



## MHD FREE CONVECTION BOUNDARY LAYER FLOW OF A NANO FLUID OVER A PERMEABLE SHRINKING SHEET WITH NTH ORDER CHEMICAL REACTION

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### Abstract:

*An analysis is presented to study the free convective unsteady magnatohydrodynamic boundary layer flow of a Nano fluid over a permeable shrinking sheet in the presence of nth order chemical reaction. Magnetic field of varying strength is applied normal to the sheet. The Nano fluid model under consideration includes Brownian motion, thermophoresis effects and nth order chemical reaction. The governing partial differential equations are transformed into a set of ordinary differential equations by applying the local similarity transformations and then the highly coupled nonlinear differential equations are solved by the method of lines. The effect of various controlling flow parameters on the dimensionless velocity, temperature and nanoparticle volume fraction profiles are analyzed.*

**Keywords:** Nanofluid; Shrinking Sheet; Magnetic Field; Thermal Radiation; Chemical Reaction; Method of Lines.

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### 1. Introduction

The flow over a shrinking surface is an important problem in many engineering processes with applications in industries such as the wire drawing, hot rolling and glass wire production. In nature, the occurrence of uncontaminated air or water is not possible. Some foreign mass may be present either naturally or mixed with the air or water. The current trend in the field of magnetic potency analysis is to give a mathematical model for the system to forecast the reactor routine. A huge amount of research work has been reported in this field. Especially in the study of heat and mass transfer with magnetic effect is of considerable importance in chemical and hydrometallurgical industries.

Bhattacharyya and Gupta [1], Gupta and Gupta [2] and Cheng and Lin [3] presented the nonlinear MHD boundary layer flow of heat and mass transfer in various situations. The study of

boundary layer flow over a shrinking surface is an ideal concept in several industrial processes. Such situations take place in polymer dispensation, manufacturing of glass sheets, paper manufacture, in textile industries and many others. The most common applications of shrinking sheet problems in engineering and industries are shrinking film. In wrapping of bulk products, shrinking film is very useful as it can be unwrapped easily with adequate heat. Crane [4] initiated a study on the boundary layer flow of a viscous fluid towards a linear stretching sheet. A precise similarity solution for the dimensionless differential system was obtained. Carragher and Carane [5] analyzed heat transfer on a continuous stretching sheet. Afterwards, many investigations [6 to 12] were made to examine flow over a stretching/shrinking sheet under dissimilar aspects of MHD, suction / injection, heat and mass transfer etc. Wang and Pop [13] reported on an analysis of the flow of a power-law fluid film on an unsteady stretching surface by applying homotopy analysis method. In these attempts, the boundary layer flow owing to stretching/shrinking has been reported. Magyari and Keller [14] provided both analytical and numerical solutions for boundary layer flow over an exponentially stretching surface with an exponential temperature distribution. The joint effects of viscous dissipation and mixed convection on the flow of a viscous fluid over an exponentially stretching sheet were studied by Partha et al. [15]. Elbashbeshy [16] numerically analyzed flow and heat transfer over an exponentially stretching surface with wall mass suction. Macha Madhu and Naikoti Kishan [17] studied the Magneto hydrodynamic mixed convection stagnation-point flow of a power-law non-Newtonian nanofluid towards a stretching surface in the presence of thermal radiation and heat source/sink.

The list of significance of flows in fluid mechanics is the flow over a shrinking sheet and is a new field of research at present and few literatures is available on this area of research now. Wang [18] was the first one to study a specific shrinking sheet problem. Recently, Miklavcic and Wang [19] obtained the existence and uniqueness of the solution for steady viscous hydrodynamic flow over a shrinking sheet with mass suction. Hayat et al. [20] derived exact and series solution using HAM describing the magneto hydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. Sajid and Hayat [21] solved the problem of MHD viscous flow due to a shrinking sheet by using HAM.

It is interesting to note that the Brownian motion of nanoparticles at molecular and nanoscale levels are a key nanoscale mechanism governing their thermal properties. Based on the size of the particle in the nanometer scale, Brownian motion and its effect on the surrounding liquids play an important role in the heat transfer. On account of these applications, Nield and Kuznetsov ([22, 23]) analyzed the free convective boundary layer flows in a porous medium saturated by nanofluid by taking Brownian motion and thermophoresis effects into concern. In the first article, the authors have assumed that nanoparticles are suspended in the nanofluid by using either surfactant or surface charge technology and they have accomplished that this prevents particles from agglomeration and deposition on the porous matrix. Chamkha *et al.* [24] carried out a boundary layer analysis for the natural convection past an isothermal sphere in a Darcy porous medium saturated with a nanofluid. The cross-diffusion in nanofluids was investigated by Nield and Kuznetsov [25] with an aim of making a detailed comparison with regular cross diffusion effects and the cross-diffusion effects peculiar to nanofluids and at the same time investigating the interaction between these effects when the base fluid of the nanofluid is itself a binary fluid such as salty water. Recent studies on boundary layer analysis for the

natural convection past a horizontal plate in a porous medium saturated with a Nano fluid is analyzed by Gorla and Chamkha [26].

Kishan et.al [27], studied the unsteady MHD flow of heat and mass transfer of Cu-water and TiO<sub>2</sub>-water Nano fluids over stretching sheet with a non-uniform heat source or sink by taking in to account the effects of viscous dissipation and chemical reaction.

On Nano fluids the applied magnetic field has significant effects and also has substantial applications in science and engineering. These include cooling of incessant filaments, in the process of drawing, annealing and thinning of copper wire. Drawing the strips through an electrically conducting fluid which is subject to a magnetic field can control the rate of cooling and stretching, there by furthering the desired characteristics of the final item for consumption. Magneto hydro magnetic flow of an incompressible viscous fluid caused by deformation of a surface was discussed by Pavlov [28]. In further work, Jafar et al. [29] stated the effects of magnetohydrodynamic (MHD) flow and heat transfer due to a stretching/shrinking sheet with an external magnetic field, viscous dissipation and Joule effects. Samirkumar et al. [30] studied the forced convection in unsteady boundary layer flow of a Nano fluid over a permeable shrinking sheet in the presence of thermal radiation recently. Thermal Radiative Transfer Properties of a fluid was reported by Brewster [31]. Vajravelu et al. [32] analyzed the MHD flow and heat transfer of an Ostwald-de Waele fluid over an unsteady stretching surface

Recently Anuradha and Priyadharshini [33] investigated MHD Free Convection Boundary Layer Flow of a Nano fluid over a Permeable Shrinking Sheet in the Presence of Thermal Radiation and Chemical Reaction. This paper investigated the effects of nth order chemical reaction and heat transfer due to the unsteady two dimensional laminar flow of an incompressible viscous Nano fluid due to a permeable shrinking sheet with thermal radiation effects. The results are discussed focus on how the chemical reaction parameter, thermal radiation, magnetic field, Brownian motion, thermophoresis effects of the heat transfer and characteristic of the flow.

## 2. Mathematical Formulation

Consider unsteady two-dimensional laminar boundary-layer flow of incompressible electrically conducting viscous nanofluid past a permeable shrinking sheet. The fluid flow is subjected to a transverse magnetic field of strength  $B_0$  which is assumed to be applied parallel to the y axis and chemical reaction with thermal radiation is considered in the flow region. It is assumed that the velocity of the shrinking sheet is  $u_w(x,t)$  and the velocity of the mass transfer is  $v_w(x,t)$ , where  $x$  is the coordinate measured along the shrinking sheet and  $t$  is the time. Also it is assumed that the constant surface temperature and concentration of the sheet are  $T_w$  and  $C_w$ , while the uniform temperature and concentration far from the sheet are  $T_\infty$  and  $C_\infty$  respectively. Further the induced magnetic field, the external electric field is negligible due to polarization of charges. Under the above assumptions, the boundary layer equations prevailing the flow, thermal and concentration fields can be written in dimensional form in the presence of magnetic field towards a permeable shrinking sheet in terms of the Cartesian coordinates  $x$  and  $y$  as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k_p} u - \frac{\sigma_e B_0^2 u}{\rho} + (1 - C_\infty) \rho f_\infty \beta g (T - T_\infty) - (\rho_f - \rho_{f_\infty}) g (C - C_\infty) \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) + \tau \left\{ D_B \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{Q_0}{(\rho c)_f} (T - T_\infty) - \frac{1}{(\rho c)_f} \left( \frac{\partial q_r}{\partial y} \right) \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} - K_r (C - C_\infty)^n \quad (4)$$

The relevant Boundary conditions are:

$$u = u_w(x, t) = -\frac{cx}{(1-\lambda t)}, v = v_w(x, t), T = T_w(x, t), C = C_w(x, t) \text{ at } y = 0$$

$$u = 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (5)$$

The wall mass transfer velocity then becomes

$$v_w(x, t) = -\sqrt{\frac{c}{(1-\lambda t)}} s \quad (6)$$

Where  $s$  is the constant wall mass transfer parameter with  $s > 0$  for suction and  $s < 0$  for injection, respectively.

Where  $u$  and  $v$  are velocity components along  $x$  and  $y$  directions,  $\alpha$  is a thermal diffusivity,  $Q_0$  is a heat generation coefficient,  $\rho$  is the density of Nano fluid,  $\rho_p$  is the nanoparticle density,  $\rho_c$  is specific heat of Nano fluid at constant pressure,  $\tau$  is the ratio of nanoparticle heat capacity,  $\sigma_e$  is the electrical conductivity,  $C_p$  is the specific heat and constant pressure,  $\beta$  is volumetric thermal expansion coefficient,  $\mu$  is the thermal viscosity,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoresis diffusion coefficient and  $K_r$  is the rate of chemical reaction.

By using Rosseland approximation the Radiative heat flux term is given by

$$q = \frac{4\sigma^*}{3k_1^*} \frac{\partial T^{*4}}{\partial y^*} \quad (7)$$

Where  $\sigma^*$  and  $k_1^*$  are the Stefan-Boltzmann constant and the mean absorption coefficient in that order. It is assumed that the temperature difference within the flow are sufficiently small such that  $T^{*4}$  may be expressed as a linear function of the temperature. This is achieved by expanding in a Taylor Series about  $T_\infty^*$  and neglecting higher order terms. Thus,

$$T^{*4} \cong 4T_\infty^{*4} - 3T_\infty^{*4} \quad (8)$$

By using equation (6) and (7), into equation (3) is reduced to

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) + \tau \left\{ D_B \left( \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right\} + \frac{Q_0}{(\rho c)_f} (T - T_\infty) - \frac{1}{(\rho c)_f} \left( -\frac{16\sigma^* T_\infty^{*3}}{3k_1^*} \right) \frac{\partial^2 T^*}{\partial y^2} \quad (9)$$

The equations (2), (4) and (9) can be transformed into the ordinary differential equation by using the following similarity transformations.

$$\eta = y \sqrt{\frac{c}{v(1-\lambda t)}}, \psi = \sqrt{\frac{cv}{v(1-\lambda t)}} x f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{10}$$

And the stream function  $\psi(x, y)$  is defined such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{11}$$

By applying the above transformations the non-dimensional, nonlinear, coupled differential equations are obtained as:

$$f''' + ff'' - f'^2 - A \left( f' + \frac{\eta}{2} f'' \right) - Mf' - \delta f + Ra_x(\theta - Nr\varphi) = 0 \tag{12}$$

$$\frac{1}{Pr_{eff}} \theta'' + f \theta' - A \frac{\eta}{2} \theta' + Nb\varphi' \theta' + Nt \theta'^2 = 0 \tag{13}$$

$$\varphi'' + Le \left( f' - \frac{\eta}{2} f'' \right) \varphi' + \left( \frac{Nt}{Nb} \right) \theta'' + \gamma \varphi^n = 0 \tag{14}$$

Where

$Pr_{eff} = \frac{Pr}{\left(1 + \frac{4R}{3}\right)}$ (Prandtl Number)	$A = \frac{\lambda}{c}$ (Heat Source Parameter)
$\lambda = \frac{Q_0 x^{1/2}}{(\rho c)_f \sqrt{(1-C_\infty)g\beta(T_w - T_\infty)}}$ (Heat source parameter)	$M = \frac{\sigma_e B_0^2}{(\rho C)_f}$ (Magnetic Parameter)
$\delta = \frac{\mu}{k_\rho} u$ (Permeable Parameter)	$\nu = \frac{\mu}{\rho_f}$ (Kinematic Viscosity)
$Le = \frac{\alpha}{D_B}$ (Lewis Number)	$R = \frac{4\sigma^* T_\infty^3}{k\alpha_m \rho_f c}$ (Radiation Parameter)
$Ra_x = \frac{(1-C_\infty)\beta g f_\infty (T_w - T_\infty)}{c^2 x}$ (Local Rayleigh Number)	$Re_x = \frac{ax^2}{\nu_\infty}$ (Local Reynolds Number)
$Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$ (Thermophoresis Parameter)	$\gamma = \frac{ku(C_w - C_\infty)}{\nu}$ (Chemical Reaction Parameter)

$$Nr = \frac{(\rho_p - \rho_{f_\infty}) \beta g f_\infty (C_w - C_\infty)}{(1 - C_\infty) f_\infty \beta (T_w - T_\infty)} \text{ (Buoyancy Ratio Parameter)} \quad Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu} \text{ (Brownian motion Parameter)}$$

The corresponding Boundary conditions are

$$\left. \begin{aligned} f = s, f' = -1, \theta = 1, \varphi = 1 \text{ at } \eta = 0 \\ f = 0, \theta = 0, \varphi = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (15)$$

The physical quantities of Skin friction ( $C_f$ ), the local Nusselt number ( $Nu$ ), and the local Sherwood number ( $Sh$ ) are calculated by the following equations:

$$C_f (Re_x)^{-1/2} = f''(0) \quad Nu (Re_x)^{-1/2} = -\theta'(0) \quad Sh (Re_x)^{-1/2} = -\varphi'(0) \quad (16)$$

### 3. Numerical Analysis

The set of non-dimensional, non-linear coupled boundary layer equations (12) – (14) subject to boundary conditions (15) are non-linear and possess no analytical solution and must be solved numerically. The governing equations are solved by using Method of line. In this study, we get the validity of the present computations which has been confirmed via benchmarking with several earlier studies. Exceptional convergence was achieved for all the results.

### 4. Results and Discussion

In this paper we investigate MHD flow of the nanofluids over a permeable shrinking sheet using boundary layer flow with heat and mass transfer problem in the presence of thermal radiation and chemical reaction. The velocity, temperature and concentration fields have been established by assigning some numerical values to the various influential parameters, including Thermophoresis parameter (Nt), Brownian motion parameter (Nb) and Chemical reaction parameter  $\Upsilon$  respectively. The graphical results of the numerical computations are given by the figures.1-18.

Fig.1, Fig.2 & Fig.3 represents the influence of Brownian motion parameter (Nb) on the velocity temperature and concentration fields. In these figures shows that, all fields are decreases with an increasing values of Brownian motion parameter (Nb).

Fig.4, Fig.5 & Fig.6 represents the influence of Thermophoresis parameter (Nt) on the velocity temperature and concentration fields. In these figures shows that, the thermophoresis parameter same as Brownian motion parameter and all fields are decreases with an increasing values of thermophoresis parameter (Nt).

Fig.7, Fig.8 & Fig.9 represents the effect of order chemical reaction parameter on the velocity, temperature and concentration profiles respectively. It is observed from these figures that an increase in the chemical reaction parameter increases the temperature and concentration profiles and decreases the velocity profile.



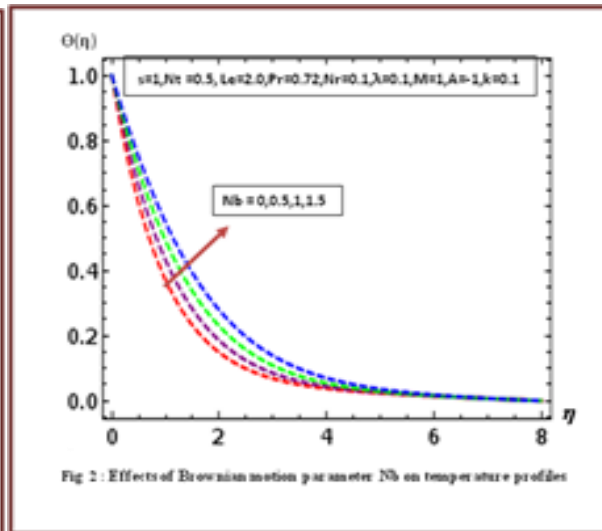
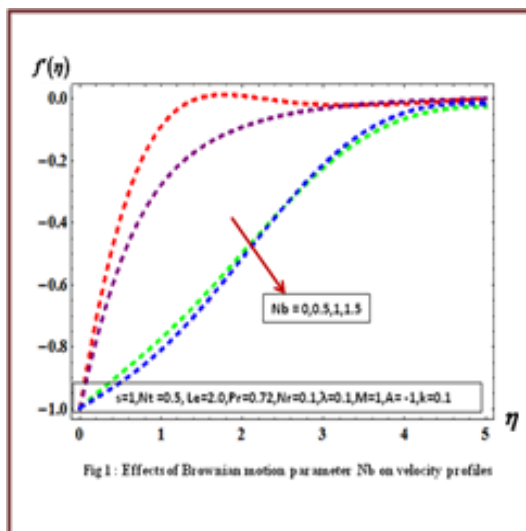
Fig.10, Fig.11 & Fig.12 represents the effect of Brownian motion parameter (Nb) on the skin friction, nusselt number and Sherwood number respectively. It is observed from these figures that an increase in Brownian motion parameter (Nb) increases nusselt number and Sherwood number and decreases skin friction.

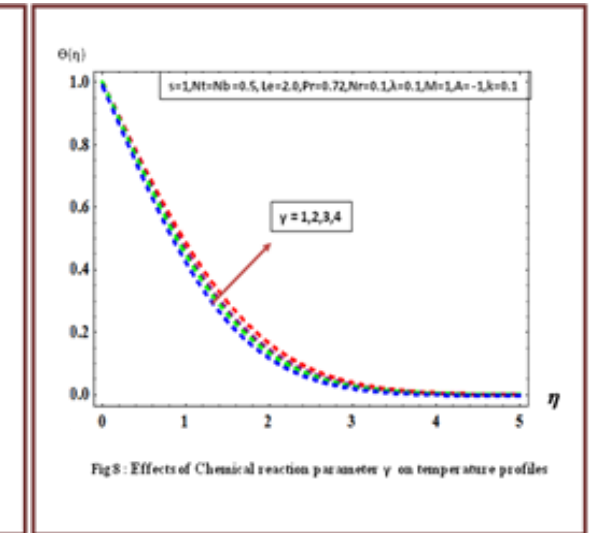
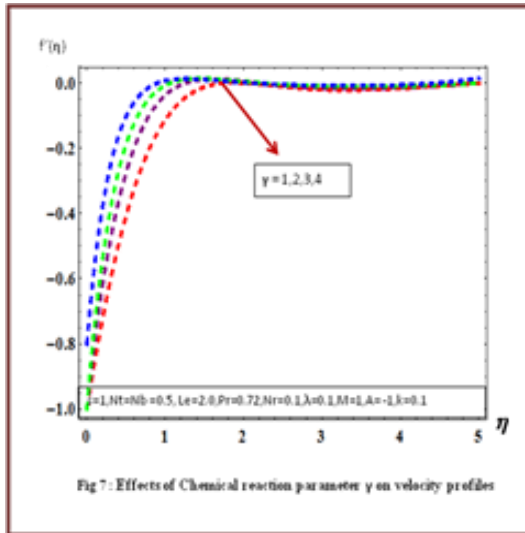
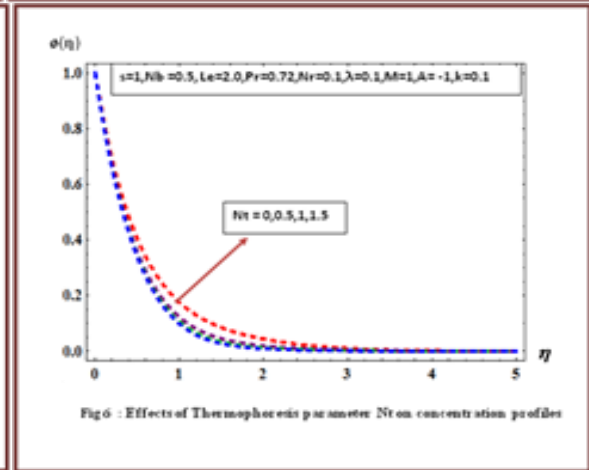
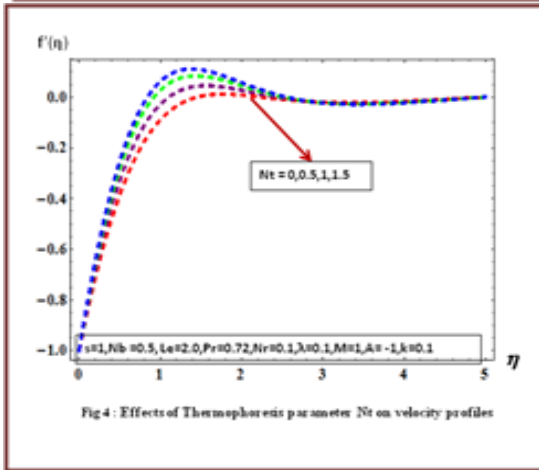
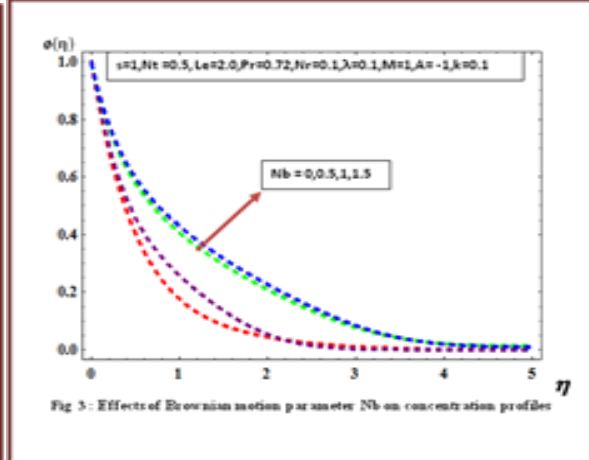
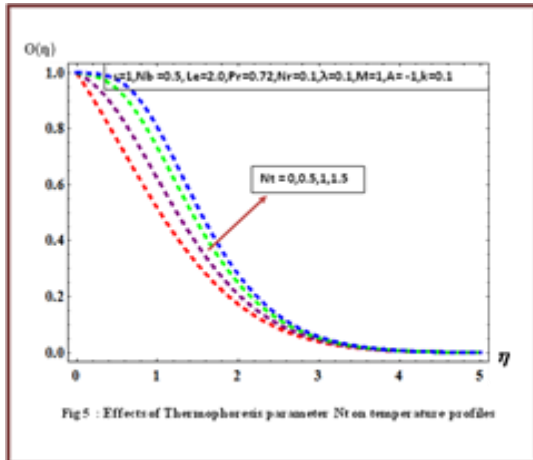
Fig.13, Fig.14 & Fig.15 represents the effect of Thermophoresis parameter (Nt) on the skin friction, nusselt number and Sherwood number respectively. It is observed from these figures that an increase in Thermophoresis parameter (Nt) increases nusselt number and Sherwood number and decreases skin friction.

Fig.16, Fig.17 & Fig.18 represents the effect of Thermophoresis parameter (Nt) on the skin friction, nusselt number and Sherwood number respectively. It is observed from these figures that an increase in Thermophoresis parameter (Nt) increases nusselt number and Sherwood number and decreases skin friction.

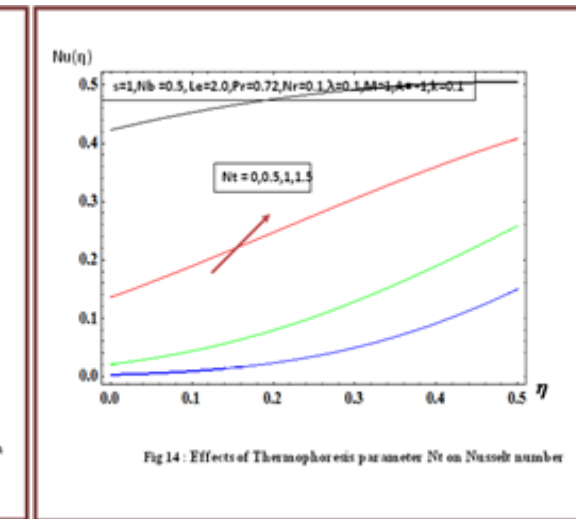
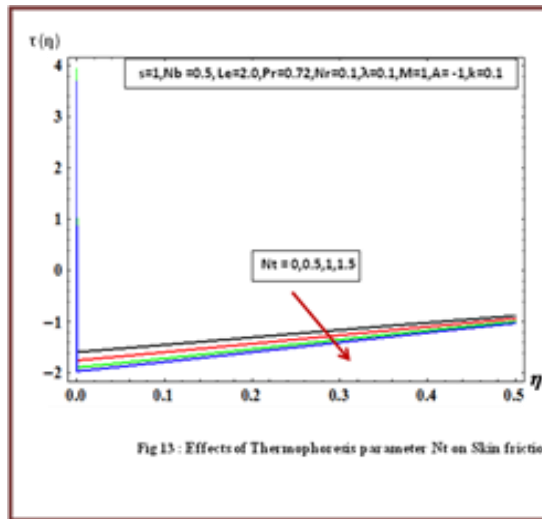
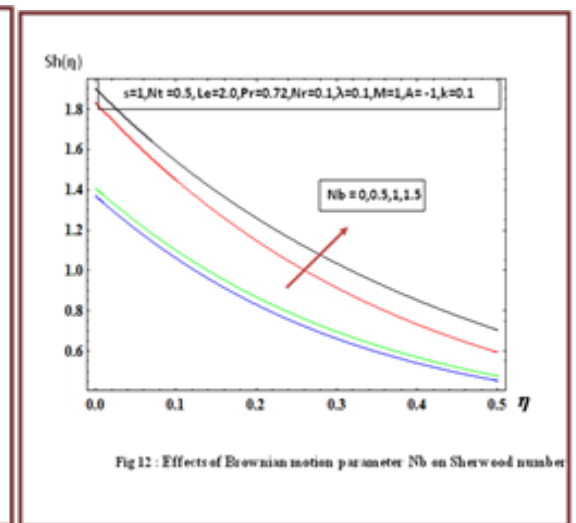
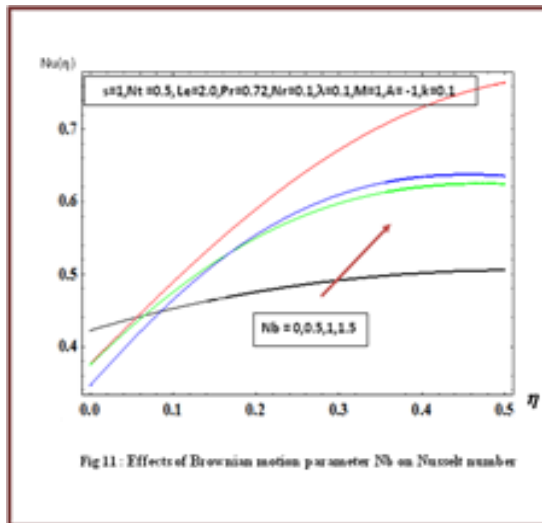
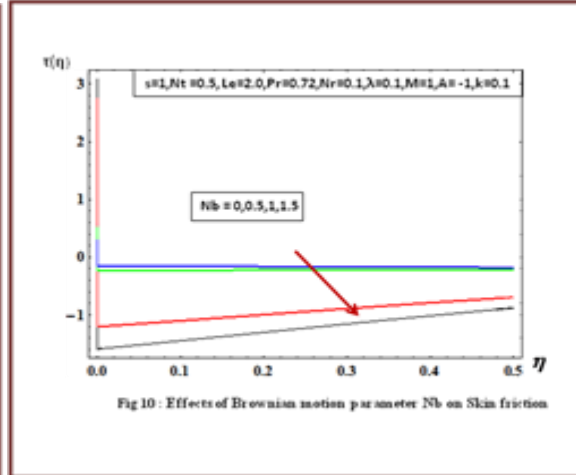
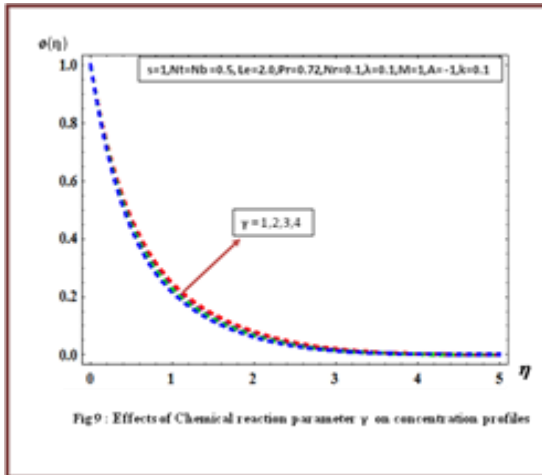
### 5. Conclusion

This paper presents MHD flow of the Nano fluids over a permeable shrinking sheet using boundary layer flow with heat and mass transfer problem in the presence of thermal radiation and chemical reaction. The dimensionless governing equations are transformed by applying the local similarity transformations and then the highly coupled nonlinear differential equations are solved by the method of lines. The effect of various controlling flow parameters on the dimensionless velocity, temperature and concentration profiles is analyzed. The present study serves as the scientific tool for understanding nth order chemical reaction through velocity, temperature and concentration profiles.









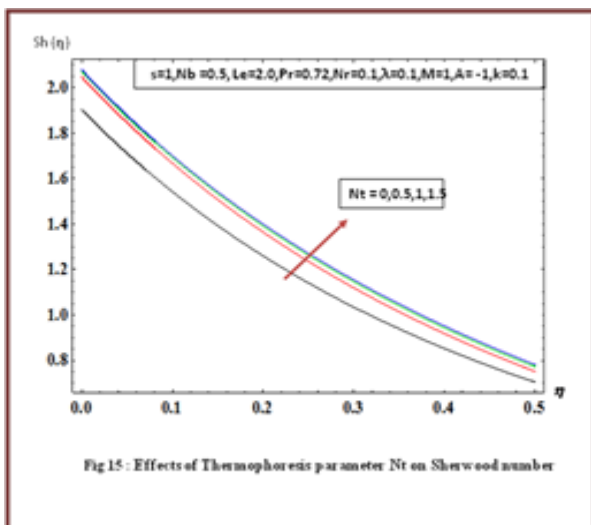


Fig 15 : Effects of Thermophoresis parameter  $Nt$  on Sherwood number

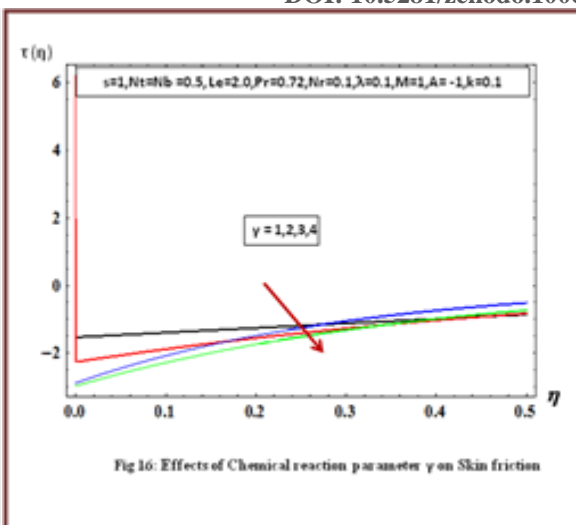


Fig 16: Effects of Chemical reaction parameter  $\gamma$  on Skin friction

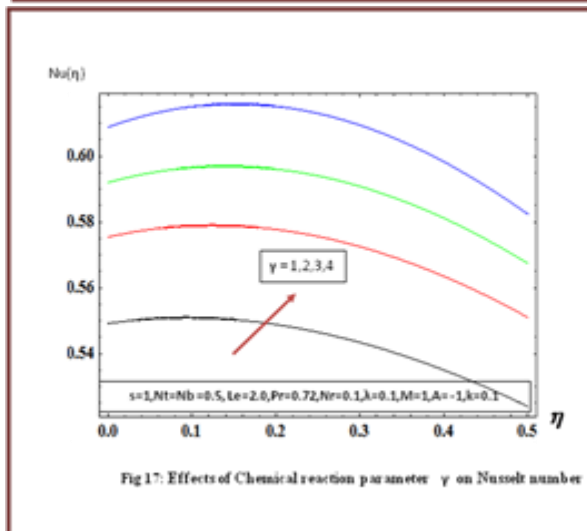


Fig 17: Effects of Chemical reaction parameter  $\gamma$  on Nusselt number

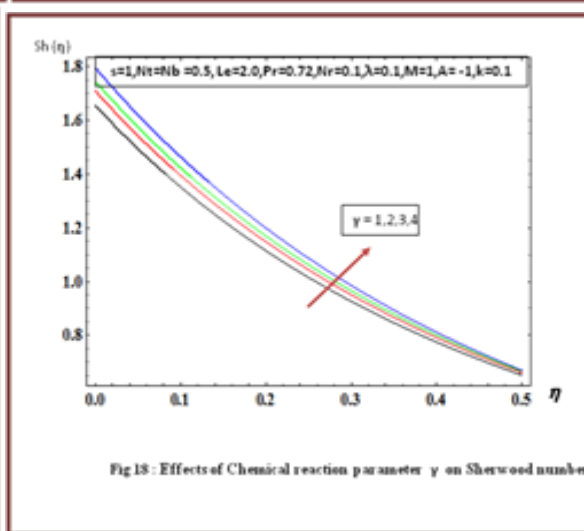


Fig 18 : Effects of Chemical reaction parameter  $\gamma$  on Sherwood number

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