose that the implementation of the Gowin V in digital environments allows a more dynamic integration between the conceptual and procedural components of knowledge.

The main objective of the article is to propose a structured model that facilitates students' understanding and resolution of physics problems. To this end, the Gowin V Diagram is used, which is a graphical representation that helps visualize the relationship between theoretical and empirical concepts in the problem-solving process. The V Diagram organizes knowledge into two main parts: the left part, which is related to context and fundamental concepts, and the right part, which focuses on applying those concepts to solve the problem.

The article describes how this tool can be used by both teachers and students to foster a more reflective and organized approach in the problem-solving process. Through the implementation of the V Diagram, students can identify and connect physical theories with experimental data and mathematical solutions, promoting a deeper understanding of concepts and improving their ability to solve problems effectively.

Additionally, it is noted that the Gowin V Diagram can be used to promote active learning, in which students develop critical and autonomous thinking skills, improving their ability to apply knowledge in practical and real situations. Some pedagogical applications and examples of how this tool can be used in the classroom are also discussed. The article concludes that implementing didactic tools based on the Gowin V Diagram can be beneficial for physics teaching, as it provides students with a clear and coherent structure to address complex problems and fosters a deeper understanding of physical principles through a visual and structured approach.

5. WHAT IS THE POLYA METHOD

The Polya method, developed by Hungarian mathematician George Polya, is a systematic strategy for solving mathematical problems that consists of four fundamental stages. This method is designed to guide students and professionals in solving problems in a structured and methodical manner, encouraging critical thinking and creativity in finding solutions. The four main stages are: understanding the problem, designing a plan, executing the plan, and performing a retrospective review. In the first stage, the aim is to completely understand the statement, identifying the data, unknowns, and problem conditions. In the second phase, potential strategies for solving the challenge are developed, selecting appropriate methods and organizing information. The third stage involves the concrete implementation of the plan, executing the previously designed steps with precision and care. Finally, in the review stage, the solution is verified, its coherence is checked, and reflection on the followed process is undertaken, which allows not only validating the result but also learning from the method used. The Polya method transcends the simple resolution of mathematical problems and has become a valuable tool for developing logical thinking, problem-solving in various fields, and fostering a systematic and reflective approach to intellectual challenges.

n the article "Problem Resolution in Mathematics", Rafael Bracho López (2003) addresses the methodology of solving mathematical problems from a practical and didactic perspective. The work focuses on deepening the strategies developed by George Pólya, with the main objective of transforming problem solving from an abstract activity to a systematic and understandable process for students.

The author analyzes in detail the four fundamental steps of Pólya's method: understanding the problem, designing strategies, executing the plan and verifying the results. His approach seeks to provide teachers with concrete tools to implement these strategies in the classroom, translating theoretical principles into effective didactic interventions.

The most significant contribution of the article lies in its ability to connect mathematical theory with pedagogical practice. Bracho López (2003) offers specific examples of application, facilitating teachers' understanding and implementation of techniques to develop students' mathematical thinking. Her work highlights the importance of seeing problem solving not as an end in itself, but as a process of developing cognitive and strategic skills.

The article proposes a methodology that goes beyond the simple mechanical resolution of exercises, promoting an approach that stimulates critical reasoning, creativity, and the analytical capacity of students. In doing so, Bracho López contributes significantly to the understanding of how mathematics can be taught more effectively and comprehensively.

5.1. HOW GOWING'S METHOD HELPS IN LEARNING COMPLEX ANALYSIS

In a study carried out by Lorena Salazar (2018) in which she approaches complex analysis through connections with real analysis in which she represents significant research in the field of mathematics didactics. The work focuses on establishing pedagogical connections between real analysis and complex analysis, seeking to develop strategies that facilitate the understanding of advanced mathematical concepts. Castillo-Garsow (2020) also addresses the conceptual difficulties that university students experience when understanding the complex number system. The study focuses on analyzing the cognitive processes, mental representations, and epistemological obstacles that students encounter when transitioning from real numbers to the complex number system. Similarly, Trigueros and Oktaç (2018) in their research "Construction of meanings for complex numbers at higher levels" explore how students at higher educational levels develop understanding and meaning of complex numbers. The research focuses on the cognitive processes involved in the interpretation and conceptualization of these numbers.

5.2. THE MAIN POINTS OF THE STUDY INCLUDE

Analysis of the difficulties that students face when understanding complex numbers, an abstract and complex mathematical concept.

Investigation of how students construct meanings and mental representations of complex numbers beyond their traditional algebraic representation.

Exploration of the different ways in which higher level students interpret and make sense of complex numbers in different mathematical contexts.

The study seeks to contribute to a better understanding of advanced mathematical learning processes, specifically in the area of complex numbers, by providing input on how students develop their conceptual understanding of this complex mathematical topic.

The author proposes an innovative approach that seeks to build conceptual bridges between two traditionally separately studied mathematical branches, with the aim of simplifying the learning and deep understanding of these complex mathematical disciplines, thus contributing to the development of more effective teaching methodologies in the university setting.

Gowin's V can be very useful in learning complex analysis by providing a structured framework for understanding and organizing mathematical knowledge. In the context of complex analysis, this tool helps students to:

Conceptual integration: It allows to visualize the connection between fundamental theoretical concepts of complex analysis (such as complex functions, limits, series, conformal transformations) and the mathematical procedures used to solve them.

Knowledge structure: It helps to break down the complexity of complex analysis into more manageable components, showing how theories relate to resolution methods.

Learning process: Facilitates deep understanding by showing the path from basic concepts to the practical application of theorems and methods.

Critical thinking: Encourages reflection on how mathematical knowledge is constructed, not just memorizing formulas but understanding their origin and meaning.

Problem-solving planning: Allows students to organize their approach to tackling complex problems, clearly identifying necessary concepts, theories, and methods.

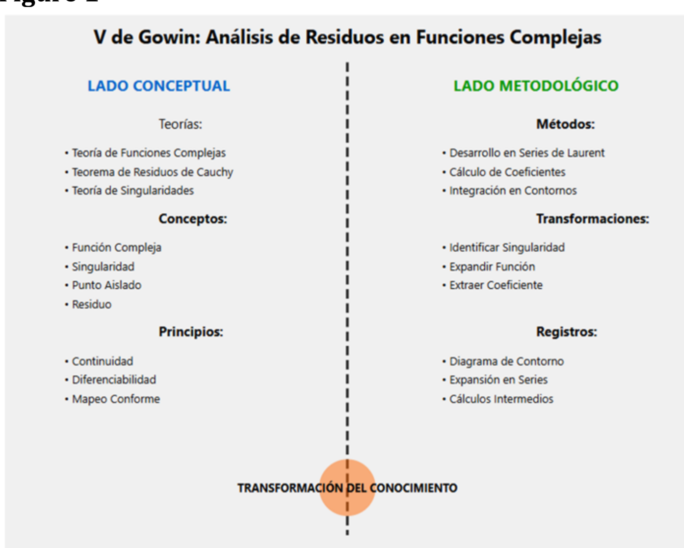
Theory-practice connection: Shows how theoretical concepts of complex analysis (such as the Riemann mapping theorem or Laurent series) translate into practical methods of solving.

In essence, Gowin's V acts as a conceptual map that helps students navigate the complexity of complex analysis, providing a structure that goes beyond simple memorization and promotes a deeper and more meaningful understanding of the discipline. Likewise, "Gowin's V is consolidated as a meta-cognitive instrument that transcends the simple collection of data, constituting a strategy for deep understanding of scientific knowledge" (Martínez-Jiménez (2019)). Also, Ruiz-Ortiz & González-García (2020) state that the adaptation of Gowin's V to digital contexts represents a methodological challenge that expands the traditional boundaries of knowledge construction.

For his part, Moreno-Armella, L. (2019) details in his research that the teaching of mathematics, the learning of abstract concepts, needs a certain structure in order to be understood. The author focuses on explaining a relevant approach, process, or theory, highlighting main findings, such as the importance of a pedagogical strategy or the results of a study. In addition, some particular aspect is discussed such as difficulties in teaching, implications for educational practice, or proposals for improvement.

For example, the diagram shows on its left side the conceptual part of complex analysis and on its right side all the methodological aspects of complex analysis.

Figure 1



How Polya's method helps in solving problems in complex analysis.

Polya's method is an extremely effective tool for solving problems in complex analysis. Below are five examples where Polya's method is used for the understanding and explanation of functions in complex analysis.

Solving Complex Integrals In the problem of calculating the integral $\oint C$ f(z) dz, where C is a closed contour surrounding the function $f(z) = 1/(z^2 + 1)$, Polya's method is applied. First, the problem is understood by identifying the singular points at $z = \pm i$. A plan is designed using the residue theorem, selecting a semicircular contour that includes these points. When executing the plan, the residues at z = i and z = -i are calculated, using the Laurent series expansion. The final result involves summing the residues multiplied by $2\pi i$, allowing the integral to be evaluated without direct integration calculations.

Singularity Analysis Considering the function f(z) = 1/(z(z-1)(z-2)), the Pólya method is used to understand the nature of its singularities. The plan includes identifying singular points at z = 0, z = 1, and z = 2. It is executed by developing Laurent series at each point, classifying the singularities: a simple pole at z = 0, another at z = 1, and a third simple pole at z = 2. Verification involves analyzing the behavior of the function in the neighborhood of each singularity, confirming its local and global properties.

Conformal Mappings By transforming the upper half-plane region $H = \{z \in C: Im(z) > 0\}$ using the function w = (z-i)/(z+i), Pólya is applied. The problem of mapping regions of the complex plane is understood. The plan involves selecting a function that preserves angles and analytic properties. The execution shows how this mapping transforms the upper half-plane into the unit disk, verifying conformity by preserving local angles and analyticity of the transformation.

Evaluating Complex Series For the series $\sum(n=1 \text{ to } \infty)\ z^n/n^2$, Pólya is used to determine its convergence. The problem is understood by analyzing the behavior of the series in different regions of the complex plane. The plan involves using the criterion of the radius of convergence. The execution calculates this radius by the limit of $|a_{n+1}/a_n|$, finding that the series converges for $|z| < 1$. The verification includes analyzing the behavior at the boundary of the convergence disk.

Fixed Point Theorem Considering the function $f(z) = z^2 + c$, where c is a complex constant, Pólya's method is applied to find fixed points. The problem is understood by defining a fixed point as z such that f(z) = z. The scheme uses the Banach fixed point theorem in metric spaces. The execution involves iterating the function f(z) starting from different initial values, observing the convergence or divergence towards fixed points or Julia sets, depending on c value.

6. GOWIN'S V METHOD IN COMPLEX ANALYSIS

Complex Integration The problem of evaluating the integral $\oint C\ z^2/(z^2 + 1)\ dz$ represents a mathematical challenge that Gowin's V structures systematically. The central event involves understanding the fundamentals of the residue theorem, identifying singularities at $z = \pm i$ as critical points. The transformations involve mapping the complex contour, applying principles of analyticity. The method focuses on computing the residues at these singular points, recording each step of the process. The fundamental principles of complex functions guide the solution, allowing the value of $2\pi i$ to be obtained as the final result.

Singularity Analysis The classification of singularities for the function f(z) = 1/(z(z-1)(z-2)) reveals the complexity of complex analysis. The main event seeks to understand the nature of singular points through algebraic transformations.

Laurent series concepts allow the function to be expanded around critical points, identifying simple poles at z = 0, z = 1, and z = 2. The method develops local series, recording the analytical behavior in each neighborhood. The principles of local behavior of complex functions guide the interpretation, resulting in an accurate classification of the singularities encountered.

Conformal Mapping The transformation from the upper half-plane to the unit disk by $w = (z-i)/(z+i)$ illustrates the depth of conformal mappings. The central event explores how complex functions can preserve geometric properties. The transformations analyze the complex function, applying methods that preserve local angles and structures. Conformal principles guide the analysis, recording each stage of the transformation. The method investigates mapping properties, resulting in a deep understanding of how complex functions can modify geometric regions while maintaining their fundamental characteristics.

Complex Series The convergence analysis for the series $\sum(n=1 \text{ to } \infty)\ z^n/n^2$ represents a characteristic problem of complex analysis. The main event explores the concept of radius of convergence through mathematical transformations. The principles of power series guide the quotient limit method, recording each step of the analysis. The transformations involve applying convergence criteria, exploring the asymptotic behavior of the series. The final result reveals the region of convergence, demonstrating how systematic methods can unravel complex properties of infinite series.

Fixed Point The search for a fixed point for $f(z) = z^2 + c$ illustrates the application of Gowin's V in complex dynamic problems. The central event involves understanding the iterative behavior of complex functions. The transformations recursively apply the function, recording each iteration. The principles of the fixed point theorem guide the method, exploring convergence and limiting behavior. The concepts of complex iteration allow us to analyze how different values of c generate different dynamic behaviors, resulting in a deep understanding of complex nonlinear systems.

7. CONCLUSIONS

Complex analysis, a mathematical branch of great depth and abstraction, finds in Gowin's V and Polya's method revolutionary pedagogical tools to transform its understanding. These methodologies represent more than simple resolution strategies; they constitute true epistemological frameworks that allow complex mathematical concepts to be broken down into systematic, understandable and meaningful processes for students.

Research reveals that complex analysis faces fundamental challenges in its teaching, mainly related to its high level of abstraction, the difficulty in visualizing transformations in the complex plane, and the disconnection between theoretical concepts and geometric representations. Faced with these obstacles, Gowin's V emerges as a fundamental tool that integrates conceptual and methodological aspects, facilitating the structuring of mathematical knowledge and fostering critical and reflective thinking.

Polya's method, on the other hand, offers a systematic approach to addressing complex mathematical problems. Its four-stage structure - understand, design, execute, and review - transcends the mere mechanical resolution of exercises, becoming a strategy that develops higher cognitive skills. This methodology not only allows for solving problems, but also educates in the very process of mathematical thinking, promoting creativity and logical reasoning.

The applications of these methodologies extend beyond complex analysis, finding resonance in fields as diverse as theoretical physics, quantum mechanics, string theory, and fractal dynamics. Their versatility suggests that they represent fundamental epistemic tools for understanding complex systems in multiple scientific disciplines.

The research also highlights the importance of emerging pedagogical trends, which integrate computational tools, interactive simulations and multi-representational approaches. These strategies seek to overcome the traditional limitations of mathematics teaching, offering more dynamic, visual and meaningful learning experiences.

From an epistemological perspective, Gowin's V and Polya's method are configured as metacognitive instruments that go beyond simple problem solving. They constitute strategies for the construction of scientific knowledge, developing in students logical, critical and creative thinking capacities that are fundamental in the training of mathematical researchers and professionals.

The recommendations derived from the study point towards a pedagogical transformation that integrates conceptual and procedural aspects, promotes the visualization of abstract concepts and uses innovative technological tools. The final objective is to evolve from a mathematics teaching model based on memorization to one based on deep and meaningful understanding.

In conclusion, the article presents these methodologies not as simple techniques, but as true philosophies of approaching mathematical knowledge. Their value lies in their ability to break down complexity, reveal hidden connections, and train students capable of creatively and rigorously navigating the challenges of advanced mathematical thinking.

CONFLICT OF INTERESTS

None.

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REFERENCES

Campos-Silva, E. (2017). Metacognitive Strategies in Social Research. Ibero-American Journal of Education, 75(2), 33-54.

Castillo-Garsow, C. (2020). Conceptual Understanding of Complex Numbers in University Students. International Journal of Mathematical Education in Science and Technology, 51(4), 567-589.

Chen, L., & Wu, Y. (2021). Interactive Visualization Techniques for Complex Analysis. International Journal of Educational Technologies, 45(3), 112-129.

González, F. J., & Pérez, J. L. (2013). Proposal for a Teaching Tool Based on Gowin's V for Solving Physics Problems. Brazilian Journal of Physics Teaching, 35(2), 2402. Retrieved from http://www.sbfisica.org.br

González-Martínez, R. (2019). Cognitive Strategies for Understanding Complex Functions [PhD thesis, University of Barcelona]. Institutional repository.

Hernández-Mosqueda, J. S. (2016). Qualitative Methods of Research Analysis. Current Research in Education, 16(3), 1-22.

Kimura, H., & Nakamura, S. (2020). New Pedagogical Approaches for Complex Analysis. Teaching Advanced Mathematics, 28(2), 75-94.

Martínez-Jiménez, P. (2019). Epistemological tools in Scientific Research. Journal of Education XXI, 22(3), 45-67.

Rodríguez-Pérez, M. (2018). Epistemological Obstacles in Complex Analysis [PhD thesis, National Autonomous University of Mexico]. Digital Archive of Mathematical Research.

Ruiz-Ortiz, M. A., & González-García, J. (2020). Heuristic Methodologies in Digital Environments. Pixel-Bit. Journal of Media and Education, 18(4), 89-110.

Salazar S., L. (2018). Approaching Complex Analysis Through Connections with Real Analysis. Mathematics, Education and Internet, 18(2), 1-15. https://tecdigital.tec.ac.cr/revistamatematica/

Santos-Lima, F. (2017). Mental Processes in the understanding of Complex Functions [PhD thesis, Federal University of Rio de Janeiro]. Database of Research in Mathematical Neuroeducation.

Triana-Vargas, O. E., & Castaño-Vélez, N. M. (2022). Virtual Learning Environments in Higher Education. Espacios Magazine, 43(6), 112-125.

Trigueros, M., & Oktaç, A. (2018). Construction of Meanings for Complex Numbers at Higher Levels. Mathematics Education.