ESTIMATION OF PROCESS CAPABILITY IN BAYESIAN PARADIGM

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ABSTRACT

The process capability index (PCI), Cp examines the capability of control charts. Bayesian techniques to estimate Cp are desirable when prior information about a process characteristic is available. In this paper, an estimator of Cp under normality with process variance having conjugate prior in Bayesian scenario is proposed. Its performance is studied and compared with Bayesian estimator developed by Cheng and Spiring (1989) An illustrative example is provided.

Keywords: Bayesian Estimator, Conjugate Prior, Posterior Distribution, Process Capability Index, Process Variance

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1. INTRODUCTION

Quality is a prudent characteristic in manufacturing industries. Cp plays pivotal role in deciding about capability of a process preset to meet the quality requirements. It is given by

$$C_p = \frac{USL-LSL}{UCL-LCL} = \frac{USL-LSL}{6\sigma} \qquad \textbf{Equation 1}$$

where σ is process standard deviation (sd), UCL, LCL are upper, lower control limits and USL, LSL are upper, lower specification limits of a control chart. Cp is a unitless measure and a process is considered as capable if $C_p > k$ where k is a positive constant ≥ 1. When σ is estimated by sample sd 's', an estimator for Cp is

given by $\hat{C}_p = \frac{USL-LSL}{6s}$. **Equation 2**

Kane (1986) compared various PCIs and Montgomery (1996) carried out a detailed discussion on PCIs along with their illustrations. Bayesian procedures for PCI use prior information about the process parameters involved. Cheng and Spiring (1989) using Bayesian approach propose an estimator $\hat{C}_{pn}$, under normal model when process sd has noninformative prior. The posterior distribution of $\hat{C}_{pn}$ derived by them is

$$\pi(y|C_p) = \left[\Gamma\left(\frac{n-1}{2}\right)\right]^{-1} 2^{1-\left(\frac{n-1}{2}\right)}\left((n-1){C_p}^2\right)^{\frac{n-1}{2}} y^{-n}\ e^{-\frac{(n-1){Cp}^2}{2y^2}}, y > 0$$ **Equation 3**

where $\hat{C}_{pn}$ is realized by y. They investigate the performance of their measure in terms of minimum value of $\hat{C}_{pn}$ required to ensure the probability that process achieves the desired specifications along with an illustration.

Chan et al. (1988) proposed a measure to process capability which accounts both target value and process variation simultaneously. They examined sampling distribution of the proposed measure with its practical applications to industrial data. Spiring (1995) outlined assessment of process capability as a tool of management. Shiau et al. (1999) studied Bayesian procedure for process capability by assuming noninformative and gamma priors for ${C_p}^2$. Kotz and Johnson (2002) reviewed some articles on PCIs studied during 1992 - 2000 from widely

scattered sources and record their interpretations along with comments. Pearn and Wu (2005) studied estimation of C_p by Bayesian approach using multiple samples.

In this paper, we establish an estimator of C_p in Bayesian paradigm under normality when process variance has conjugate prior. In section 2, we propose $\hat{C}_{pc}$ and derive its posterior distribution. We study about its performance in section 3, illustrate its performance in section 4 and record our conclusions in section 5. The computed values supporting performance of $\hat{C}_{pc}$ are given in tables provided in appendix.

2. PROPOSED BAYESIAN ESTIMATOR OF C_p

In this section, we propose a Bayesian estimator $\hat{C}_{pc}$ for C_p when process variance has a conjugate prior distribution. That is, $\hat{C}_{pc}$ is given by Equation 2 with an assumption that, σ^2 has a conjugate prior.

Suppose, X_1, X_2 … X_n is a random sample of size n from $N(\mu, \sigma^2)$, then the density function of X_i is given by

$$f_{X_i}(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x, \mu < \infty \ , \sigma > 0$$ **Equation 4**

The likelihood function of the sample $\boldsymbol{X} = (\mathrm{X}_1, \mathrm{X}_2 \ \ldots \ \mathrm{X}_\mathrm{n})$ is given by

$$L(\boldsymbol{X},\mu,\sigma^2) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$ **Equation 5**

Also, $z = \frac{(n-1)s^2}{\sigma^2} \sim {\chi^2}_{(n-1)}$ where $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

We assume that $\pi(\sigma^2) \sim IG(\eta,\delta), \eta > 0, \delta > 0$ where η is shape parameter and δ is scale parameter. IG stands for inverse gamma which is a conjugate prior.

$$\pi(\sigma^2) = \frac{\delta^\eta}{\Gamma\eta} e^{-\frac{\delta}{\sigma^2}} \left(\frac{1}{\sigma^2}\right)^{\eta+1}, \sigma^2 > 0$$ **Equation 6**

Using Equation 1 and Equation 2, z can be written as

$$z = \frac{(n-1)C_p^2}{\hat{C}_{pc}{}^2}$$ **Equation 7**

Thus, realizing $\hat{C}_{pc}$ by y, we have the posterior distribution of $y^2|\boldsymbol{X}$ given by

$$\pi(y^2|\boldsymbol{X}) = \frac{\left(\frac{n-1}{2}C_p^2+\delta\right)^{\frac{n-1}{2}+\eta}}{\Gamma\left(\frac{n-1}{2}+\eta\right)} e^{-\frac{1}{y^2}\left(\frac{n-1}{2}C_p^2+\delta\right)} \left(\frac{1}{y^2}\right)^{\frac{n-1}{2}+\eta+1}$$ **Equation 8**

Using appropriate transformation, $\pi(y|\boldsymbol{X})$ is given by

$$\pi(y|\boldsymbol{X}) = \frac{2\left(\frac{n-1}{2}C_p^2+\delta\right)^{\frac{n-1}{2}+\eta}}{\Gamma\left(\frac{n-1}{2}+\eta\right)} e^{-\frac{1}{y^2}\left(\frac{n-1}{2}C_p^2+\delta\right)} \left(\frac{1}{y^2}\right)^{\frac{n-1}{2}+\eta+\frac{1}{2}} \quad y > 0$$ **Equation 9**

When $\eta = \delta = 0$, Equation 9 reduces to Equation 3 indicating that the posterior distribution of $\hat{C}_{pn}$ due to Cheng and Spiring (1989) is a particular case of posterior distribution of $\hat{C}_{pc}$ given in Equation 9. From Bhat and Gokhale (2014) Bhat and Gokhale (2016) and Gokhale (2017) we observe that,

$$\pi(y^2|\boldsymbol{X}) = \pi(\sigma^2|\boldsymbol{X})$$

And

$$\pi(y|\boldsymbol{X}) = \pi(\sigma|\boldsymbol{X})$$ **Equation 10**

where in C_p^2 in left hand side is replaced by $s2$ in right hand side.

3. PERFORMANCE OF $\hat{C}_{pc}$

In this section, we evaluate the performance of $\hat{C}_{pc}$ by obtaining minimum value of $\hat{C}_{pc}$ needed to assure $P\big((C_p > k)|\boldsymbol{X}\big)$. That is,

τ = minimum $\hat{C}_{pc}|p_c$.

Here, pc = $p_c = P(process\ is\ capable|\ sample)$

$$= P\big((C_p > k)|\boldsymbol{X}\big)$$

$$= P\left(\left(\frac{USL-LSL}{6\sigma} > k\right)|\boldsymbol{X}\right)$$

$$= P\left(\left(\frac{USL-LSL}{6k} > \sigma\right)|\boldsymbol{X}\right) \qquad \textbf{Equation 11}$$

which is equivalent to finding

$$p_c = \int_0^{\frac{USL-LSL}{6k}} \pi(y|\boldsymbol{X})dy = \int_0^{\frac{USL-LSL}{6k}} \pi(\sigma|\boldsymbol{X})d\sigma$$

$$= \int_0^{a} \frac{2\left(\frac{n-1}{2}s^2+\delta\right)^{\frac{n-1}{2}+\eta}}{\Gamma\left(\frac{n-1}{2}+\eta\right)} e^{-\frac{1}{\sigma^2}\left(\frac{n-1}{2}s^2+\delta\right)} \left(\frac{1}{\sigma^2}\right)^{\frac{n-1}{2}+\eta+\frac{1}{2}} d\sigma \qquad \textbf{Equation 12}$$

Where $a = \frac{USL-LSL}{6k}$.

By taking $t = \frac{1}{\sigma^2}\left(\frac{n-1}{2}s^2+\delta\right)$, $b = \left(\frac{n-1}{2}\right)\frac{k^2}{\hat{C}_{pc}{}^2} + \frac{\delta}{a^2}$ and $\xi = \left(\frac{n-1}{2}\right) + \eta$ and

proceeding on the lines of Chan et al. (1988), we express (12) as

$$p_c = \int_b^{\infty} \frac{1}{\Gamma\xi} t^{\xi-1}\, e^{-t} dt \qquad \textbf{Equation 13}$$

By using, Wilson-Hilferty (1931) transformation, (13) can be written as

$$p_c \cong 1 - \Phi\left\{\frac{\left(\frac{2}{n}b\right)^{1/3} - \left(1-\frac{1}{9\xi}\right)}{\frac{1}{3\xi^{\frac{1}{2}}}}\right\} \qquad \textbf{Equation 14}$$

Where $\Phi(\cdot)$ is cumulative distribution function of standard normal variate.

On simplifying Equation 14 we get

$$b = \frac{n}{2}\left(\frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1 - \frac{1}{9\xi}\right)\right)^3 \qquad \textbf{Equation 15}$$

Therefore,

$$(n-1)\frac{k^2}{\hat{C}_{pc}{}^2} = n\left(\frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1 - \frac{1}{9\xi}\right)\right)^3 - \frac{2\delta}{a^2}$$

$$\Rightarrow \hat{C}_{pc} = k\left(\frac{n-1}{n\left\{\frac{\Phi^{-1}(1-p_c)}{3\xi^{1/2}} + \left(1-\frac{1}{9\xi}\right)\right\}^3 - \frac{2\delta}{a^2}}\right)^{1/2}$$

$$= k\left(\frac{n-1}{n\left\{\frac{\Phi^{-1}(1-p_c)}{3\xi^{1/2}} + \left(1-\frac{1}{9\xi}\right)\right\}^3 - \frac{72k^2\delta}{w^2}}\right)^{1/2} \qquad \textbf{Equation 16}$$

Where $w = USL - LSL$.

In order to evaluate τ, one need to specify w, n, pc, k, η and δ. To compute minimum $\hat{C}pc$ obtained in Equation 16, the denominator has to be greater than zero.

That is, $n\left\{\frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1 - \frac{1}{9\xi}\right)\right\}^3 - \frac{72k^2\delta}{w^2} > 0$

$$\Rightarrow w > \left\{\frac{72k^2\delta}{n\left\{\frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1-\frac{1}{9\xi}\right)\right\}^3}\right\}^{1/2} \qquad \textbf{Equation 17}$$

By taking $\gamma = \left\{\frac{72k^2\delta}{n\left\{\frac{\Phi^{-1}(1-p_c)}{3\xi^{\frac{1}{2}}} + \left(1-\frac{1}{9\xi}\right)\right\}^3}\right\}^{1/2}$ in Table 1, we furnish w as $\lceil\gamma\rceil$ least

upper integer greater than γ. In Table 2, we calculate τ for higher values of w given in Table 1 n= 5, 15, 25, 50, 75, 100, pc = 0.90, 0.95, 0.99, k=1, 1.33,1.66, η = 0, 5, 10 and δ = 0, 5, 10. Using Table 2 we plot τ in Figure 1 Figure 2 and Figure 3 respectively for $\eta = \delta$, $\eta < \delta$ and $\eta > \delta$.

Figure 1

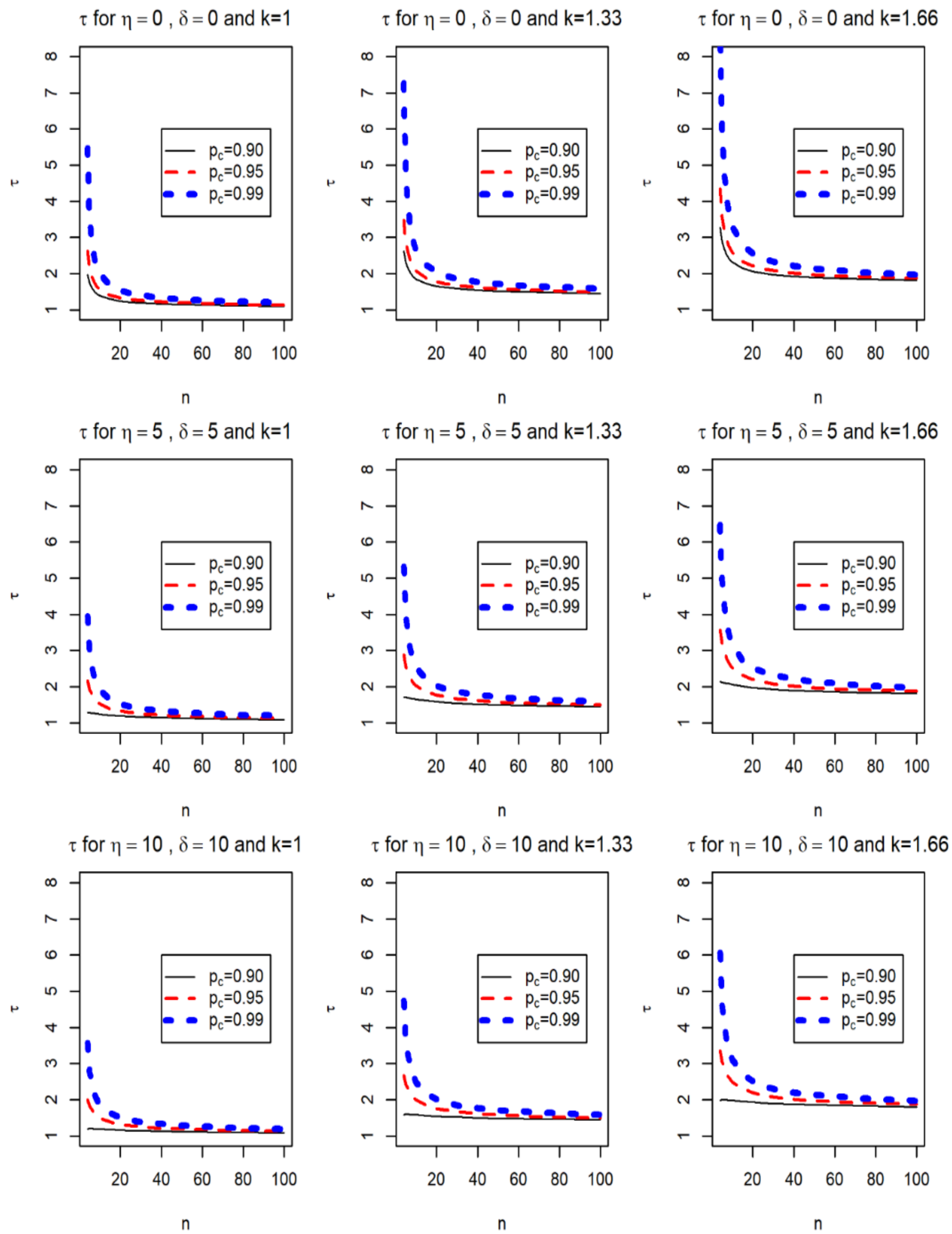

Figure 1 τ *for* $\eta = \delta$ and various values of n, p_c, k

Figure 2

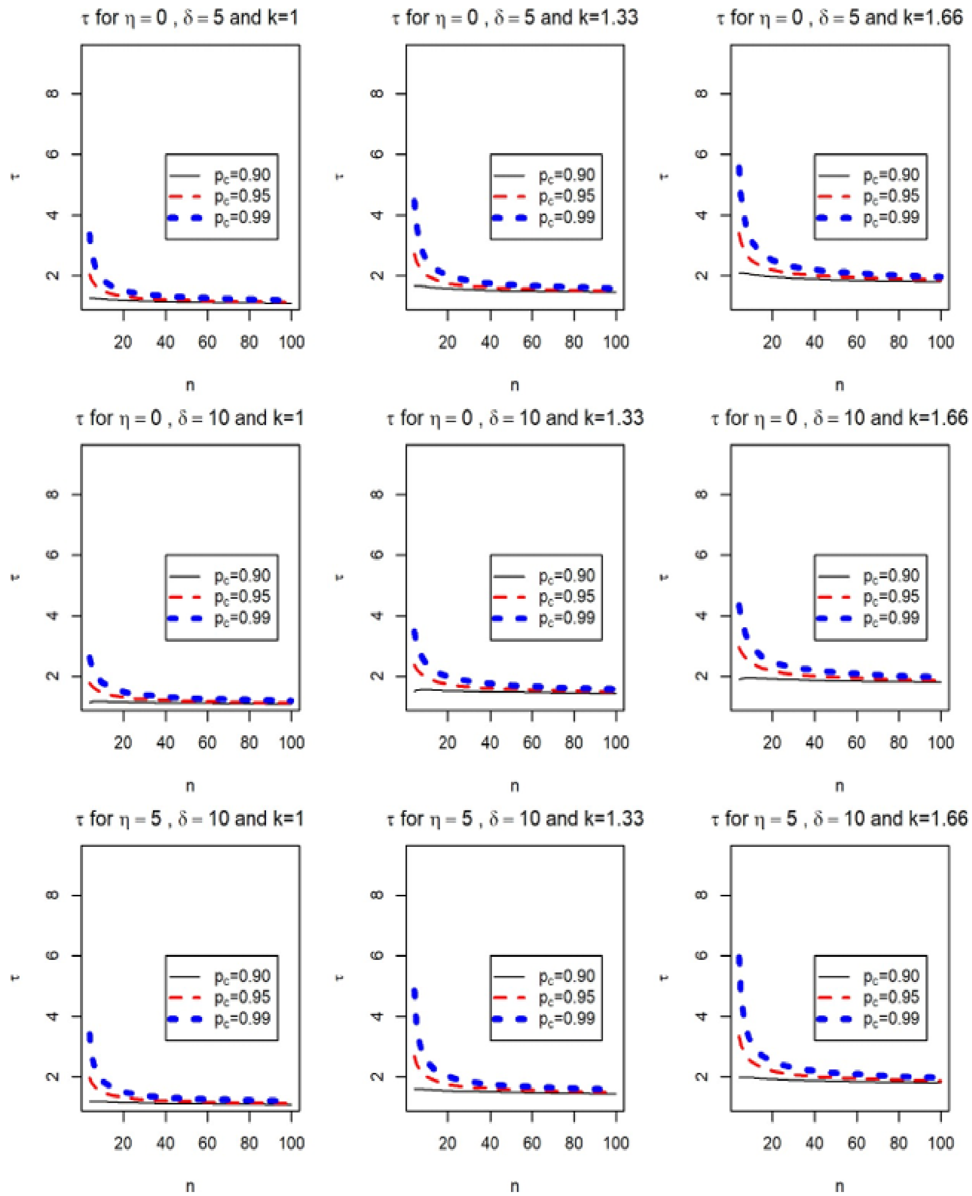

Figure 2 τ *for with* $\eta < \delta$ and various values of n, p_c, k

Figure 3

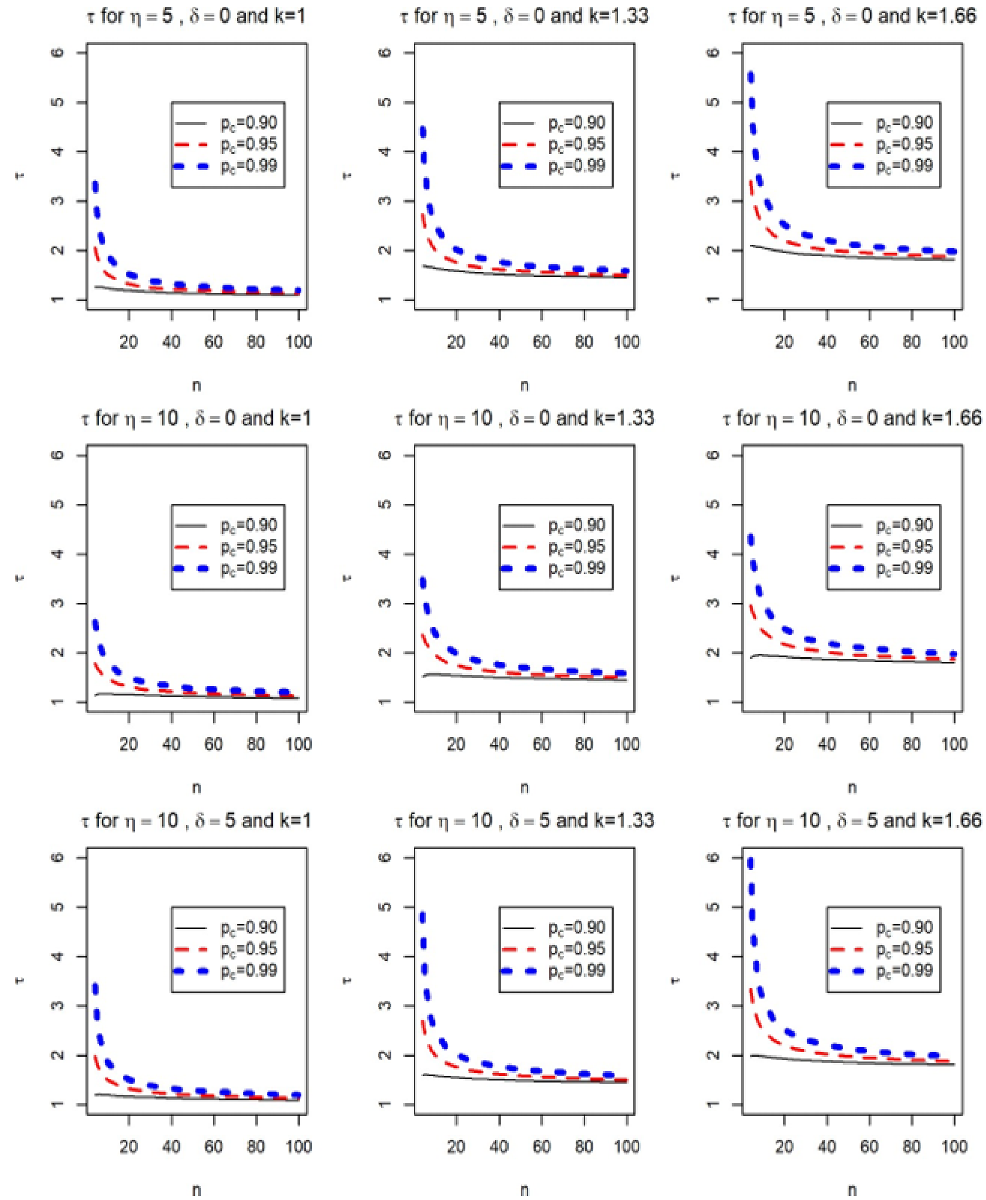

Figure 3 τ *for* $\eta > \delta$ and various values of n, p_c, k

From Table 1 we observe that, w increases as pc, k increase and decreases as n increases. For fixed η, w increases as δ increases, for fixed δ, it decreases as η increases and for $\eta = \delta$, it increases for increasing values of η and δ. From Figure 1 and Table 2 we observe that, for $\eta = \delta$, τ is higher for higher values of p_c. It is increasing for increasing value of k when n is small and remains nearly same when n is large. Also, τ is smaller for higher values of η and δ. Figure 2 and Figure 3 depict that τ decreases respectively as δ increases for $\eta < \delta$ and as η increases for $\eta > \delta$. Also, from all the three figures it is observed that, τ increases as k increases along with increase in p_c. Table 2 shows that, for fixed values of $\eta = 0$, there is no considerable change in values of τ for various values of n, k and pc for increasing δ.

4. ILLUSTRATION

In this section, we consider example given in Kane (1986) and discussed in Cheng and Spiring (1989)

Example 1

Example 1 For n=300, s=4.3, $\widehat{C}_{pn} = 1.5504$ and $p_n = P((C_p > 1)|\ \widehat{C}_{pn}) = 0.9999$. Then for $p_c = p_n$, using (16), for different values of η, δ and n, $\widehat{C}_{pc}$ is given by

			Cpc				
η	δ	n=300	n=50	η	δ	n=300	n=50
0	5	1.174	1.5479	10	0	1.167	1.4214
0	10	1.1746	1.5565	5	5	1.1707	1.4782
5	0	1.1701	1.4709	10	10	1.1682	1.4347

Example 2

Example 2 For n=79, s=7.8, $\widehat{C}_{pn} = 0.8547$ and $p_n = 0.0139$. For $p_c = p_n$, $\widehat{C}_{pc}$ is given by

			Cpc				
η	δ	n=79	n=5	η	δ	n=79	n=5
0	5	0.8453	0.5091	10	0	0.8584	0.6783
0	10	0.8462	0.5129	5	5	0.8528	0.6386
5	0	0.8519	0.6314	10	10	0.8602	0.6966

In Example 1, it is seen that for different values of η, δ and n=300, $\hat{C}_{pc}$ is lesser than $\hat{C}_{pn}$, whereas for n=50, $\hat{C}_{pc}$ is near to $\hat{C}_{pn}$ when $\eta = 0$ and is lesser than $\hat{C}_{pn}$ for other values of η and δ. In Example 2, $\hat{C}_{pc}$ is near to $\hat{C}_{pn}$ for n=79 when $\delta = 0$, whereas for n=5, $\hat{C}_{pc}$ is lesser than $\hat{C}_{pn}$ for various values of η and δ. It is also observed that, sample sd is smaller in Example 1 when compared to sample sd in Example 2

5. CONCLUSIONS

In this section, we furnish our conclusions based on our observations.

- Under Bayesian approach, the proposed estimator $\hat{C}_{pc}$ includes $\hat{C}_{pn}$ due to Cheng and Spiring (1989) as its particular case in the sense that, the posterior distribution of $\hat{C}_{pc}$ reduces to that of $\hat{C}_{pn}$ when hyper parameters η and δ are zero.
- For all the values of η and δ under consideration, τ the minimum value of $\hat{C}_{pc}$ needed to assure p_c, the probability that process is capable given the sample, increases along with increasing values of k and pc for smaller values of n.
- For $\eta = \delta$, τ is decreasing as η and δ are increasing.
- For $\eta < \delta$, τ decreases as δ increases and for $\eta > \delta$, it decreases as η increases.
- $\hat{C}_{pc} < \hat{C}_{pn}$ when sample sd is small, n is large and also when sample sd is large, n is small.

6. APPENDIX

Table 1

Table 1 $w = \lceil \gamma \rceil$ for various values of n, p_C, k, η and δ

η,δ	k	n / p_c	5	15	25	50	75	100	η,δ	k	n / p_c	5	15	25	50	75	100
0,5	**1**	**0.9**	17	7	5	4	3	3	**0,10**	**1**	**0.9**	24	10	7	5	4	3
		0.95	21	8	5	4	3	3			**0.95**	29	13	8	5	4	4
		0.99	35	9	6	4	3	3			**0.99**	49	13	8	5	4	4
	1.33	**0.9**	22	9	7	5	4	3		**1.33**	**0.9**	31	13	9	6	5	4
		0.95	28	10	7	5	4	3			**0.95**	39	14	10	7	5	5
		0.99	46	12	8	5	4	4			**0.99**	65	17	11	7	6	5
	1.66	**0.9**	28	11	8	6	5	4		**1.66**	**0.9**	39	16	12	8	6	5
		0.95	34	12	9	6	5	4			**0.95**	48	17	12	8	6	6
		0.99	57	15	10	6	5	4			**0.99**	80	21	14	9	7	6
5,5	**1**	**0.9**	12	7	5	4	3	3	**5, 10**	**1**	**0.9**	17	9	7	5	4	3
		0.95	18	8	5	4	3	3			**0.95**	25	10	8	5	4	4
		0.99	27	9	6	4	3	3			**0.99**	38	12	8	5	4	4
	1.33	**0.9**	16	9	7	5	4	3		**1.33**	**0.9**	23	12	9	6	5	4
		0.95	24	10	7	5	4	3			**0.95**	34	15	10	7	5	5
		0.99	35	12	8	5	4	4			**0.99**	50	16	11	7	6	5
	1.66	**0.9**	20	11	8	6	5	4		**1.66**	**0.9**	29	15	11	8	6	5
		0.95	30	12	9	6	5	4			**0.95**	42	17	12	8	6	6
		0.99	44	14	10	6	5	4			**0.99**	62	20	14	9	7	6
10,5	**1**	**0.9**	12	6	5	4	3	3	**10,10**	**1**	**0.9**	16	9	7	5	4	4
		0.95	17	7	5	4	3	3			**0.95**	23	10	7	5	4	4
		0.99	23	9	6	4	3	3			**0.99**	32	12	8	5	4	4
	1.33	**0.9**	15	8	6	5	4	3		**1.33**	**0.9**	21	12	9	6	5	4
		0.95	22	10	7	5	4	3			**0.95**	31	14	10	7	5	5
		0.99	30	11	8	5	4	4			**0.99**	42	16	11	7	6	5
	1.66	**0.9**	19	10	8	6	5	4		**1.66**	**0.9**	26	14	11	8	6	5
		0.95	27	12	9	6	5	4			**0.95**	38	17	12	8	6	6
		0.99	38	14	10	6	5	4			**0.99**	53	20	13	9	7	6

Table 2

Table 2 τ for various values of η, δ, k, p_c and n

η,δ	n	k=1			k=1.33			k=1.66		
		p_c=0.90	p_c=0.95	p_c=0.99	p_c=0.90	p_c=0.95	p_c=0.99	p_c=0.90	p_c=0.95	p_c=0.99
0,0	5	1.7372	2.1531	3.5876	2.3105	2.8636	4.7715	2.8838	3.5741	5.9555
	15	1.2947	1.4108	1.6818	1.722	1.8764	2.2368	2.1493	2.342	2.7918
	25	1.2128	1.29	1.4589	1.613	1.7157	1.9404	2.0132	2.1414	2.4218
	50	1.142	1.1897	1.2886	1.5188	1.5822	1.7139	1.8957	1.9748	2.1391
	75	1.1133	1.1502	1.2251	1.4807	1.5298	1.6294	1.8481	1.9093	2.0337
	100	1.0969	1.1279	1.19	1.4589	1.5001	1.5827	1.8209	1.8723	1.9754
0,5	5	1.7382	2.1549	3.596	2.3127	2.8678	4.7912	2.8881	3.5823	5.9938
	15	1.2949	1.411	1.682	1.7223	1.8767	2.2373	2.1498	2.3426	2.7929
	25	1.2129	1.2901	1.459	1.6131	1.7158	1.9406	2.0135	2.1417	2.4223
	50	1.142	1.1897	1.2887	1.5189	1.5823	1.7139	1.8958	1.9749	2.1393

	75	1.1134	1.1502	1.2251	1.4808	1.5298	1.6294	1.8482	1.9094	2.0337
	100	1.0969	1.1279	1.19	1.4589	1.5001	1.5827	1.821	1.8723	1.9754
0,10	5	1.7382	2.1549	3.5961	2.3128	2.8679	4.7916	2.8882	3.5825	5.9946
	15	1.2949	1.411	1.682	1.7223	1.8768	2.2374	2.1498	2.3427	2.7929
	25	1.2129	1.2901	1.459	1.6132	1.7158	1.9406	2.0135	2.1417	2.4223
	50	1.142	1.1897	1.2887	1.5189	1.5823	1.7139	1.8958	1.975	2.1393
	75	1.1134	1.1502	1.2251	1.4808	1.5298	1.6294	1.8482	1.9094	2.0337
	100	1.0969	1.1279	1.19	1.4589	1.5001	1.5827	1.821	1.8723	1.9754
5,0	5	1.2617	1.8611	2.7631	1.678	2.4753	3.6749	2.0944	3.0895	4.5868
	15	1.2072	1.3868	1.6384	1.6056	1.8445	2.1791	2.0039	2.3022	2.7197
	25	1.1718	1.2809	1.4436	1.5585	1.7035	1.92	1.9452	2.1262	2.3964
	50	1.1275	1.187	1.2844	1.4996	1.5787	1.7082	1.8717	1.9704	2.1321
	75	1.1055	1.1488	1.223	1.4704	1.528	1.6266	1.8352	1.9071	2.0302
	100	1.0919	1.127	1.1887	1.4522	1.4989	1.581	1.8126	1.8709	1.9732
5,5	5	1.2805	1.9233	2.9799	1.7049	2.5641	3.9862	2.1237	3.1859	4.9218
	15	1.2118	1.3939	1.65	1.6122	1.8545	2.1957	2.0112	2.3131	2.7379
	25	1.1743	1.2841	1.4482	1.562	1.7081	1.9266	1.949	2.1313	2.4036
	50	1.1286	1.1882	1.286	1.5011	1.5805	1.7105	1.8733	1.9723	2.1345
	75	1.1062	1.1496	1.2239	1.4713	1.529	1.6279	1.8362	1.9083	2.0316
	100	1.0924	1.1276	1.1893	1.4529	1.4997	1.5818	1.8133	1.8717	1.9742
5,10	5	1.2801	1.922	2.9751	1.7001	2.548	3.9265	2.1296	3.2062	4.9975
	15	1.2117	1.3938	1.6498	1.611	1.8528	2.1928	2.0126	2.3153	2.7415
	25	1.1742	1.284	1.4481	1.5614	1.7073	1.9254	1.9498	2.1323	2.405
	50	1.1286	1.1882	1.2859	1.5009	1.5801	1.7101	1.8737	1.9727	2.135
	75	1.1062	1.1496	1.2239	1.4712	1.5289	1.6277	1.8364	1.9085	2.0319
	100	1.0924	1.1275	1.1893	1.4528	1.4996	1.5817	1.8135	1.8719	1.9744
10,0	5	1.1598	1.6933	2.3537	1.5425	2.2521	3.1304	1.9252	2.8109	3.9071
	15	1.1649	1.366	1.6012	1.5494	1.8168	2.1296	1.9338	2.2676	2.658
	25	1.1467	1.2724	1.4295	1.5251	1.6923	1.9013	1.9035	2.1122	2.373
	50	1.1165	1.1844	1.2803	1.4849	1.5752	1.7028	1.8534	1.9661	2.1253
	75	1.0991	1.1475	1.221	1.4618	1.5262	1.6239	1.8245	1.9049	2.0268
	100	1.0875	1.1262	1.1874	1.4464	1.4978	1.5793	1.8053	1.8695	1.9711
10,5	5	1.1961	1.8128	2.7109	1.6002	2.4441	3.7191	1.9943	3.0402	4.6058
	15	1.1751	1.3825	1.6279	1.5654	1.8429	2.1719	1.9531	2.2989	2.7087
	25	1.1523	1.2801	1.4405	1.534	1.7045	1.9186	1.9142	2.1268	2.3938
	50	1.119	1.1874	1.2841	1.4889	1.58	1.7088	1.8582	1.9718	2.1325
	75	1.1007	1.1493	1.2232	1.4643	1.5291	1.6273	1.8275	1.9083	2.031
	100	1.0887	1.1275	1.1889	1.4483	1.4999	1.5817	1.8075	1.8719	1.974
10,10	5	1.2009	1.8295	2.768	1.5969	2.4323	3.678	1.9974	3.0509	4.6435
	15	1.1764	1.3846	1.6314	1.5645	1.8414	2.1695	1.9539	2.3002	2.7109
	25	1.153	1.2811	1.4419	1.5335	1.7038	1.9176	1.9146	2.1274	2.3946
	50	1.1193	1.1878	1.2846	1.4887	1.5797	1.7085	1.8584	1.972	2.1328
	75	1.1009	1.1496	1.2234	1.4641	1.5289	1.6272	1.8276	1.9085	2.0311
	100	1.0888	1.1276	1.1891	1.4482	1.4998	1.5815	1.8076	1.872	1.9741

CONFLICT OF INTERESTS

None.

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REFERENCES

Bhat, S. V., and Gokhale, K. D., (2014). Posterior Control Charts for Process Variance Based on Various Priors. Journal of the Indian Society for probability and statistics. 15, 52-66.

Bhat, S. V., and Gokhale, K. D., (2016). Posterior Control Chart for Standard Deviation based on Conjugate Prior. Journal of Indian Statistical Association. 54(1-2), 157- 166.

Chan, L. K., Cheng, S. W., and Spiring, F. A., (1988). A New Measure of Process Capability : Journal of Quality Technology. 20(3), 162-175. https://doi.org/10.1080/00224065.1988.11979102

Cheng, S. W., and Spiring, F. A., (1989). Assessing process capability : a Bayesian approach, IIE Transactions. 97-98. https://doi.org/10.1080/07408178908966212

Gokhale, K. D. (2017). Studies in statistical quality control using prior information. An Unpublished thesis submitted to the Karnatak, University Dharwad. https://shodhganga.inflibnet.ac.in/handle/10603/221506

Kane, V. K. (1986). Process capability indices, Journal of Quality Technology, 41-52. https://doi.org/10.1080/00224065.1986.11978984

Kotz, S., Johnson, N. L. (2002). Process capability indices--a review, 1992-2000. Journal of Quality Technology, 34(1), 1-19. https://doi.org/10.1080/00224065.2002.11980119

Montgomery, D. C. (1996). Introduction to statistical quality control (6th Edition ed.). John Wiley and Sons.

Pearn, W. L. and Wu, C. W. (2005). A Bayesian approach for assessing process precision based on multiple samples, European Journal of Operational Research. 165(3), 685-695. https://doi.org/10.1016/j.ejor.2004.02.009

Shiau, J. H., Chiang, C., and Hung, H. (1999). A Bayesian procedure for process capability assessment, Quality and Reliability Engineering International 15, 369-378.

Spiring, F. A. (1995). Process capability : a total quality management tool, Total Quality Management 6, 21-33. https://doi.org/10.1080/09544129550035558

Wilson, E. B., Hilferty, M. M. (1931). The Distribution of Chi-Square. Proceedings of the National Academy of Sciences. 17, 684-688. https://doi.org/10.1073/pnas.17.12.684