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THE CORDIAL LABELING FOR THE CARTESIAN PRODUCT BETWEEN PATHS AND CYCLES

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Abstract

A graph is said to be cordial if it has a 0-1 labeling that satisfies certain properties. In this paper we show the Cartesian product of a path and a cycle or vice versa are always cordial under some conditions. Also, we prove that the Cartesian product of two paths is cordial.

Keywords: Cartesian Product; Cordial; Cycle; Graph; Labeling; Path.

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1. Introduction

It is known that graph theory and its branches have become interest topics for almost all fields of mathematics and also other area of science such as chemistry, biology, physics, communication, economics, and computer science. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. There are many contributions and different kinds of labeling [6-10,11]. Suppose that $G = (V, E)$ is a graph, where V is the set of its vertices and E is the set of its edges Throughout, it is assumed G is connected, finite, simple and undirected. A binary vertex labeling of G is a mapping $f : V \longrightarrow \{0,1\}$ in which $f(u)$ is said to be the labeling of $u \in V$ .For an edge $e = uv \in E$,where $u, v \in V$, the introduced edge labeling $f^*:E \longrightarrow \{0,1\}$ is defined by the formula$f^*(uw) = |f(v) + f(w)|\ (mod\ 2)$.Thus,for any edge e, $f^*(e) = 0$ if its two vertices have the same labeling and $f^*(e) = 1$ if they have different labeling.Let us denote V_0 and V_1 be the numbers of vertices labeled by 0 and 1in V respectively,and let E_0 and E_1 be the corresponding numbers of edges in E labeled by 0 and 1 respectively .A binary vertex labeling $f$of G is said to be cordial if $|V_0$ - $V_1| \leq 1$ and $|E_0 - E_1| \leq 1$.A graph G is cordial if it has cordial labeling . The cordial graphs were introduced by Cahit [1] as a weaker version of both graceful and harmonious graphs [7-9]. A lot of operations on graphs have been applied on the cordial

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labeling[2-5].The Cartesian product of two graphs is that Boolean operation $G = G_1 \times G_2$ in which for any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$, the line uv is in X whenever $[u_1 = v_1 and\, u_2v_2 \in X_2$] or [$u_2 = v_2$ and $u_1v_1 \in X_1$]where X,X_1 ,X_2 are the sets of edges of G, $G_1 and\ G_2$respectively [12]. It follows from the definition of the Cartesian product that the graph $G_1 \times G_2$) has $n_1.n_2$vertices and $n_1m_2 + n_2m_1$ edges where G_1 has n_1, m_1 $vertices\ and\ edges\ and G_2$ has n_2, m_2 vertices and edges. In this paper we show that $P_n \times C_m$,$C_n \times P_m$and $P_n \times P_m$ are cordial for some$n$ and m.

2. Terminologies and Notation

A path with n vertices and $n - 1$ edges is denoted by P_n ,and a cycle with n vertices and n edges is denoted by C_n .Given a path or a cycle with $4r$ vertices ,we let L_{4r} denote the labeling 0011 0011…0011(repeated r-times).In most cases ,we modify this by adding symbols at one end or the other (or both);thus $010L_{4r}$ denotes the labeling 010 00110011 ... 0011 of the path P_{4r+3}(or the cycle C_{4r+3})when $r \geq 1$ and 010 when $r = 0$,and so on [3-6].We let O_r denote the labeling 00 ... 0(zero repeated r times), 1_r denote the labeling 11 ... 1 (one repeated r times) ,$M_r$denotes to the labeling 0101…01if r is even and 0101 ... 010if r is odd . The Cartesian product of two graphs is that Boolean operation $G = G_1 \times G_2$) in which for any two points u =(u_1,u_2) and v =(v_1,v_2), the edgeuv is in X whenever [$u_1 = v_1\ and\, u_2v_2 \in X_2$] or [$u_2 = v_2$ and $u_1v_1 \in X_1$] where X, X_1 ,X_2 are the sets of edges of G, $G_1 and\ G_2$respectively [12].See Figure 2.1. It follows from the definition of the Cartesian product that the graph$G_1 \times G_2$ has $n_1n_2$vertices and $n_1m_2 + n_2m_1$ edges ,whereG_1 has n_1, m_1vertices and edges, G_2has n_2, m_2 vertices and edges.It is easy to see that $G_1 \times G_2$ is not isomorphic to $G_2 \times G_1$.

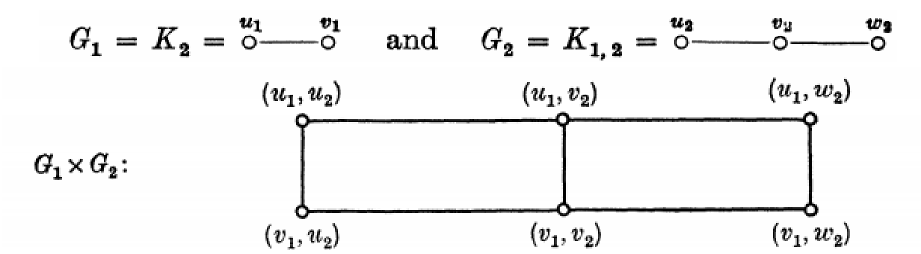

Figure 2.1.

3. The Cordiality of The Product of a Path and a Cycle

In this section, we show that the Cartesian product graphs $P_n \times C_m$is cordial for some n.

Lemma 3.1: The graph $P_n \times C_n$ is cordial if n is an even number.

Proof: We shall study two cases for $n \equiv i(mod\ 4)$ and $i = 0$,2.Suppose that $n = 2s$
Case (1): At $i = 0$ we choose the following labeling:
The[1^{st},3^{rd},5^{th},...,$(2s-3)^{th}$]rows are labeled byO_{2s} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s-2)^{th}$]rows are labeled by1_{2s} .
The $(2s-1)^{th}$row is labeled by M_{2s} .

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The $(2s)^{th}$row is labeled by L_{2s}.In this case:
$V_0 = V_1$ =$[(2s)(s-1)] + [\ s + s] = \ 2s^2$,
E_0 =the vertical edges plus the horizontal edges $= [s+s]\ + [2s(2s-2)\ + (s)\ ]\ = s(4s-1)$
E_1 =the vertical edges plus the horizontal edges $= [(2s)(2s-3) + s + (s)] + [2s\ + s] = s(4s-1)$.Therefore$|E_0 - E_1| = 0$ and$|v_1 - v_1| = 0$which satisfies the cordiality conditions

Case (2): For$i = 2$we can apply the following labeling:
The[1^{st}, 3^{rd},5^{th},...,$(2s-3)^{\text{th}}$]rows are labeled by O_{2s} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s-2)^{\text{th}}$]rows are labeled by1_{2s} .
The $(2s-1)^{th}$ row is labeled by M_{2s} .
The $(2s)^{th}$ row is labeled by $L_{2s-2}10$ .In this situation:
$V_0 = V_1 = [(2s)(s-1)] + [\ s + s] = 2s^2$,
E_0 =thehorizontal edges plus thevertical edges $= [2s(2s-2)\ + (s+1)\ ]\ + [(s-1) + s]\ = s(4s-1)$
E_1 =thehorizontal edges plus thevertical edges $= [2s\ + (s-1)\ ]\ + [(2s)(2s-3) + s\ + (s+1)] =\ s(4s-1)$Therefore,$|E_0 - E_1| = 0$,$|v_1 - v_1| = 0$so $P_n \times C_n$ where n is even is cordial as we wished to show. The following examples demonstrate some examples for the previous cases:

$$P_4 \times C_4\ \ (for\ s = 2)\ v_0 = v_1 = 8\ \ , E_0 = E_1 = 14\ \ i.e\ |\ E_0 - E_1| = 0,\ \ |v_1 - v_1| = 0\ .$$

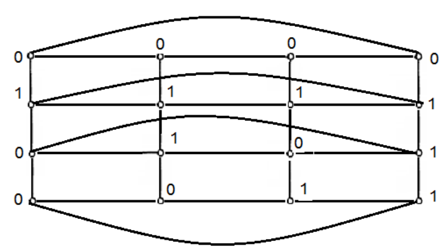

Figure 3.1.

$P_6 \times C_6$(for $s = 3$)

$$v_0 = v_1 =\ \ 18\ , E_0 = E_1 =\ 33\ , i.e.|E_0 - E_1| = 0,\ \ |v_0 - v_1| = 0.$$

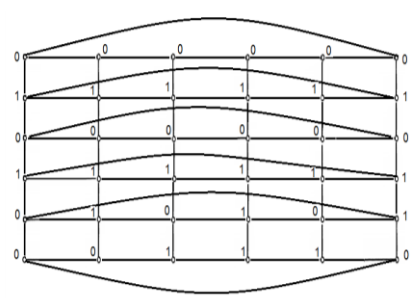

Figure 3.2.

Lemma 3.2: The graph $P_n{\times}C_m$ is cordial for all n is even and m is odd such that $n - m = -1$ also it is cordial for all n is odd and m is even which satisfy $n - m =\ 1$.

Proof: We divide the proof into two cases:

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Case (1): Suppose that $n = 2s$ and$m = 2s + 1$that satisfy the previous condition i.e. $n - m = -1$. Then we can use the following labeling:
The[1^{st}, 3^{rd},5^{th},...,$(2s-3)^{th}$] rows are labeled byO_{2s+1} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s-2)^{th}$] rows are labeled by1_{2s+1} .
The $(2s-1)^{th}$row is labeled byM_{2r+1} .
The $(2s)^{th}$ row is labeled by$M_{2r}1$ .Then we will get : $v_0 = v_1 = [(2s+1)(s-1)] + [s+1+s] = s(2s+1)$,
E_0 =the vertical edges plus the horizontal edges =[s+2s] +$[(2s+1)(2s-2) + (1+1)] = s(4s+1)$
E_1 =the vertical edges plus the horizontal edges= $[s(2s-3) + (2s-1) + s(2s-2)] = s(4s+1) - 1$.

i.e. $|E_0 - E_1| = 1$, $|v_0 - v_1| = 0$ so $P_{2s} \times C_{2s+1}$ is cordial as we wanted to show
For example:$(for\ s = 2) P_4 \times C_5$
$v_0 = v_1 = 10$, $E_0 = 18, E_1 = 17$,i.e.$|E_0 - E_1| = 1$, $|v_0 - v_1| = 0$

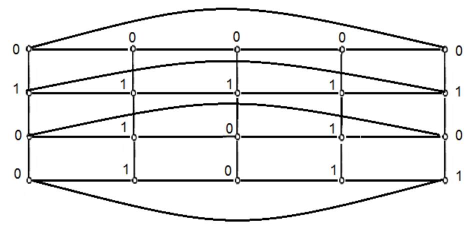

Figure 3.3.

Case (2): Suppose that $n = 2s + 1$ and $m = 2s$**.**It is obvious that $n - m = 1$.

By using the following labeling:
The[1^{st}, 3^{rd},5^{th},...,$(2s-1)^{th}$] rows are labeled byO_{2s} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s)^{th}$] rows are labeled by1_{2s} .
The $[(2s+1)^{th}$ row is labeled by M_{2s} .Therefore,

$$v_0 = v_1 = [(2s)(s)] + [s] = s(2s+1) ,$$

E_0 =the vertical edges plus the horizontal edges = $[s] + [(2s)(2s)] = s(4s+1)$.
E_1 =the vertical edges plus the horizontal edges = $[s(2s) + s(2s-1)] + [2s] = s(4s+1)$.
i.e. $|E_0 - E_1| = 1$, $|v_0 - v_1| = 0$ so $P_{2s+1} \times C_{2s}$ is cordial as we wanted to show.
For example,**:** (for $s = 2) P_5 \times C_4$
$v_0 = v_1 = 10$, $E_0 = E_1 = 18$,$|E_0 - E_1| = 1$, $|v_0 - v_1| = 0$.

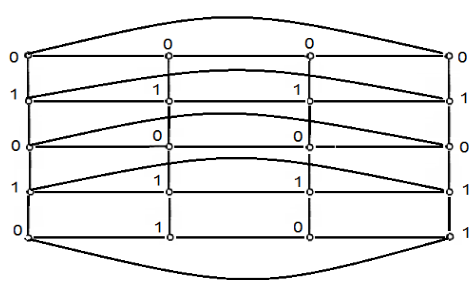

Figure 3.4.

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Lemma 3.3: The graph $P_n \times C_n$ is cordial if n is an odd number.

Proof: Suppose that $n = 2s + 1$.Then one can take the following labeling:
The$[1^{st}, 3^{rd}, 5^{th}, ..., (2s-1)^{th}]$ rows are labeled byO_{2s+1} .
The $[2^{nd}, 4^{th}, 6^{th}, ..., (2s)^{th}]$ rows are labeled by1_{2s+1} .
The$(2s+1)^{th}$ row is labeled by M_{2s+1} .Therefore,
$v_0 = [(2s+1)(s)] + [s+1] = 2s(s+1) + 1, v1 = [(2s+1)(s)] + [s] = 2s(s+1)$,
E_0 =the vertical edges plus the horizontal edges $= [s] + [2s(2s+1) + 1] = s(4s+3) + 1$.
E_1 =the vertical edges plus the horizontal edges $= [(s+1) + (2s+1)(2s-1)] + [2s] = s(4s+3)$.

i.e.$|E_0 - E_1| = 1$, $|v_0 - v_1| = 1$ so $P_{2s+1} \times C_{2s+1}$is cordial as we wanted to prove.
For example: (for s=2) $P_5 \times C_5$
$v_0 = 13$, $v_1 = 12$, $E_0 = 23$, $E_1 = 22$ $i.e.$ $|E_0 - E_1| = 1$, $|v_0 - v_1| = 1$.

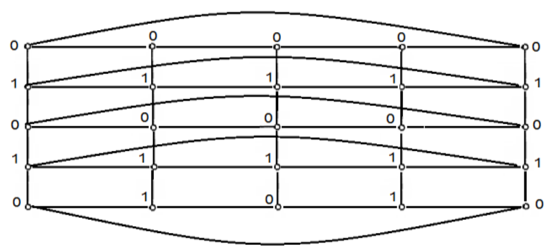

Figure 3.5.

Now, from the previous lemmas we have the following conclusion:

Theorem 3.1: The Cartesian product graph $P_{2s+i} \times C_{2s+j}$is cordial for all i and j where ($i, j = 0,1$).

4. The Cartesian Product of Two Paths

In this section, we study the cordiality of Cartesian product between two paths.

Lemma 4.1: The graph $P_n \times P_n$ is cordial if n is an even number.

Proof: Suppose that $n = 2s$, then we choose the following labeling:
The$[1^{st}, 3^{rd}, 5^{th}, ..., (2s-1)^{th}]$ rows are labeled by O_{2s} .
The $[2^{nd}, 4^{th}, 6^{th}, ..., (2s)^{th}]$ rows are labeled by 1_{2s} .Then we get : $v_0 = v_1 = s(2s)$,
E_0 =the vertical edges plus the horizontal edges$= 0 + [2s(2s-1)] = 2s(2s-1)$
E_1 =the vertical edges plus the horizontal edges$= [2s(2s-1)] + 0 = 2s(2s-1)$
i.e. . $|E_0 - E_1| = 0$, $|v_0 - v_1| = 0$. So, $P_{2s} \times P_{2s}$is cordial as we wanted to show.

For example , $P_4 \times P_4$:

$$v_0 = v_1 = 8, E_0 = E_1 = 12 \ , i.e. \ |E_0 - E_1| = 0, \ \ |v_0 - v_1| = 0 \ .$$

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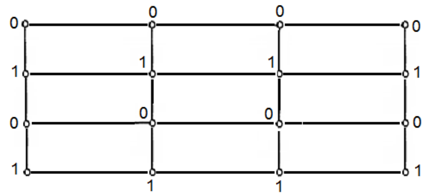

Figure 4.1.

Lemma 4.2: The graph $P_n \times P_m$is cordial for all n, where n is even and m is odd such that $n - m = -1$ also it is cordial for all n is odd and m is even such that$n - m = 1$.

Proof: We divide the proof into two cases:

Case (1): Suppose that $n = 2s$ and $m = 2s + 1$which satisfy the previous condition $n - m = -1$ .Therefore, we can use the following labeling**:**
The[1^{st}, 3^{rd},5^{th} ,...,$(2s - 1)^{\text{th}}$] rows are labeled by O_{2s+1} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s)^{\text{th}}$]rows are labeled by 1_{2s+1} .In this case **:**
$v_0 = v_1 = s(2s + 1)$,
E_0 = the vertical edges plus the horizontal edges= $[0] + [2s(2s)] = 4s^2$,
E_1 =the vertical edges plus the horizontal edges= $[(2s + 1)(2s - 1)] + 0 = (2s + 1)(2s - 1)$, i.e. $|E_0 - E_1| = 1,\ |v_0 - v_1| = 0$ so $P_{2s} \times P_{2s+1}$is cordial as we wanted to show

For example $P_4 \times P_5$**:**

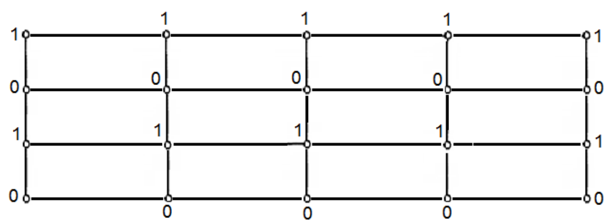

Figure 4.2.

Case (2): Suppose that $n = 2s + 1$ and $m = 2s$ **.**It is obvious that $n - m = 1$.Therefore, we use the following labeling:
The[1^{st}, 3^{rd},5^{th} ,...,$(2s - 1)^{\text{th}}$] columns are labeled by O_{2s+1} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s)^{\text{th}}$] columns are labeled by 1_{2s+1} . In this case:
$v_0 = v_1 = s(2s + 1)$,
E_0 =the vertical edges plus the horizontal edges= $[2s(2s)] + 0 = (4s^2)$,
E_1 =the vertical edges plus the horizontal edges= $0 + [(2s + 1)(2s - 1)] = (2s + 1)(2s - 1)$,i.e.$|E_0 - E_1| = 1,\ |v_0 - v_1| = 0$ so $P_{2s+1} \times P_{2s}$is cordial as we wanted to show .

For example $P_5 \times P_4$**:**

$$v_0 = v_1 = 10, \quad E_0 = 16, E_1 = 15, i.e. |E_0 - E_1| = 1, \quad |v_0 - v_1| = 0.$$

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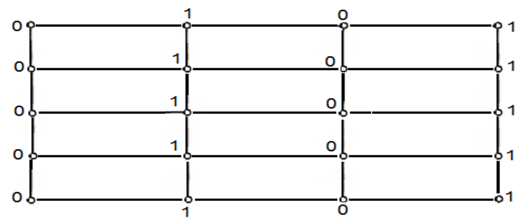

Figure 4.3.

Lemma 4.3: The graph $P_n \times P_n$ is cordial for all odd numbers n.

Proof: Suppose that $n = 2s + 1$**.** Therefore, one we take the following labeling**:**
The [1st,2^{nd}] columns are labeled by the sequence $[1_s 0_{s+1}]$.

The [3rd,5th ,...,$(2s + 1)^{\text{th}}$] columns are labeled by the sequence$[0_s 1_{s+1}]$.
The [4th,6th ,...,$(2s)^{\text{th}}$] columns arelabeled by the sequence $[1_s 0_{s+1}]$.Then:
$v_0 = (s + 1)(s + 1) + s(s) = 2s(s + 1) + 1$, $v_1 = s(s + 1) + s(s + 1) = 2s(2s + 1)$,
E_0 =the vertical edges plus the horizontal edges= $[(2s - 1)(s + 2) + (2s - 1)(s - 1)] + [(2s + 1)] = 2\,s(2s + 1)$,
E_1 =the vertical edges plus the horizontal edges= $[(2s + 1)] + [(2s - 1)(2s + 1)] = 2s(2s + 1)$, i.e.$|E_0 - E_1| = 0$, $|v_0 - v_1| = 1$.So $P_{2s+1} \times P_{2s+1}$is cordial as we wanted to show .
For example $P_5 \times P_5$**:**
$v_0 = 13$, $v_1 = 12$, $E_0 = E_1 = 20$, i.e.$|E_0 - E_1| = 0$, $|v_0 - v_1| = 1$.

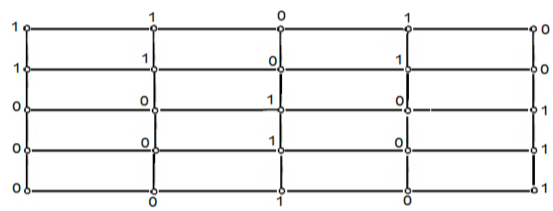

Figure 4.4.

The prove of the following theorem comes directly from the previous lemmas

Theorem4.1: The Cartesian product graph $P_{2s+i} \times P_{2s+j}$is cordial for all i and j where $(i, j = 0,1)$.

5. The Cartesian Product of a Cycle and A Path

In this section, we will study the cordiality of a cycle and a path.

Lemma 5.1: The graph $C_n \times P_n$ is cordial if n is an even number.

Proof: Suppose that $n = 2s$**.** Therefore, we study two cases for $n \equiv i (mod\ 4)$ and $i = 0,2$**.**

Case (1): At $i = 0$ we choose the following labeling:
The[1^{st},3^{rd},5^{th},...,$(2s - 3)^{\text{th}}$] columns are labeled byO_{2s} .

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The $[2^{nd}, 4^{th}, 6^{th}, \ldots, (2s-2)^{th}]$ columns are labeled by1_{2s} .

The$(2s-1)^{th}$column is labeled byM_{2s}. The$(2s)^{th}$ column is labeled by L_{2s} .Then**:**
$V_0 = V_1 = [(2s)(s-1)] + [s+s] = 2s^2$,
E_0 =the vertical edges plus the horizontal edges $= [2s(2s-2) + (s)] + [s+s] = s(4s-1)$
E_1 =the vertical edges plus the horizontal edges $= [2s + s] + [(2s)(2s-3) + s + (s)] = s(4s-1)$ Then:$|E_0 - E_1| = 0, |v_0 - v_1| = 0$ which satisfies the cordiality conditions.

For example: $C_4 \times P_4$(for $s = 2$)

$$v_0 = v_1 = 8 \ , E_0 = E_1 = 14 \ \ i.e: |E_0 - E_1| = 0, |v_1 - v_1| = 0.$$

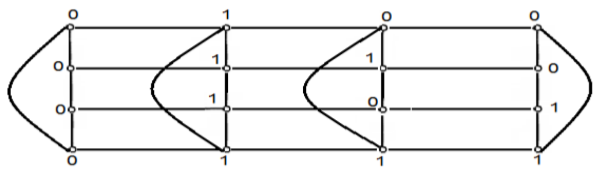

Figure 5.1.

Case (2): For $i = 2$we can apply the following labeling**:**

The$[1^{st}, 3^{rd}, 5^{th}, \ldots, (2s-3)^{th}]$columns are labeled by$O_{2s}$.

The $[2^{nd}, 4^{th}, 6^{th}, \ldots, (2s-2)^{th}]$columns are labeled by$1_{2s}$.

The $(2s-1)^{th}$column is labeled byM_{2s} .

The $(2s)^{th}$ column is labeled by $L_{2s-2}10$ .In this case**:**
$V_0 = V_1 = [(2s)(s-1)] + [s + (s-1) + 1] = 2s^2$,
E_0 =the vertical edges plus the horizontal edges $= [2s(2s-2) + 0 + (s+1)] + [(s-1) + s] = s(4s-1)$
E_1 =the vertical edges plus the horizontal edges $= [2s + (s-1)] + [(2s)(2s-3) + s + (s+1)] = s(4s-1)$.Then: $|E_0 - E_1| = 0$,$|v_0 - v_1| = 0$ which is cordial.

For example: $C_6 \times P_6$
$v_0 = v_1 = 18 \ , E_0 = E_1 = 33, \ i.e. |E_0 - E_1| = 0, |v_0 - v_1| = 0.$,

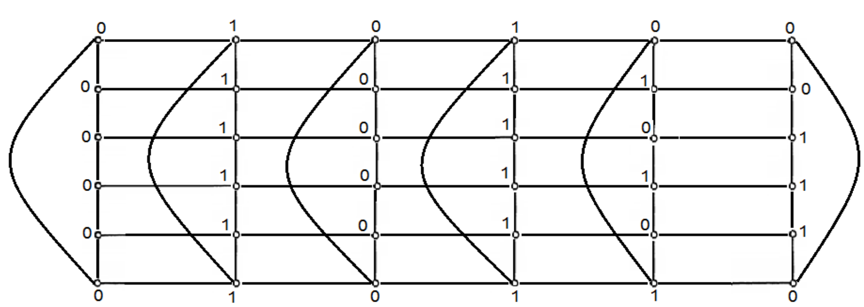
Figure 5.2

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Lemma 5.2: The graph $C_n \times P_m$is cordial for all n is even and m is odd such that $n - m = -1$ also it is cordial for all n is odd and m is even which satisfy $n - m = 1$.

Proof: we divide the proof into two cases:
Case (1): Suppose that $n = 2s$ and $m = 2s + 1$which satisfy the previous condition $n - m = -1$. we can use the following labeling:
The$[1^{st}, 3^{rd}, 5^{th}, ..., (2s-1)^{th}]$columns are labeled by$O_{2s}$.
The $[2^{nd}, 4^{th}, 6^{th}, ..., (2s)^{th}]$columns are labeled by$1_{2s}$.
The $(2s+1)^{th}$column is labeled byM_{2s} .In this case
$V_0 = V_1 = (s)(2s) + s = s(2s+1)$,
E_0 =the vertical edges plus the horizontal edges $= [2s(2s) + (s)] = s(4s+1)$
E_1 =the vertical edges plus the horizontal edges $= [2s] + [(2s)(s) + (2s-1)(s)] = s(4s+1)$
Then: $|E_0 - E_1| = 0, |v_0 - v_1| = 0$.So $C_{2s} \times P_{2s+1}$is cordial as we wished.

For example:$C_4 \times P_5$

$$v_0 = v_1 = 10, E_0 = E_1 = 18 \;\; i.e. |E_0 - E_1| = 0, |v_0 - v_1| = 0.$$

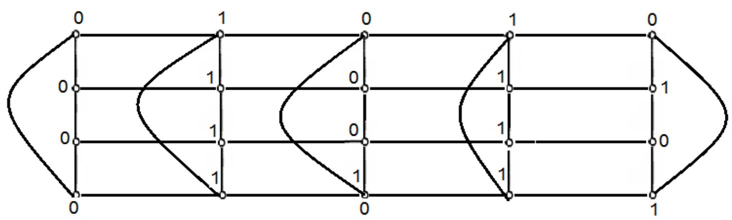

Figure 5.3

Case (2): Suppose that $n = 2s + 1$ and $m = 2s$ **.**It is obvious that $n - m = 1$.
We use the following labeling:
The$[1^{st}, 3^{rd}, 5^{th}, ..., (2s-3)^{th}]$columns are labeled by$O_{2s+1}$.

The $[2^{nd}, 4^{th}, 6^{th}, ..., (2s-2)^{th}]$column are labeled by$1_{2s+1}$.

The $(2s-1)^{th}$column is labeled byM_{2s+1} .

The $(2s)^{th}$column is labeled by$M_{2s}1$. In this situation**:**

$$v_0 = v_1 = (2s+1)(s-1) + s + (s+1) = s(2s+1) \;,$$

E_0 =the vertical edges plus the horizontal edges $= [(2s+1)(2s-2) + 1 + 1] + [0 + s + 2s] = s(4s+1)$,
E_1 =the vertical edges plus the horizontal edges $= [2s + 2s] + [s(2s-3) + s(2s-2) + (2s-1)] = s(4s+1) - 1$. Then$|E_0 - E_1| = 1$**,**$|v_0 - v_1| = 0$which satisfy cordiality conditions.

For example: $C_5 \times P_4$

$$v_0 = v_1 = 10, E_0 = 18, E_1 = 17 \; then \; |E_0 - E_1| = 0, |v_0 - v_1| = 0.$$

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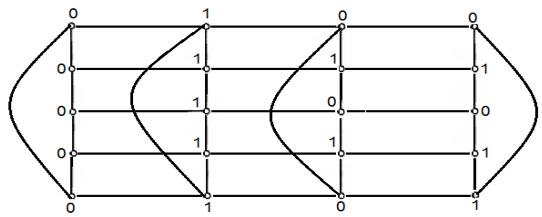

Figure 5.4.

Lemma 5.3. The graph $C_n \times P_n$ is cordial for all n is an odd number.

Proof: Suppose that $n = 2s + 1$we take the following labeling**:**
The[1^{st}, 3^{rd},5^{th},...,$(2s-1)^{\text{th}}$]columns are labeled by O_{2s+1} .
The [2^{nd} ,4^{th} ,6^{th},...,$(2s)^{\text{th}}$]columns are labeled by1_{2s+1} .
The $(2s+1)^{th}$column is labeled by M_{2s+1}.

Then:$v_0 = (2s+1)(s) + (s+1) = 2s(s+1)+1$,$v_1 = (2s+1)(s) + s = 2s(s+1)$,
E_0 =the vertical edges plus the horizontal edges $= [(2s+1)(2s)+1] + [s] = s(4s+3)+1$,
E_1 =the vertical edges plus the horizontal edges $= [2s] + [(2s-1)(2s+1)+(s+1)] = s(4s+3)$ then$|E_0 - E_1| = 1$,$|v_1 - v_1| = 1$. So $C_{2s+1} \times P_{2s+1}$is cordial as we wanted to show**.**

For example: $C_5 \times P_5$
$v_0 = 13$,$v_1 = 12$,$E_0 = 23$, $E_1 = 22$ then $|E_0 - E_1| = 1, |v_0 - v_1| = 1$.

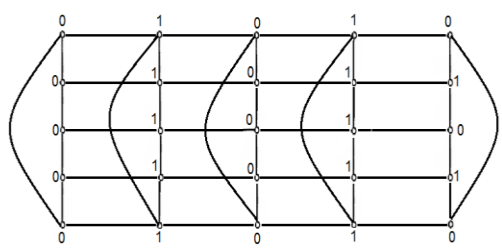

Figure 5.5.

As a direct consequence of the last three lemmas we have

Theorem 5.1: The Cartesian product graph $C_{2s+i} \times P_{2s+j}$is cordial for alli and j where (i , j = 0,1) .

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