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FRICTION ANALYSIS IN CASE OF THE CARRIAGE- FEED SHAFT CINEMATIC COUPLING AT CNC LATHE

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Abstract:
The paper presents a mathematical model for analysis of friction between the tool bearing saddle and conductor at CNC lathe. The analysis of longitudinal advance movement laws is performed taking into account the appearance and development of disturbing harmonic forces created by auto-vibrations determined by the interaction between the partial elastic systems of tool and workpiece. The friction force is emphasized as product of two components depending on the sliding speed and on the normal disturbing force. By establishing the dynamic response of the system, when the normal force depends linearly on speed, acceleration and mobile ensemble position, the premises are created for stability analysis of the friction movement, obtaining the limit speeds under which the stick-slip phenomenon occurs. Thus, it is provided for a rational design of CNC lathe elastic structure, in order to improve the surface quality and the dimensional precision.

Keywords: Friction Analysis; Carriage; Feed Shaft; Cinematic Coupling; CNC Lathe; Routh-Hurwitz Criterion.

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1. Introduction

It is a known fact from specialty literature that the movement stability of the dynamic system of machine tools in the absence of auto-vibrations or jerky sliding movements is evaluated in conformance with known stability requirements. Using Nyquist stability criterion requires determining the frequency characteristic of the equivalent "open" system. The equivalent "closed" system is stable if the amplitude-phase characteristic does not include the point with $(-1, i_0)$ coordinates [1].

In the absence of cutting, the dynamic system of the machine tool becomes a typical friction system as part of the mechanical system of various machines or mechanisms.

Sliding friction is a complex technological process which includes both elastic and plastic deformation of the superficial layers of the bodies in contact. The friction process presents proper stability in the absence of jamming.

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Usually the friction force is proportional with the active load, i.e. with the normal contact deformation (in *z* axis) of the friction surfaces. The friction force appears during the probable tangential movement. The dynamics of the sliding friction process makes the variation of the normal contact deformation dependent on the variation of the friction force.

For determining movement stability of friction systems it is largely used the single degree of freedom model [2].

Figure 1a presents the friction system diagram while figure 1b presents the mixed friction process characteristics:

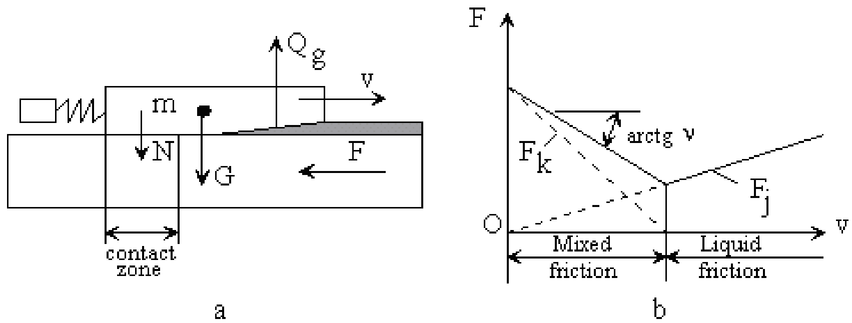

Figure 1

Movement stability for the falling zone of the friction force characteristic depending on speed is determined by comparing the characteristic slope with the damping coefficient. This kind of analysis can only be used in case of dry friction.

According to fig. 1, the falling zone of the friction characteristic is inside the mixed friction zone. Decreasing the friction force together with increasing the speed *v* is accomplished through the action of the hydrodynamic force O_g which unloads the contact zone. In the space between carriage and guide there appears either absorption or pushing out of the greasing fluid. This leads to appearance of viscous resistance, of considerable value in case of emersion (immersion) or in case of friction force change. When evaluation the movement stability, this resistance is added to the damping resistance of the driving system and of the greasing fluid, proportional with the sliding speed.

The interaction between the main components of the elastic system of the CNC lathe refers to analysis of the longitudinal advance movement of the system composed of the tool bearer, workpiece grip and the workpiece.

The movement may have constant speed above a critical limit, which usually determines the stick-slip phenomenon. This phenomenon is characterized by intermittent sliding or jamming of bodies in contact, being met in several mechanical systems [3], [4]. Its effect is destabilizing when the movement of the system is performed at small speeds.

In research performed up to now, a factor usually ignored was the effect of the normal force an of the friction force, whose behavior follows state variable laws.

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2. Establishment of The Mathematical Model for Friction Analysis

The analysis of the saddle-conductor coupling is performed under the assumptions that the friction force and normal force are functions depending on the system state.

Their magnitude is determined not only by the sliding speed, but also by the movement past evolution.

The mathematical model is a single degree-of-freedom elastic system, corresponding to figure 2:

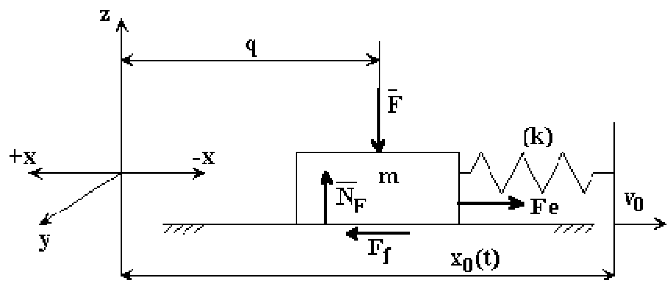

Figure 2

Applying the fundamental law of dynamics [4]:

$$m\vec{a} = \vec{R} \tag{1}$$

Considering the general model, for which $\lambda \neq 0$ and projecting (1) an axes, we obtain (figure 3):

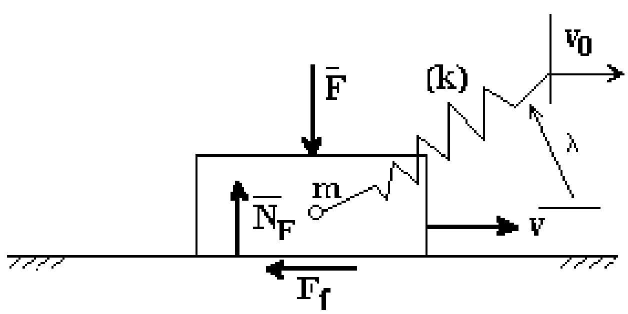

Figure 3

$$m\ddot{q} + kq = k(x_0 + x \cdot tg\alpha) - F_f \tag{2}$$

Considering the spring initial position given by $x_0 = x_0(t)$, we have:

$$F_f = F_f(\dot{q}) + F_f(F) \tag{3}$$

Where: $F_f(\dot{q})$ = friction force component depending on sliding speed
$F_f(F)$ = friction force component depending on normal disturbing force

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At their turn, these components have the expressions:

$$F_f(\dot{q}) = a\dot{q} - \int_0^t h(t-t')\,\dot{q}(t')dt' \qquad (4)$$

Where:$a\dot{q}$ = friction force component proportional with the sliding speed
$\int_0^t h(t-t')\,\dot{q}(t')dt'$ = component which depends on the previous displacement variation

In fig. 4 it is depicted the response of the friction force component at the step change of the sliding speed.

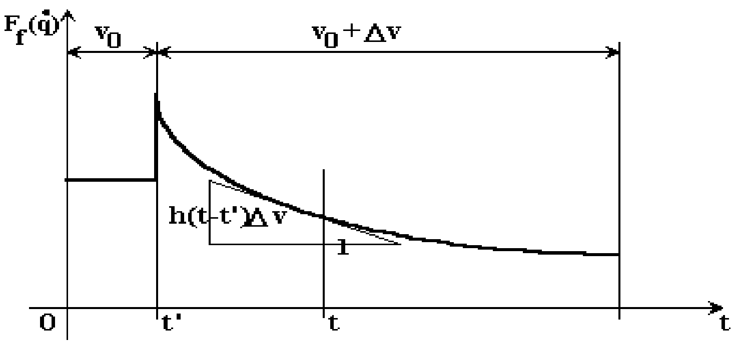

Figure 4

Also,

$$F_f(F) = bF + \int_0^t g(t-t')\,F(t')dt' \qquad (5)$$

Where:bF = component proportional with the disturbing force

$\int_0^t g(t-t')\,F(t')dt$ = component which depends on the previous variation of the normal force

In figure 5 it is presented the response of the friction force for changes of the normal force:

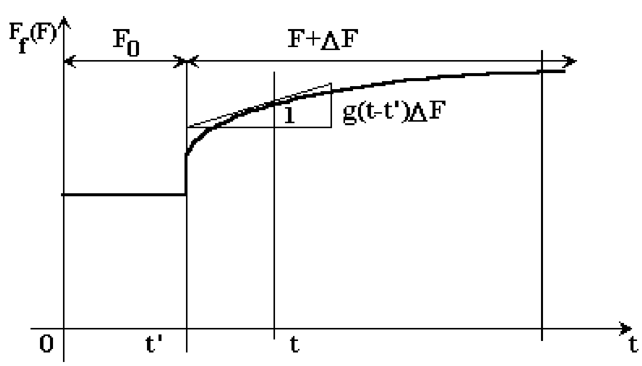

Figure 5

The mathematical model becomes:

$$m\ddot{q} + kq = k[x_0(t) - x(t)\cdot tg\alpha] - a\dot{q} + \int_0^t h(t-t')\,\dot{q}(t')dt' - bF - \int_0^t g(t-t')\,F(t')dt' \quad (6)$$

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Applying the Laplace transform in (6) we get:

$$[ms^2 + k + as - s\tilde{h}(s)]\tilde{q} = k[\tilde{x}_0(s) - \tilde{x}(s) \cdot tg\alpha] - (b + \tilde{g}(s))\tilde{F}(s) \quad (7)$$

A generalized equation in which the normal disturbing force depends linearly on acceleration, speed, displacement and time variation is given by:

$$F(q, \dot{q}, \ddot{q}, t) = A\ddot{q} + B\dot{q} + Cq + D(t) \quad (8)$$

Where:

A = coefficient expressing dependence on acceleration
B = coefficient expressing dependence on speed
C = coefficient expressing dependence on mobile system position
D(t) = coefficient expressing time dependence of the normal force

For the given damping we have:

$$h(t) = c_1 e^{-rt}$$
$$g(t) = c_2 e^{-rt} \quad (9)$$

Where 1/r is the time characteristic to damping processes.

The dynamic response of the system is:

$$q(t) = L^{-1}\left[\frac{k(r+s)[\tilde{x}_0(s)+\tilde{x}(s)\cdot tg\alpha]-(br+bs+c_2)\tilde{D}(s)}{(ms^2+as+k)(r+s)-c_1 s+(br+bs+c_2)(As^2+Bs+C)}\right] \quad (10)$$

If λ = 0 and

$$P(s) = (ms^2 + as + k)(r + s) - c_1 s + (br + bs + c_2)(As^2 + Bs + C) \quad (11)$$

we can analyze the friction stability using Routh-Hurwitz criterion. The system response becomes:

$$q(t) = \frac{1}{m+bA}\left\{\left[\sum_{i=1}^{3}(krm_i + n_i)\int_0^t e^{s_i(t-t')}x_0(t')dt'\right] - \left[\sum_{i=1}^{3}((br + c_2)m_i + bn_i)\int_0^t e^{s_i(t-t')}D(t')dt'\right]\right\} \quad (12)$$

The relation (12) represents the movement law of the mass *m*, which moves with friction along the contact surface, under state variable laws and varying normal force [5].

We also have:

$$D(t) = F_y(t) + k_{y_1}y_1(t) + c_{y_1}\dot{y}_1(t) + k_y(y_1(t) - y_2(t)) + c_y(\dot{y}_1(t) - \dot{y}_2(t)) - m_1\ddot{y}_1(t) \quad (13)$$

Where: $F_y(t)$ – Ox component of the nominal cutting force
K_{y1}, k_y – rigidities of the elastic system along the coordinate axes
C_{y1}, c_y – damping coefficients along the coordinate axes

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Corresponding to fig. 6, we have:

$$F_y(t) = \lambda F_{as} \cos \omega t \sin \alpha \tag{14}$$

Where: F_{as} = nominal grinding force
λ = overlay factor between previous and current tool pass

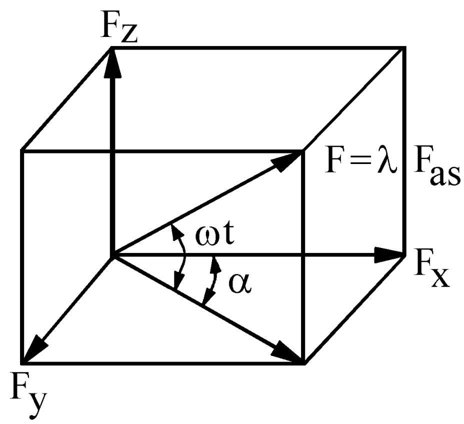

Figure 6

We have:

$$D(t) = \lambda F_{as} \cos \omega t \sin \alpha + k_1 k_{y_1} \frac{F_{01}}{m_1} e^{i\omega t} + i \frac{c_{y_1} \omega F_{01}}{m_1} k e^{i\omega t} + k_y \left[\frac{F_{01}}{m_1} k_1 - \frac{F_{02}}{m_2} k_2\right] e^{i\omega t} + $$
$$+ i\omega c_y \left[\frac{F_{01}}{m_1} k_1 - \frac{F_{02}}{m_2} k_2\right] e^{i\omega t} + \omega^2 F_{01} k_1 e^{i\omega t} \tag{15}$$

where: $k_1 = [(\omega_{n1}^2 - \omega^2)^2 + \xi_1^2 \omega^2]^{-1}$

$$k_2 = [(\omega_{n2}^2 - \omega^2)^2 + \xi_2^2 \omega^2]^{-1} \tag{16}$$

Denoting:

$$\Delta_1 = F_{01} k_1$$
$$\Delta_2 = F_{01} k_1 \left[\frac{k_{y_1}}{m_1} + \frac{k_y}{m_1} - \frac{k_y}{m_2} \frac{k_2}{k_1} \frac{F_{02}}{F_{01}}\right]$$
$$\Delta_3 = c_{y_1} k_1 \frac{F_{01}}{m_1} + c_y \left(k_1 \frac{F_{01}}{m_1} - k_2 \frac{F_{02}}{m_2}\right)$$
$$\frac{k_r m_i + n_i}{(m + bA) s_i^2} = C_{li}$$
$$\frac{[(b_r + c_2) m_i + b n_i] \left[\frac{\lambda F_{as} \sin \alpha + i\omega \Delta_3}{s_i^2 + \omega^2}\right]}{m + bA} = C_{2i}$$
$$\frac{\omega [(b_r + c_2) m_i + b n_i](i\omega + s_i)(\Delta_2 + \omega^2 \Delta_1)}{(m + bA)(s_i^2 + \omega^2)} = C_{3i}$$

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$$\frac{\omega[(b_r+c_2)m_i+bn_i]\Delta_3}{(m+bA)(s_i^2+\omega^2)} = C_{4i} \tag{17}$$

The movement equation becomes:

$$q(t) = v_0\sum_{i=1}^{3} C_{li}(e^{s_it} - l - s_it) - \sum_{i=1}^{3} C_{2i}(s_ie^{s_it} + \omega\sin\omega t - s_i\cos\omega t) -$$
$$-\sum_{i=1}^{3} C_{3i}(e^{s_it} - e^{i\omega t}) - \sum_{i=1}^{3} C_{4i}(\omega e^{s_it} - s_i\sin\omega t - \omega\cos\omega t) \tag{18}$$

Under these circumstances, the equations for speed and acceleration become:

$$v(t) = v_0\sum_{i=1}^{3} C_{li}(e^{s_it} - l - s_it) - \sum_{i=1}^{3} C_{2i}(s_ie^{s_it} + \omega\sin\omega t - s_i\cos\omega t) -$$
$$-\sum_{i=1}^{3} C_{3i}(e^{s_it} - e^{i\omega t}) - \sum_{i=1}^{3} C_{4i}(\omega e^{s_it} - s_i\sin\omega t - \omega\cos\omega t) \tag{19}$$

$$a(t) = v_0\sum_{i=1}^{3} C_{li}(s_ie^{s_it} - s_i) - \sum_{i=1}^{3} C_{2i}(s_i^2e^{s_it} + \omega^2\cos\omega t + \omega s_i\sin\omega t) -$$
$$-\sum_{i=1}^{3} C_{3i}(s_ie^{s_it} - i\omega e^{i\omega t}) + \sum_{i=1}^{3} C_{4i}(\omega s_ie^{s_it} - \omega s_i\cos\omega t + \omega^2\sin\omega t) \tag{20}$$

The stop condition is:

$$v(t) = 0 \tag{21}$$
from where we obtain the limit value v_0:

$$v_0 = \frac{\sum_{i=1}^{3} C_{2i}(s_i^2e^{s_it} + \omega^2\cos\omega t + \omega s_i\sin\omega t) + \sum_{i=1}^{3} C_{3i}(s_ie^{s_it} - i\omega e^{i\omega t})}{\sum_{i=1}^{3} C_{li}(s_ie^{s_it} - s_i)} -$$
$$-\frac{\sum_{i=1}^{3} C_{4i}(\omega s_ie^{s_it} - \omega s_i\cos\omega t + \omega^2\sin\omega t)}{\sum_{i=1}^{3} C_{li}(s_ie^{s_it} - s_i)} \tag{22}$$

Below this value v_0 occurs the stick-slip phenomenon. The condition to have constant velocity movement is:

$$a(t) = 0 \tag{23}$$

The value v_0 for which the movement is performed with constant velocity is:

$$v_0 = \frac{\sum_{i=1}^{3} C_{2i}(s_i^3e^{s_it} - \omega^3\sin\omega t + \omega^2 s_i\cos\omega t) + \sum_{i=1}^{3} C_{3i}(s_i^2e^{s_it} + \omega^2e^{i\omega t})}{\sum_{i=1}^{3} C_{li}(s_i^2e^{s_it})} -$$
$$-\frac{\sum_{i=1}^{3} C_{4i}(\omega s_i^2e^{s_it} + \omega^2 s_i\sin\omega t + \omega^3\cos\omega t)}{\sum_{i=1}^{3} C_{li}(s_i^2e^{s_it})} \tag{24}$$

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3. Conclusions

- Based on the model with a single mechanical degree of freedom, when turning takes place in special dynamic conditions, it is necessary to analyze the friction as a result of state variable laws.
- It is emphasized the dynamic response of the system, based on the evolution of the normal force determined by the dynamic component of the cutting force at turning, as well as by its linear dependency as function of speed, acceleration and spatial position.
- Friction stability analysis is performed using the Routh-Hurwitz criterion [6].
- There are revealed the speed and acceleration limit values, below which the movement becomes unstable.
- Friction analysis offers the possibility towards a rational design of mobile components of the CNC lathe, in order to improve the processing precision and to increase the productivity.

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