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INHOMOGENEOUS COSMOLOGICAL PERFECT FLUID MODELS IN MODIFIED THEORY OF GENERAL RELATIVITY WITH TIME DEPENDENT-TERM

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ABSTRACT

The cosmological term is achieved with non-static inhomogeneous cosmological models when a perfect fluid generates the gravitational field's source. Einstein's field equations are solved for three physically significant examples (the vacuum cosmological model, the radiating cosmological model, and the Zeldevich model) using the gamma law equation of state.

Keywords: Inhomogeneous Cosmological Model, Perfect Fluid, Radiating Cosmological Model, Zeldevich Model

1. INTRODUCTION

Barber (1982) proposed his second self-creation theory of gravity, which modifies general relativity, in an effort to outperform Einstein's theory. The ideal fluid in this simplified theory of general relativity just splits the matter tensor as a reciprocal of the gravitational constant G, rather than gravitating directly. The local impacts shown in observational studies are predicted by this idea. Furthermore, by analysing the behaviour of photons and degenerate matter entities, this theory may be supported or denied. An accurate measurement of the deflection of light and radio waves passing near the sun, along with the discovery of anomalous precessions in pulsar orbits over central masses, would validate or invalidate such a notion. The theory predicts the perihelia of planets with the same precision as general relativity and in that regard; it is in agreement with observation to within 1%. In the limit $\lambda \rightarrow 0$.

In every way, this revised theory is similar to Einstein's theory. Many authors have examined the modified theory of general relativity from various perspectives. The Friedman-Barber field equations have been solved by Pimentel (1985) using the power law dependency of the scalar field on the scale factor as an assumption. In generalising Pimentel's work Pimentel (1985), Venkateswarlu & Reddy (1990), Soleng (1987) has obtained solutions for the vacuum-dominated, dust-filled universe of the flat FRW spase-time. Bianchi cosmological solutions of VIo type are found in the works of Reddy & Venkateswarlu (1989), both in vacuum and with perfect fluid pressure equivalent to the energy density. When the source of the gravitational field is a perfect fluid, Venkateswarlu & Reddy (1990) have also built spatially homogenous and anisotropic Bianchi type-1 cosmological macro models.

Space homogeneous and anisotropic Bianchi type-II and III cosmological models have been obtained by Shanthi & Rao (1991) in both vacuum and stiff fluid conditions. Carvalho (1996) obtained a homogenous and isotropic model of the primitive universe in which the gamma parameter of the "gamma law" state equation continually varies with cosmological time. He also presented a unified description of the primitive universe between the inflationary period and the epoch dominated by radiation. Shri Ram & Singh (1998) have obtained a spatially homogeneous and isotropic R-W model of the universe in the presence of perfect fluid by using the ‘gamma law” equation of state. Mohanty et al. (2000) have obtained vacuum and Zeldovich fluid models for plane symmetric anisotropic homogeneous space-time. Mohanty et al. (2002), Mohanty et al. (2003) have obtained an anisotropic homogeneous Bianchi Type-1 cosmological micro model in Barber’s second theory of gravitation wherein the scalar field describes the elementary particles and their interactions Srivastav & Sinha (1998). Also, they have obtained a micro and macro cosmological model in the presence of a massless scalar field interacted with perfect fluid. Panigrahi & Sahu (2003), Panigrahi & Sahu (2002), Panigrahi & Sahu (2003), Panigrahi & Sahu (2004) have obtained plane symmetric inhomogeneous macro models in Barber’s second theory of gravitation. Sahu and Bianchi Type-1 vacuum models have been obtained by Sahu & Panigrahi (2003). Sahu & Panigrahi (2006), Sahu et al. (2010) have investigated Masonic perfect fluid models in modified theory of general relativity.

The vitality energy tensor of matter, which is produced by a idealize liquid, is ordinarily the subject of examination for relativistic models. But to get more reasonable models, one must consider the consistency component in cosmology has pulled in the consideration of numerous analysts because it can account for tall entropy of the display universe Weinberg (1971), Weinberg (1972). The tall entropy by baryon and the momentous degree of isotropy of microwave infinite foundation radiation recommends that dissipative impacts in cosmology ought to be considered. Furthermore, it's over here. Thick impacts are anticipated to happen due to a few forms. These are the decoupling of neutrinos amid the radiation time and the decay of matter and radiation amid the recombination time Kolb & Turner (1990), gravitational string generation Turok (1988) and Barrow (1988) and molecule creation impact within the terrific unification time. Murphy (1973) illustrated that the presentation of bulk thickness can anticipate the peculiarity of the enormous bang. therefore, one would need to consider the nearness of fabric dissemination other than the idealize liquid to get practical cosmological models (see Gron (1990) for an audit of cosmological models with bulk consistency).

To our information none of the creators has examined the altered hypothesis of common relativity for plane symmetric inhomogeneous space time in nearness of idealize liquid with time subordinate term Λ . In this paper, we have examined the consistency of this theory in the context of a perfect fluid.

2. FIELD EQUATIONS

Here we consider the space time portrayed by inhomogeneous metric of the frame

$$ds^2 = dt^2 - A^2 dx^2 - B^2\left(dy^2 + dz^2\right) \qquad \textbf{Equation 1}$$

Where A, B are functions of 'x' and' t'.

The field equations in Barbers second self-creation theory with time dependent cosmological constant are

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R + \Lambda(t) g_{ij} = -8\pi\phi^{-1} T_{ij} \qquad \textbf{Equation 2}$$

Where $\Box\phi = \frac{8\pi\lambda}{3} T$, **Equation 3**

The vitality force tensor T_{ij} for idealize liquid is given by

$$T_{ij} = (p+\rho) u_i u_j - p g_{ij} \qquad \textbf{Equation 4}$$

Together with $g_{ij} u_i u_j = 1$ **Equation 5**

In commoving co-ordinate system the surviving components of the field equations (2)-(5) for the space time (1) are

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{B'^2}{A^2 B^2} - \Lambda(t) = -8\pi\phi^{-1} p \qquad \textbf{Equation 6}$$

$$B_4' - \frac{A_4 B'}{A} = 0 \qquad \textbf{Equation 7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{B''}{A^2 B} + \frac{A'B'}{AB} - \Lambda(t) = -8\pi\phi^{-1} p \qquad \textbf{Equation 8}$$

$$\frac{2B''}{A^2 B} - \frac{2A'B'}{AB} + \frac{B'^2}{A^2 B^2} - \frac{2A_4 B_4}{AB} - \frac{B_4^2}{B^2} - \Lambda(t) = -8\pi\phi^{-1} p \qquad \textbf{Equation 9}$$

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{2B_4}{B}\right)\phi_4+\left(\frac{A'}{A^3}-\frac{2B'}{A^2B}\right)\phi'-\frac{\phi''}{A^2}=\frac{8\pi\lambda}{3}\left(p-3\rho\right) \quad \textbf{Equation 10}$$

Here after wards the prime (') and the subscript "4" denotes partial differentiation w.r.to x and t respectively.

In order to solve the field equations for obtaining solutions in explicit forms, we may consider different equation of state. As the metric potentials are functions of x and t, it is difficult to solve the field equations (6)-(10) for non-static case. Hence, we consider the following particular cases.

Case-1: $A=A\left(x\right)\ \ and\ \ B=B\left(t\right).$

In this case the field equations (6)-(10) reduces to

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}-\Lambda\left(t\right)=-8\pi\phi^{-1}p \quad \textbf{Equation 11}$$

$$\frac{B_{44}}{B}-\Lambda\left(t\right)=-8\pi\phi^{-1}p \quad \textbf{Equation 12}$$

$$\frac{B_4^2}{B^2}+\Lambda\left(t\right)=8\pi\phi^{-1}p \quad \textbf{Equation 13}$$

$$\phi_{44}+\frac{2B_4}{B}\phi_4+\frac{A'}{A^3}\phi'-\frac{\phi''}{A^2}=\frac{8\pi\lambda}{3}\left(p-3\rho\right) \quad \textbf{Equation 14}$$

Here we have the system of four equations in five unknowns. In order to make the system consistent, we take the help of gamma law equation of state

$$p=\nu\rho,\ \ 0\le\nu\le1$$

2.1. VACUUM MODEL $\left(p=\rho=0\right)$

For this case equations (11)-(14) reduce to

$$\frac{2B_{44}}{B}+\frac{B_4^2}{B^2}-\Lambda\left(t\right)=0 \quad \textbf{Equation 15}$$

$$\frac{B_{44}}{B}-\Lambda\left(t\right)=0 \quad \textbf{Equation 16}$$

$$\frac{B_4^2}{B^2}+\Lambda\left(t\right)=0 \quad \textbf{Equation 17}$$

$$\phi_{44} + \frac{2B_4}{B}\phi_4 + \frac{A'}{A^3}\phi' - \frac{\phi''}{A^2} = 0 \ .$$ **Equation 18**

Using equation (16) in equation (15), we obtain

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{B_{44}}{B} = 0$$

$$\Rightarrow \frac{B_{44}}{B} + \frac{B_4^2}{B^2} = 0$$

$$\Rightarrow \frac{B_{44}}{B_4} + \frac{B_4}{B} = 0$$

$$\Rightarrow \ln\left(BB_4\right) = \ln\left(\frac{k_1}{2}\right)$$

$$\Rightarrow BB_4 = \frac{k_1}{2}$$ **Equation 19**

Integrating equation (19), we obtain

$$B = \sqrt{k_1 t + k_2}$$ **Equation 20**

Where k_1 *and* k_2 are constants of integration.

Now using equation (20) in equation (17) we get

$$\frac{B_4^2}{B^2} + \Lambda\left(t\right) = \frac{k_1^2}{4B_4} + \Lambda\left(t\right) = 0$$

$$\Lambda\left(t\right) = \frac{k_1^2}{4B^4} = \frac{-k_1^2}{4\left(k_1 t + k_2\right)^2}$$ **Equation 21**

For simplification if we consider ϕ is a function of 't' only then equation (18) reduce to

$$\phi_{44} + \frac{2B_4}{B}\phi_4 = 0$$ **Equation 22**

With the help of equation (20), equation (22) reduces to

$$\phi_{44} + \frac{2B_4}{B}\phi_4 = \phi_{44} + \frac{k_1}{B^2}\phi_4 = 0$$

$$\Rightarrow \phi_{44} + \frac{k_1}{k_1 t + k_2}\phi_4 = 0$$

$$\Rightarrow \frac{\phi_{44}}{\phi_4} + \frac{k_1}{k_1 t + k_2} = 0$$

Integrating we get

$$\phi_4\left(k_1t+k_2\right)=m_1$$

$$\Rightarrow \phi_4=\frac{m_1}{k_1t+k_2}$$

Again integrating

$$\Rightarrow \phi=\frac{m_1}{k_1}\ln\left(k_1t+k_2\right)+m_2 \qquad \textbf{Equation 23}$$

Where $m_1 \neq 0$ *and* m_2 are constants of integration.

If we consider f is a function of 'x' only then equation (18) reduce to

$$\frac{A'}{A^3}\phi'-\frac{\phi''}{A^2}=0$$

$$\Rightarrow \frac{\phi'}{A^2}\left[\frac{A'}{A}-\frac{\phi''}{\phi'}\right]=0$$

$$\frac{A'}{A}-\frac{\phi''}{\phi'}=0 \qquad \textbf{Equation 24}$$

Integrating equation (2.4), we obtain

$$\ln\left(\frac{A}{\phi'}\right)=\ln\left(\frac{1}{m_3}\right)$$

$$\Rightarrow \frac{A}{\phi'}=\frac{1}{m_3}$$

$$\Rightarrow \phi'=m_3A \qquad \textbf{Equation 25}$$

Again integrating equation (25), we get

$$\Rightarrow \phi=m_3\int A\left(x\right)dx+m_4 \qquad \textbf{Equation 26}$$

Where $m_3\left(\neq 0\right)$ and m_4 are constants of integration.

If we consider f is a separable function of x and t and in the form of $f_1\left(x\right)+f_2\left(t\right)$ with zero separable constant then the Barbers scalar f From equation (18) can be obtained as

$$\Rightarrow \phi=\frac{m_1}{k}\ln\left(k_1t+k_2\right)+m_3\int A\left(x\right)dx+m_5 \qquad \textbf{Equation 27}$$

Where m_5 is a constant of integration.

Further, if we are consider ϕ is a separable function of x and t and in the form of $f_1(x)\cdot f_2(t)$ with zero separable constant, then Barbers scalar ϕ from equation (18),can be obtained as

$$\Rightarrow \phi = \left[\frac{m_1}{k_1}\ln\left(k_1 t + k_2\right) + m_2\right]\cdot\left[m_3\int A(x)dx + m_5\right]. \qquad \textbf{Equation 28}$$

Hence the Vacuum cosmological model in second self creation theory of Barber can determined for any arbitrary metric potential $A = A(x)$.

2.2. RADIATING MODEL: $(p = 3\rho)$

In this case, equating equation (11) and (12), we find

$$\frac{B_{44}}{B} + \frac{B_4}{B^2} = 0 \qquad \textbf{Equation 29}$$

On integration, equation (29) yields

$$B = \sqrt{k_1 t + k_2} \qquad \textbf{Equation 30}$$

where $k_1 (\neq 0)$ and k_2 are constants of integration. Now using (30) in equation (13), we obtain $\frac{B_4}{B^2} + \Lambda(t) = 8\pi\phi^{-1}\rho$

$$\frac{k_1^2}{4(k_1 t + k_2)^2} + \Lambda(t) = 8\pi\left(\frac{\rho}{\phi}\right)$$

$$\rho = \frac{1}{8\pi}\left[\frac{k_1^2}{4(k_1 t + k_2)^2} + \Lambda(t)\right]\phi. \qquad \textbf{Equation 31}$$

Further using equation (30) and $\rho = 3p$ in equation (14), we get

$$\phi_{44} + \frac{k_1}{k_1 t + k_2}\phi_4 + \frac{A'}{A^3}\phi' - \frac{\phi''}{A^2} = 0 \qquad \textbf{Equation 32}$$

For the simplification, if we consider $\phi = \phi(t)$ then equation (32) reduces to

$$\phi_{44} + \frac{k_1}{k_1 t + k_2}\phi_4 = 0 \qquad \textbf{Equation 33}$$

This yield on intrgration

$$\phi = \frac{m_1}{k_1}\ln\left(k_1 t + k_2\right) + m_2 \qquad \textbf{Equation 34}$$

Where m_1 (¹ 0) and m_2 are constants of integration.

If we consider f = f (x), then equation (32) reduces to

$$\frac{A'}{A^3}\phi' - \frac{\phi''}{A^2} = 0$$ **Equation 35**

Which on integration yields

$$\Rightarrow \phi = m_3 \int A(x)dx + m_4$$ **Equation 36**

Where m_3 (¹ 0) and m_4 are constants of integration.

Using (34) and (36) in equation (31), we get

$$\Rightarrow \rho = (3p) = \frac{1}{8\pi}\left[\frac{k_1^2}{4(k_1t+k_2)} + \Lambda(t)\right]\cdot\left[\frac{m_1}{k_1}\ln(k_1t+k_2) + m_2\right]$$ **Equation 37**

Or

$$\Rightarrow \rho = (3p) = \frac{1}{8\pi}\left[\frac{k_1^2}{4(k_1t+k_2)} + \Lambda(t)\right]\cdot\left[m_3\int A(x)dx + m_4\right]$$ **Equation 38**

Also, if ϕ is a separarable function of x and t and in the form of $f_1(x) + f_2(t)$ with zero separable constant then equation (32) yields

$$\Rightarrow \phi = \frac{m_1}{k_1}\ln(k_1t+k_2) + m_3\int A(x)dx + m_5$$ **Equation 39**

Where m_5 is a constant of integration.

Further, if we consider $\phi = f_1(x) + f_2(t)$, then equation (32) yields

$$\Rightarrow \phi = \left[\frac{m_1}{k_1}\ln(k_1t+k_2) + m_3\right]\cdot\left[m_3\int A(x)dx + m_4\right]$$ **Equation 40**

Hence the radiating cosmological model in second self creation theory of Barber can be determined for any arbitrary metric potential $A = A(x)$.

Using (39) and (40) in equation (31), we can get another two values of $\rho = (3p)$

2.3. ZELDEVICH MODEL: $(P = \rho)$

In this case the model doesn't exist.

3. CONCLUSION

In this paper a plane symmetric inhomogeneous cosmological model has been constructed by taking perfect fluid along with time-depended cosmological constant term. Also, I have studied the consistency of this theory to the case of a perfect fluid in three different cases. The Vacuum and radiating cosmological model exists in second self creation theory of Barber and can be determined for any arbitrary metric potential, but in case of Zel'dovich model it doesn't exist.

CONFLICT OF INTERESTS

None.

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