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DESIGN, SIMULATION AND MATHEMATICAL MODELLING OF THE DYNAMIC BEHAVIOR OF A VEHICLE TIRE AND CHASSIS SYSTEM AT A TURN

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Abstract:
Road security has become with time a topic of concern in our society as per the increasing number of accidents and deaths occurring on the highways. Regulatory experts on road users have constantly been working for ways to solve this problem and thence better the lives of the citizens. This paper is aimed at proposing a mathematical model integrating specific parameters, describing the dynamic lateral behavior of a vehicle's tire and chassis systems and enabling to state a relationship between road characteristics and vehicle dynamics. To achieve this, we made used of the fundamental theorems of dynamics for the modeling of the vehicle's suspended and non-suspended masses and load transfers, then we associated this with the Pacejka Tire model to obtain a complete vehicle model. After the particularization of a global model, a simulator was realized named "DYNAUTO SIMULATOR" which iterates the given variables to produce a consistent result. After an experimental research made on the Ndokoti – PK 24 road section we could, thanks to our simulator determine the maximum speed to have at every turn of this road section and also understand the effect of the modification of a vehicle's center of gravity on its stability. This work will be an important tool which can be recommended to the regulatory board as a major asset in the road construction policy and also in the improvement of road safety measures.

Keywords: Simulation; Modelling; Dynamic; Behavior; Vehicle; Road; Tire; Speed.

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1. Introduction

It is undisputable that the automobile has been a breakthrough in the engineering domain and also the most used way of transportation in the world today. Nevertheless, the rate at which people die of car accidents per year or the rate at which people are mutilated and property are destroyed by

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vehicles each year are as facts as well. As a measure to palliate this dreadful phenomenon, the study of a vehicle's behavior at a turn in order to illustrated all the risk has been conducted over the years by numerous engineers.

Our contribution to this is by basically designing a simulator which integrate the various vehicle, road and tire parameters and with the help of equations derived based on an analysis of vehicle's lateral dynamics [1] iterate and calculate the vehicle's specific speed at a particular turn on a road section.

A mathematical model is first of all designed from the fundamental theorems of dynamics and the variables which directly affect the Lateral dynamics of a vehicle which are the yaw speed and the center of gravity drift are sorted out and these will be the backbone of this software.

2. Materials and Methods

2.1. Materials

This work is centered on the programming of computer software and hence no physical material is required; all the tests were done in the computer and the various data were immediately implemented on the software. For this research paper, apart from the data collected from a research laboratory, the various materials were used such as:
1) DELL® Latitude E6220 with 250GB HDD and 4GB RAM (Laptop).
2) The Analytical software MATLAB 2016 version.

2.2. Methods

2.2.1. Elaboration of the General Model

Before calculation and derivation of equations, it is important to set a foundation for these actions. Hence, we need to establish the reference frames which will be considered at any point in time. These frames are;

- The inertial frame (relates the vehicle trajectory to road surface) $R_0(O_0, \overrightarrow{x_0}, \overrightarrow{y_0}, \overrightarrow{z_0})$
- The body frame (with respect to the suspended mass) $R_1(G, \overrightarrow{x_1}, \overrightarrow{y_1}, \overrightarrow{z_1})$
- The intermediary frame (with respect to the non-suspended mass) $R_{10}(M, \overrightarrow{x_{10}}, \overrightarrow{y_{10}}, \overrightarrow{z_{10}})$
- The intermediary reference frame for rolling $\dot{R}_{10}(O_1, \overrightarrow{x_{10}}^*, \overrightarrow{y_{10}}^*, \overrightarrow{z_{10}}^*)$
- The tires frame $R_t(B_1, \overrightarrow{x_t}, \overrightarrow{y_t}, \overrightarrow{z_t})$

The vehicle is basically a giant physical system and as such is subjected to the laws of Physics also known as Newton's Laws.

$$\overrightarrow{\Sigma F}^0 = M\vec{\gamma}_G^0 \tag{1}$$

$$\sum \overrightarrow{M_{dyn}} = \vec{\Gamma}_G^0 \tag{2}$$

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While considering the first part, the forces acting at the reference points after being derived from (1), yield the following set of equations;

$$\begin{cases} M\left[\frac{dV}{dt} - \delta V\left(\dot{\delta} + \dot{\psi}\right)\right] = F_{xfL} + F_{xfR} + F_{xrL} + F_{xrR} + Mg\,sin(\alpha_0) \\ M\left[V\left(\dot{\delta} + \dot{\psi}\right) + \delta\frac{dV}{dt}\right] - M_s + \ddot{\theta} = F_{yfL} + F_{yfR} + F_{yrL} + F_{yrR} \\ Mg\,cos(\alpha_0) = F_{zfL} + F_{zfR} + F_{zrL} + F_{zrR} \end{cases} \tag{3}$$

The Kinematic moment of the car body on G is obtained by supposing that the center of inertia of the car body is taken as the center of inertia of the whole vehicle and also that variation of the center of gravity's position due to the wheels' action is negligible. This is given by the expression:

$$\mu_G^0 = I_{sm}\vec{\Omega}_1^0$$

This expression is expanded and written as a matrix and hence the kinematic moment of the vehicle's body on G is given by:

$$\mu_G^0 = \begin{bmatrix} I_{xx}^{sm}\dot{\theta} + I_{xz}^{sm}\dot{\psi} \\ 0 \\ I_{xz}^{sm}\dot{\theta} + I_{zz}^{sm}\dot{\psi} \end{bmatrix} \tag{4}$$

The Kinematic moment of the 4 tires is obtained from determining the kinematic moment of one tire and then summing it to 4. The expression for the kinematic moment of one tire is derived from the expression below;

$$[\mu_B^0]_i^{tire} = I_i^{tire}\vec{\Omega}_{R_i}^0$$

This expression is further expanded and written as a matrix so that the kinematic moment of one tire is represented by the equation;

$$[\mu_B^0]_i^{tire} = \begin{bmatrix} 0 \\ I_{yy}^{tire}\eta_i \\ I_{zz}^{tire}\dot{\psi} \end{bmatrix}_{10}$$

Hence, summing the matrix above to the 4 tires, we get the expression for the kinematic moment of the 4 tires given as:

$$\sum_{i=1}^4 [\mu_G^0]_i^{tire} = \begin{bmatrix} 0 \\ \sum_{ij=1}^4 I_{yy}^{tire}\eta_{ij} \\ \sum_{i=1}^4 \left(I_{zz}^{tire} + m^{tire}({a_i}^2 + {b_i}^2)\right)\dot{\psi} \end{bmatrix}_{10} \tag{5}$$

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The Kinematic moment of the axles is obtained in a similar manner as seen above with that of the 4 tires. Hence of summing the matrix of the kinematic moment of one axle to 4, we get the expression:

$$\sum_{i=1}^{4}[\mu_G^0]_i^{axle} = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4}\left(I_{zz}^{axle} + m^{axle}({a_i}^2 + {b_i}^2)\right)\dot{\psi} \end{bmatrix}_{10} \tag{6}$$

Hence, the kinematic moment of the entire vehicle at the center of gravity, G which is the combination of the individual kinematic moments derived above is given by the expression;

$$\mu_G^0 = \begin{bmatrix} I_{xx}^{sm}\dot{\theta} + I_{xz}^{sm}\dot{\psi} \\ \sum_{ij=1}^{4} I_{yy}^{tire}\eta_{ij} \\ I_{xz}^{sm}\dot{\theta} + I_{zz}^{sm}\dot{\psi} + \sum_{i=1}^{4}\left(I_{zz}^{tire} + m^{tire}({a_i}^2 + {b_i}^2)\right)\dot{\psi} + \sum_{i=1}^{4}\left(I_{zz}^{axle} + m^{axle}({a_i}^2 + {b_i}^2)\right)\dot{\psi} \end{bmatrix}_{10}$$

This kinematic moment is further simplified to get the expression;

$$\mu_G^0 = \begin{bmatrix} I_{xx}\dot{\theta} + I_{xz}\dot{\psi} \\ \sum_{ij=1}^{4} I_{yy}^{tire}\eta_{ij} \\ I_{xz}\dot{\theta} + I_{zz}\dot{\psi} \end{bmatrix}_{10} \tag{7}$$

The dynamic moment of a system at a point is defined as the derivative of the kinematic moment of the system at that point. That is;

$$\vec{\Gamma}_G^0 = \frac{d^0}{dt}\vec{\mu}_G^0$$

Hence, the dynamic moment of the vehicle at the point G is given by the expression;

$$\vec{\Gamma}_G^0 = \begin{bmatrix} I_{xx}\ddot{\theta} + I_{xz}\ddot{\psi} - \sum_{ij=1}^{4} I_{yy}^{tire}\eta_{ij}\dot{\psi} \\ -I_{xx}\dot{\theta}\dot{\psi} - I_{xz}\dot{\psi}^2 + \sum_{ij=1}^{4} I_{yy}^{tire}\eta_{ij} \\ I_{xz}\ddot{\theta} + I_{zz}\ddot{\psi} \end{bmatrix}_{10} \tag{8}$$

It is imperative to define the following symbols used in the various expressions, presented in the table below;

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Table 1: Symbols used

Symbols	**Designation**	**Unit**
ΔF_{zi}	Vertical Load variation	N
M_Z	Auto-alignment moment	N.m
$\dot{\psi}$	Yaw speed	Rad/s
δ_g	Center of gravity drift	Degrees
λ	Slip angle	Degrees
γ_t	Tire trail angle	Degrees
E_1et E_2	Front and back wheel track	m
h_{CG} and h	The Center of gravity height (CG) to the ground and the active roll height.	m
M_s	Suspended Mass	Kg
$C_{\theta 1}$	Front anti-roll stiffness	N/rad
$C_{\theta 2}$	Rear anti-roll stiffness	N/rad
$k_{\delta 1}$	Front drift rigidity	N/rad
$k_{\delta 2}$	Rear drift rigidity	N/rad
I_{sm}	The Inertial matrix of the suspended mass	Kgm^2
μ	Maximum traction coefficient	
$\vec{\Gamma}_G^0$	The dynamic moment of the vehicle acting on the CG.	
$\vec{\gamma}_G^0$	The CG acceleration in the frame R_0.	m/s^2

The next step is determining expression for the vertical loads of the axles which are the front and rear axles, the values for these loads greatly influence derivability. These expressions are given below in terms of the transversal acceleration;

At the Front axle;

$$\Delta F_{zf} = \frac{M_f h_1 + Mh\left(\frac{C_{\theta 1}}{C_\theta}\right)\theta}{E_1}\gamma_t \tag{9}$$

At the rear axle;

$$\Delta F_{zr} = \frac{M_r h_2 + Mh\left(\frac{C_{\theta 2}}{C_\theta}\right)\theta}{E_2}\gamma_t \tag{10}$$

Finally, determining these vertical loads at each of the tires is important in the understanding of the vehicle's dynamic behavior. This is an accurate interpretation of the Pacejka model [16] as it clearly illustrates the tire/road relationship. Hence for each of the tires, their vertical loads are;

Front left tire: $F_{zfL} = \frac{1}{2}M_f g\cos\alpha_0 - \frac{1}{E_1}\left(M_f h_1 + Mh\frac{C_{\theta 1}}{C_\theta} + A_{\theta 1}\dot{\theta}\right)(\gamma_t \cos\lambda - g\sin\lambda)$ (11)

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Front right tire: $F_{zfR} = \frac{1}{2} M_f g \cos \alpha_0 + \frac{1}{E_1}\left(M_f h_1 + Mh \frac{C_{\theta 1}}{C_\theta} + A_{\theta 1}\dot{\theta}\right)(\gamma_t \cos \lambda - g \sin \lambda)$ (12)

Rear left tire: $F_{zrL} = \frac{1}{2} M_r g \cos \alpha_0 - \frac{1}{E_2}\left(M_r h_2 + Mh \frac{C_{\theta 1}}{C_\theta} + A_{\theta 2}\dot{\theta}\right)(\gamma_t \cos \lambda - g \sin \lambda)$ (13)

Rear right tire: $F_{zrR} = \frac{1}{2} M_r g \cos \alpha_0 + \frac{1}{E_2}\left(M_r h_2 + Mh \frac{C_{\theta 1}}{C_\theta} + A_{\theta 2}\dot{\theta}\right)(\gamma_t \cos \lambda - g \sin \lambda)$ (14)

2.2.2. Elaboration of a Particular Model

This model is based on the general model above with the only particularity that it is centered only on the movement of the vehicle in the lateral plane (lateral dynamics). Studying lateral dynamics implies that the moment will only be considered in the x and z directions of the plane. That is;

Along x;

$$(I_{xx} + M_s h^2)\ddot{\theta} + I_{xz}\ddot{\psi} - Mh\left(V\left(\dot{\delta} + \dot{\psi}\right) + \delta \frac{dV}{dt}\right) = M_{\theta 1}(\theta) + M_{\dot{\theta} 1}(\dot{\theta}) + M_g(\theta)$$

Along z;

$$I_{xz}\ddot{\theta} + I_{zz}\ddot{\psi} + Mh\theta \frac{dV}{dt} = L_1(F_{yfL} + F_{yfR}) - L_2(F_{yrL} + F_{yrR})$$

In this case, the vehicle speed is a constant because the longitudinal dynamics is not considered. Hence for this model only the yaw speed and center of gravity drift equations from the general model equations;

$$MV\left(\dot{\delta} + \dot{\psi}\right) = F_{yfL} + F_{yfR} + F_{yrL} + F_{yrR} \quad (15)$$

$$I_{zz}\ddot{\psi} = L_1(F_{yrL} + F_{yfR}) - L_2(F_{yrL} + F_{yrR}) \quad (16)$$

In this study, we introduced the drift coefficient which is the linear relation between the transversal force and the drift angle of the vehicle. It is hence given by;

$$F_{yi}(\delta_i) = -k_{\delta_i}\delta_i \quad (17)$$

The drift coefficient after being substituted into the two equations (15) and (16) above will modify them into;

$$MV\left(\dot{\delta}(t) + \dot{\psi}(t)\right) = -2k_{\delta 1}\left(\delta(t) + \frac{L_1}{V}\dot{\psi}(t) - \varepsilon_r(t)\right) - 2k_{\delta 2}\left(\delta(t) - \frac{L_2}{V}\dot{\psi}(t)\right) \quad (18)$$

And

$$I_{zz}\ddot{\psi}(t) = -2L_1 k_{\delta 1}\left(\delta(t) + \frac{L_1}{V}\dot{\psi}(t) - \varepsilon_r(t)\right) + 2L_2 k_{\delta 2}\left(\delta(t) - \frac{L_2}{V}\dot{\psi}(t)\right) \quad (19)$$

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In order to determine the expression for the yaw speed equation (19) is reduced and solved as a differential equation of second order. Hence, we get the expression of the yaw speed of a vehicle in a stabilized mode as;

$$\dot{\psi}(t) = \frac{VL_1k_{\delta1}}{(L_1^2k_{\delta1}+L_2^2k_{\delta2})}\left[1-\exp\left(-\frac{2(L_1^2k_{\delta1}+L_2^2k_{\delta2})t}{VI_{ZZ}}\right)\right]\frac{\pi}{180}\varepsilon_r \tag{20}$$

A block representation of this model is setup to illustrate the inputs, every parameter in play as well as the outputs which in this case are the yaw speed and the center of gravity drift.

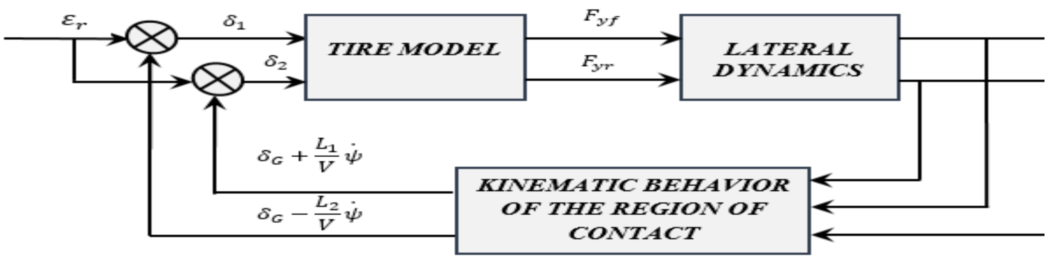

Figure 1: Block representation of the Dynamic model.

The Center of gravity drift is obtained in the same manner as the yaw speed gotten above. That is, reducing the equation to a differential equation and then solving it to get its expression.

$$\delta_g(t) = -(\alpha+K)\exp\left(-\frac{2(k_{\delta1}+k_{\delta2})}{MV}t\right) + K\exp\left(-\frac{2(L_1^2k_{\delta1}+L_2^2k_{\delta2})}{VI_{zz}}t\right) + \alpha \tag{21}$$

In addition to these two important variables which basically make up the core of our model, we introduced a variable which is often considered a factor of stability of a vehicle and it is called the Over steering rate. This rate is closely related to whether the vehicle is over steering or under steering. It is given by the expression;

$$R_{OS} = \left(\frac{d\varepsilon_s(\gamma_t)}{d\gamma_t}\right) = \frac{M_f}{k_{\delta1}} - \frac{M_r}{k_{\delta2}} \tag{22}$$

2.2.3. Simulation of the Model

The mathematical model being established, we can go forth to program the simulator that is going to implement this model and provide quantifiable results or values. This simulator is called Dynauto Simulator (a short form for Dynamic Automotive Simulator); It is a simple and user-friendly GUI (Graphical User interface) developed in MATLAB. Vehicle data, tire data, road characteristics are loaded into the interface as inputs and then the simulator runs the equations modelled using these parameters to get the yaw speed, center of gravity drift as well as the other variables that affect the vehicle's behavior at a turn. Note that this conclusion is also based on the road characteristics. To illustrate this, we are going to simulate the dynamic behavior at a particular turn of a chosen road section with a vehicle specification, case of the 1996 Nissan Almera;

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Tire Characteristics
Brand and type; Michelin 245/45R18-100WO.1
Tire trail; 0.025m
Road Section Characteristics
Slope; 7°
Transversal Slope; 2.5°
Curvature radius; 70m

3. Results and Discussion

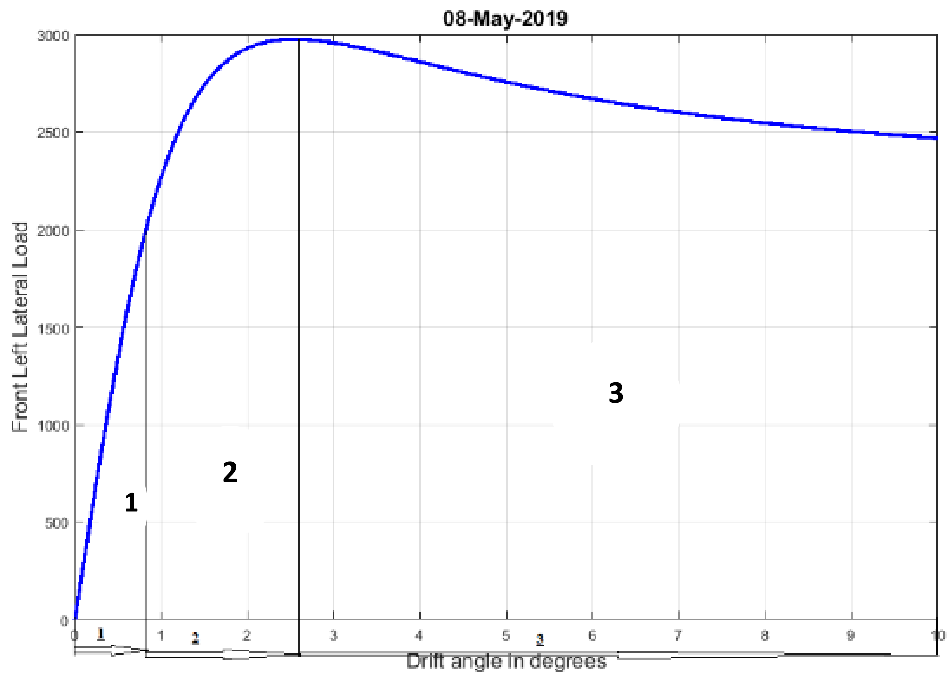

Figure 2: The Graph illustrating Lateral Load vs Drift angle for the front tire.

This curve is generated by the simulator, divided in three zones.

Zone 1: Represents the Tire adherence zone. Here the tire adheres to the road, the more the drift angle increases, the more the transversal effort generated by the tire increases.

Zone 2: Represents the Transition zone where the tire starts to slip. The curve increases in a non-linear manner before reaching a maximum point also known as saturation. Even as the drift angle increases, the transversal effort does not increase anymore.

Zone 3: Represents the slip zone where the tire has completely lost adherence. Here, the transversal effort decreases as the drift angle increases. There is tire slip; meaning a loss in derivability as the vehicle can't go on the wanted trajectory. If the driver increases the steering angle and thus the wheel camber, he increases the drift angle and consequently decreases the transversal effort generated by the tire.

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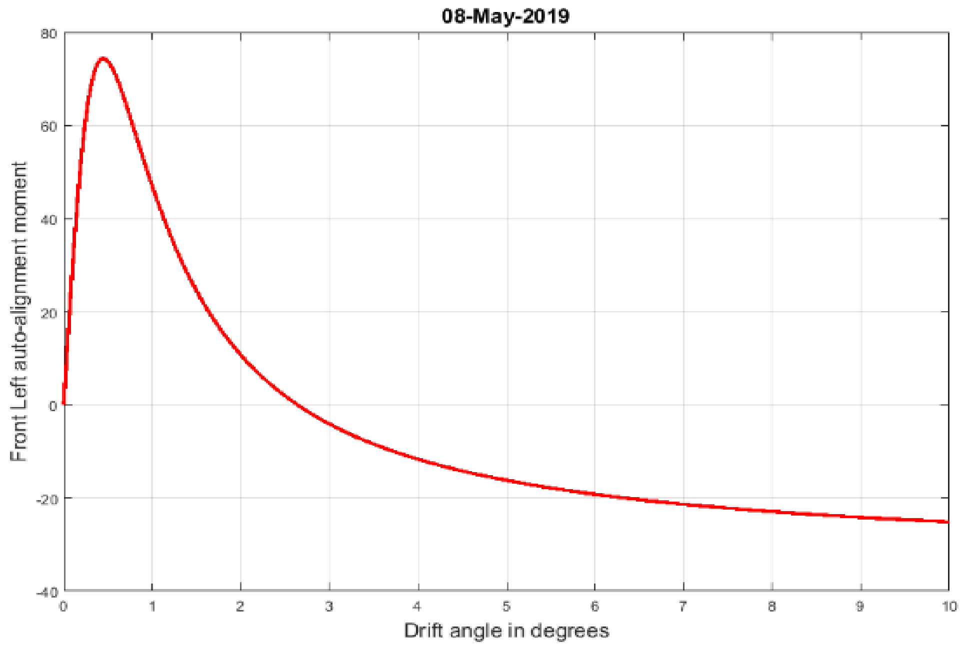

Figure 3: The Graph illustrating auto-alignment moment vs the drift angle for the front left tire. This curve reaches a maximum then decreases abruptly till the auto-alignment moment reaches zero. If the drift angle keeps increasing, the auto-alignment sign changes to negative.

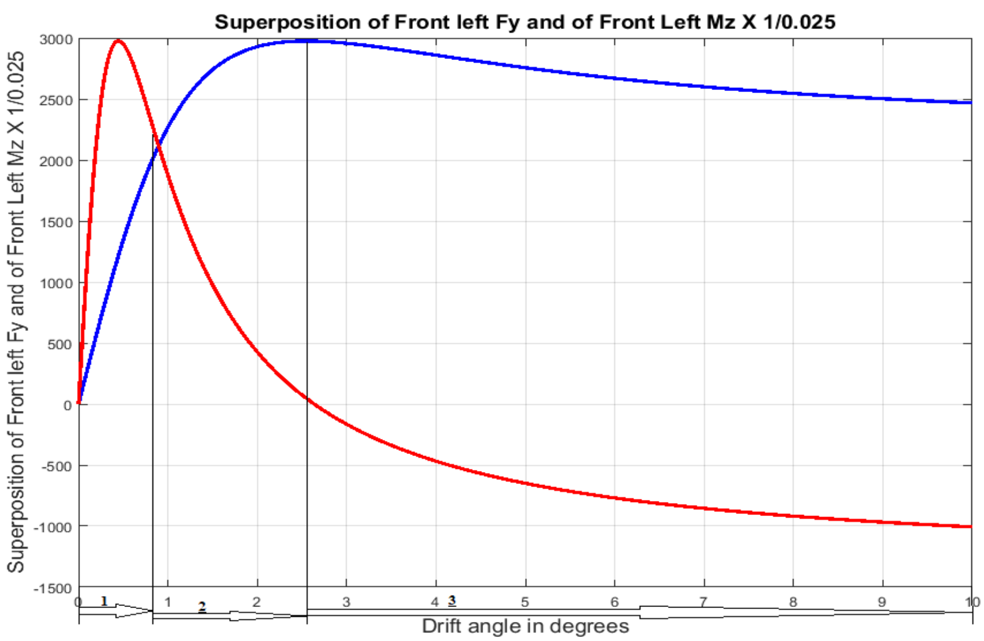

Figure 4: Superposition of lateral load to drift angle and auto-alignment moment to drift angle.

This superposition graph is generated by the simulator in order for the user to study and compare the evolution of both properties. It is also divided in three zones;
Zone 1; Adherence zone
Zone 2; Transition zone.
Zone 3; Slip zone.

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In the first zone, we have a simultaneous increase of the auto-alignment moment M_z and the lateral load F_y with respect to the drift angle. The superposition of these curves enables us to note that M_z reaches its maximum before F_y. The fall in M_z is related to the tire entry into the transition zone.

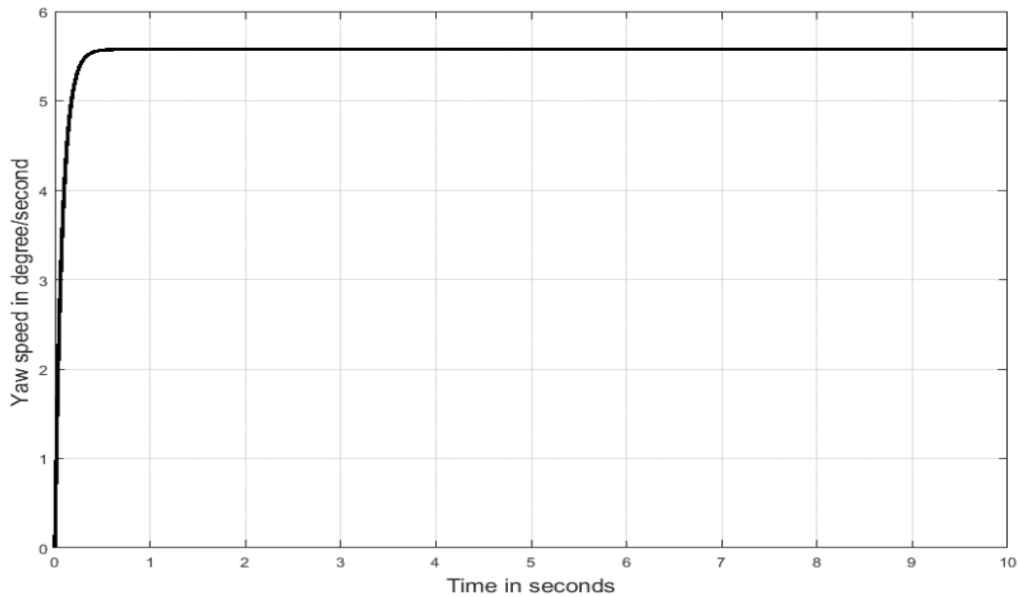

Figure 5: Vehicle's yaw speed with respect to time.

This yaw speed is specific for the vehicle's steering angle (30° in this case) and is an essential property in the study of the vehicle lateral dynamics.

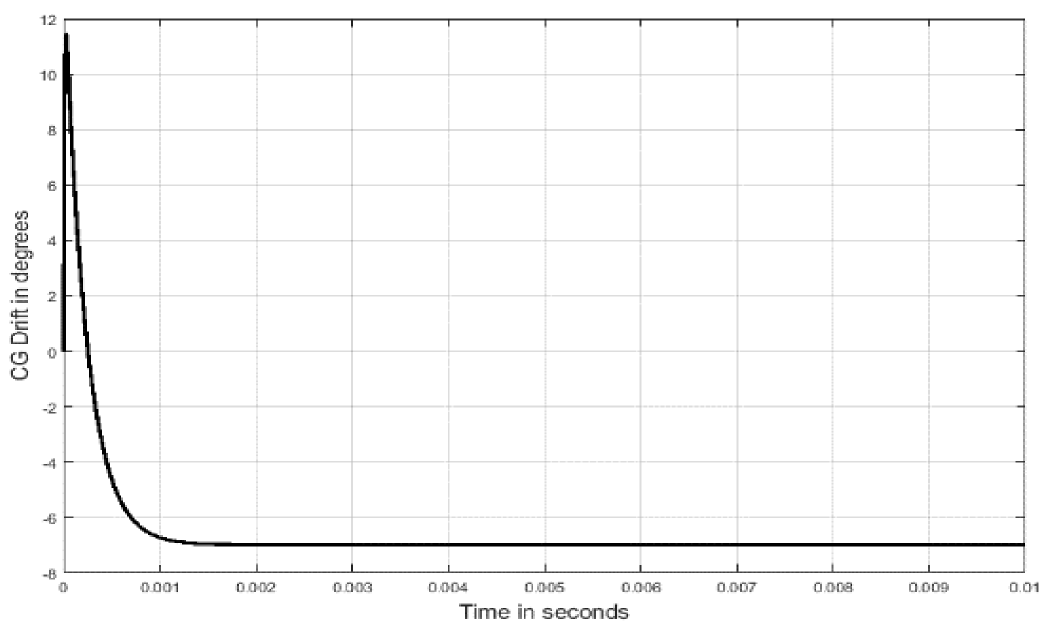

Figure 6: Vehicle's center of gravity drift with respect to time.

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The simulator is hence used to iterate the vehicle's speed on the chosen road section in order to determine the specific speed to have at a turn in that section. The table below illustrates the road parameters collected at the various turn on the Ndokoti – PK24 road section;

Table 2: Road data

Location	Slope (°)	Transversal Slope (°)	Curvature Radius (M)	Steering Angle (°)
PK 17 (A)	18.5	6	31	25
PK17 (B)	12	3	41	20
PK 18 (A)	5	2.5	80	11
PK 18 (B)	4	2.5	60	18
PK 21	7	3	22	26
PK 24	3	3.4	55	20

As said earlier, this procedure works for any brand-new car and any type of passengers' vehicle, as long as the user has all the necessary information concerning this vehicle and the type of tire used. It should be noted that a modification in a vehicle's center of gravity affects its lateral dynamic behavior and hence the speed values are not the same. In this case, the new center of gravity height should be introduced, and the iterations made again to get the new values for the modified vehicle.

4. Conclusion

The constant road accidents leading to severe loss of lives and properties has prompted the research of this article which deals with studying the dynamic behavior of a vehicle's tire and chassis systems at a turn with the focus on determining the specific maximum speed that a particular vehicle should have at a particular turn on a road section. Modelling the vehicle's dynamic behavior from its basic structural characteristics also reveals how this behavior can be influenced by a modification applied in the vehicle's structure. The simulator which takes into consideration all the various aspects of a vehicle's dynamics appears to be an essential tool in the fight against road accidents and thus the betterment of the lives of all road users in general.

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