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ANALYSIS OF PRIORITY QUEUES WITH PENTAGON FUZZY NUMBER

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Abstract:
Fuzziness is a sort of recent incoherence. Fuzzy set theory is asserted to depict vagueness. This study explores the queuing model of priority classes adopting pentagon fuzzy number with the inclusions of fuzzy set operations. A mathematical programming method is designed to establish the membership function of the system performance, in which the arrival rate and service rate of the system performance of two priority classes are utilized as fuzzy numbers. Based on α-cut approach and Zadeh's extension principle, the fuzzy queues are scaled down to a family of ordinary queues. The potency of the performance measures of the characteristics of the queuing model is ensured with an illustration and its graph.

Keywords: *Fuzzy Sets; Membership Functions; Priority Queues; Mathematical Programming; Pentagon and Trapezoidal Fuzzy Numbers; Performance Measures.*

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1. Introduction

Queues (or) waiting lines are universal. A queuing model is formulated so that queue lengths and waiting time can be envisaged. Queuing theory examines every component of waiting lists to be served, including the arrival and service process, and the number of customers and system places. The prime vitality is to find the most desirable and supreme level of service.

The most common queue discipline is the "first come, first served" (FCFS), or the "first in, first out" (FIFO) rule under which the customers are serviced in the strict order of their arrivals. Other queue disciplines are "last in, first out" (LIFO) rule according to which the last arrival in the system is serviced first, "selection service in random order" (SIRO) rule according to which the arrivals are serviced randomly irrespective of their arrivals in the system and a variety of priority schemes – according to which a customer's service is done in preference over some other customer.

In priority discipline, the service is of two types preemptive and non-preemptive. In pre-emptive priority, the customers of high priority are given service over the low priority customers. In non-premption, the highest priority customer goes ahead in the queue, but his service is started only after the completion of the service of the currently being served customer.

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Fuzzy queuing models have been described by such researchers like Li and Lee [4], Kaufmann [3], Negi and Lee [5], kao et al [1]. Chen [10,11] has analyzed fuzzy queues using Zadeh's extension principle and has developed (FM/FM/1): (∞/FCFS) and (FM/FM k/1) : (∞/FCFS) where FM denotes fuzzified exponential time based of queuing theory. Kao et al constructed the membership functions of the system characteristic for fuzzy queues using parametric linear programming. S. Thamotharan [9] studied multiserver queuing model in triangular and trapezoidal fuzzy numbers using α cuts in 2016. In recent times, Usha Madhuri and Chandan [8] designed FM/FM/1 queuing model with Pentagon fuzzy numbers using α-cuts and Ashok Kumar. V [12] using DSW algorithm.

In this paper, fuzzy set theory is employed to formulate the membership function of fuzzy priority queues in which two arrival rates and single service rate are Pentagon fuzzy numbers. α-cut and fuzzy arithmetic operations are used to derive system characteristics.

2. Preliminaries

1) **Fuzzy set:** A fuzzy set Ã is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A, \mu_{\tilde{A}}(x) \in [0,1]\}$ where $M_{\tilde{A}}(x)$ is called grade of membership of x in Ã.
2) **α - Cut:** Given a fuzzy set A in X and any real number α ∈ [0,1], then the α -level set of A denoted by a_A is the ordinary set $\alpha_A = \{x \in X : \mu_A(x) \geq \alpha\}$. The strong α-cut defined as $^{\alpha+}A$ is the crisp set $^{\alpha+}A = \{x \in X : \mu_A(x) > \alpha\}$
3) **Support:** The support of fuzzy set Ã is the set of all points x in X such that $M_{\tilde{A}}(x) > 0$.
4) **Height:** The height h (A) of a fuzzy set A is the largest membership grade obtained by any element in that set such that $h(A) = \sup_{x \in X} A(x)$. A fuzzy set A is called normal when h (A) =1, and subnormal when h (A) <1.
5) **Trapezoidal Fuzzy Number**

The trapezoidal fuzzy number is defined as Ã = (a, b, c, d) where a < b < c < d with its membership function as

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1 \quad , & b \leq x \leq c \\ R(x) = \dfrac{d-x}{d-c}, & c \leq x \leq d \\ 0, \ otherwise \end{cases}$$

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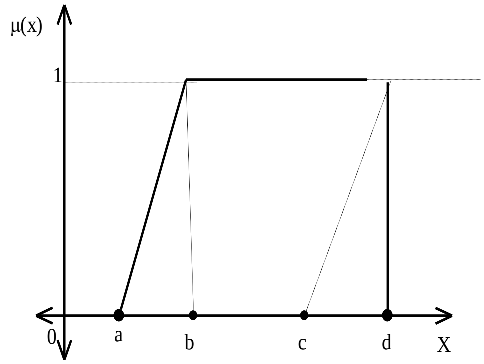
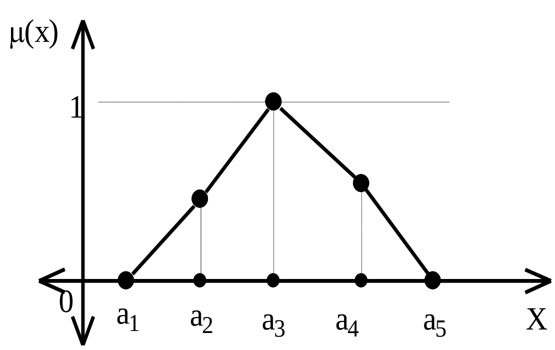


6) **Pentagon Fuzzy Number**

A pentagon fuzzy number is a 5-tuple subset of a real number having five parameters (a_1, a_2, a_3 a_4 a_5) where a_3 is the middle point and (a_1, a_2) and (a_4, a_5) are the left and right side points of a_3.

A fuzzy number $\tilde{A} = [a_1, a_2, a_3, a_4, a_5]$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5$ is said to be a pentagon fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 , & x < a_1 \\ L_1(x) = \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ L_2(x) = \dfrac{x-a_2}{a_2-a_1}, & a_2 \le x \le a_3 \\ 1 , & x = a_3 \\ R_1(x) = \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ R_2(x) = \dfrac{a_5-x}{a_5-a_4}, & a_4 \le x \le a_5 \\ 0 , & x > a_5 \end{cases}$$



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7) **Signed Distance for Pentagon Fuzzy Number**

Let $\tilde{D}$ be the signed distance of $\tilde{D}$ measured from $\tilde{O}$ as $d\left(\tilde{D},\tilde{O}\right)=\frac{1}{12}\left(a+3b+4c+3d+e\right)$

8) **Zadeh's Extension Principle**

The membership function of performance measures of the queuing model is examined in accordance to Zadeh's principle.

Let P(x, y) denote the system performance measure of interest where the arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$, $P(\tilde{\lambda},\tilde{\mu})$ are all fuzzy numbers. $P(\tilde{\lambda},\tilde{M})$ is formalized based on Zadeh's principle as $\mu_{P(\tilde{\lambda},\tilde{M})}\left(z\right)=\sup_{\substack{x\in X \\ y\in Y}}\left\{\min\left(M_{\tilde{A}}(x),\mu_{\tilde{S}}(y)\right)/z=P\left(x,y\right)\right\}$

3. Fuzzy Priority Queues

Queue discipline is based on a priority system. We elaborate a fuzzy queuing model FM/FM/1 with a priority. Each arrival unit is framed as a member of one of two priority classes. Assume the arrivals of the first or higher priority with mean rate $\tilde{\lambda}_1$ and the second or lower priority with mean rate $\tilde{\lambda}_2$ such that $\tilde{\lambda}=\tilde{\lambda}_1+\tilde{\lambda}_2$. The first or higher priority units have the right to be served ahead of the others without preemption. It is ascertained that the capacity of the system and the calling source population are interminable.

In this model, $\tilde{L}_1$ and $\tilde{L}_2$ denote the average system length of first and second priorities. $\tilde{L}_{q_1}$ and $\tilde{L}_{q_2}$ refer to the average queue length of first and second priorities. $\tilde{W}_{q_1}$ and $\tilde{W}_{q_2}$ refer to the average waiting time of units in the queue in the first and second priorities.

4. Parametric Programming Problem for Fuzzy Priority Queuing Model

Consider a queuing system with single server FM/FM/I model. The time between successive arrivals is taken as $\tilde{A}_i$, $i=1,2$ of units in the first and second priority and service time $\tilde{S}$ are represented by the following fuzzy sets.

$$\tilde{A}_i=\left\{\left(a,\mu_{\tilde{A}_i}\left(a\right)\right)\Big/a\in X\right\} \text{ Where i = 1, 2} \qquad \ldots \qquad (1)$$

$$\tilde{S}=\left\{\left(s,\mu_{\tilde{S}}\left(s\right)\right)\Big/s\in Y\right\} \qquad \ldots \qquad (2)$$

Where X and Y are crisp universal sets of the inter arrival times and service times and $\mu_{\tilde{A}_i}\left(\alpha\right)$ where i = 1,2, $\mu_{\tilde{S}}\left(s\right)$ are the membership functions

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$$\mu_{\tilde{A}_i}(\alpha) = \{a \in X / \mu_{\tilde{A}_i}(\alpha) \geq \alpha\},\ i = 1,2 \quad \ldots \quad (3)$$

$$S(\alpha) = \{S \in Y / \mu_{\tilde{S}}(s) \geq \alpha\} \text{ Are the } \alpha \text{ - level sets of } \tilde{A}_i \text{ and } \tilde{S} \quad \ldots \quad (4)$$

Where A_i (α), i = 1, 2 and S(α) are the ordinary sets. Using α-cuts, the inter arrival times and service time can be represented by different levels of confidence intervals. Hence a fuzzy queue can be reduced to a family of crisp queues with different α-cuts $\{A_i(\alpha)/0 < \alpha \leq 1\}$, i = 1, 2 and $\{S(\alpha)/0 < \alpha \leq 1\}$. These two sets form nested structures for expressing the relationship between the crisp sets and fuzzy sets.

Let the confidence intervals of the fuzzy sets $\tilde{A}_i, i = 1,2$ and $\tilde{S}$ be $[l_{Ai(\alpha)}, u_{Ai(\alpha)}]$, $i = 1,2$ and $[l_{s(\alpha)}, u_{s(\alpha)}]$. When both inter arrival time and service time are fuzzy numbers, based on Zadeh's extension principle [7], the membership function of the performance measure $P(\tilde{A}_i, \tilde{S})$, $i = 1,2$ is defined as

$$\mu_{p(\tilde{A}_i,\tilde{S})}(z) = \sup_{\substack{a\in X\\ s\in Y}} \{Min(\mu_{\tilde{A}_i}(a), \mu_{\tilde{S}}(s) / z = p(a,s))\} \text{ Where i = 1, 2,....} \quad \ldots. \quad (5)$$

Our approach is to construct the membership function $\mu_{P(\tilde{A}_i,\tilde{S})}(z)$ based on deriving the α-cut of $\mu_{P(\tilde{A}_i,\tilde{S})}$. The parametric programming technique for finding lower and upper bounds of the α-cut of $\mu_{P(\tilde{A}_i,\tilde{S})}(z)$ are $l_{P(\alpha)} = \min p(a,s)$

Such that $l_{A_i(\alpha)} \leq a \leq u_{A_i(\alpha)}$ where i = 1, 2 and $l_{s(\alpha)} \leq s \leq u_{s(\alpha)}$ .... (6)

And $u_{P(\alpha)}$ = max p (a,s) such that $l_{A_i(\alpha)} \leq a \leq u_{A_i(\alpha)}$ where i = 1, 2 $l_{s(\alpha)} \leq s \leq u_{s(\alpha)}$ .... (7)

If both $l_{P(\alpha)}$ and $u_{P(\alpha)}$ are invertible with respect to α, then the left shape function L(z) = $l^{-1}_{P(\alpha)}$ and the right shape function R(z) = $u^{-1}_{P(\alpha)}$ can be obtained, from which the membership function $\mu_{P(\tilde{A}_i,\tilde{S})}(z)$, $i = 1,2$ is constructed.

$$\mu_{p(\tilde{A}_i,\tilde{S})}(z) = \begin{cases} L_1(z) & , \quad z_1 \leq z \leq z_2 \\ L_2(z) & , \quad z_2 \leq z \leq z_3 \\ 1 & , \quad z = z_3 \\ R_1(z) & , \quad z_3 \leq z \leq z_4 \\ R_2(z) & , \quad z_4 \leq z \leq z_5 \end{cases}$$

Where $z_1 \leq z_2 \leq z_3 \leq z_4 \leq z_5$ and $L_1(z) = R_2(z) = 0$ for the pentagon fuzzy number.

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(FM/FM/1): (∞/FCFS) queues with priority can be reduced as M/M/1 queue with priority for which

$$L_{q_1} = \frac{\left(\rho\frac{\lambda_1}{\mu}\right)}{\left(1-\frac{\lambda_1}{\mu}\right)} \text{ And } L_{q2} = \frac{\left(\rho\frac{\lambda_2}{\mu}\right)}{(1-\rho)\left(1-\frac{\lambda_1}{\mu}\right)}$$

$$w_{q_1} = \frac{\lambda}{\mu(\mu-\lambda_1)} \text{ And } w_{q2} = \frac{\lambda}{(\mu-\lambda)(\mu-\lambda_1)}$$

Where λ_1 and λ_2 are the arrival rates of first and second priority units, μ is the service rate and $\lambda = \lambda_1 + \lambda_2$ with $\rho = \frac{\lambda}{\mu}$. The above procedure can be applied to find the membership functions $M_{p(\tilde{A}, \tilde{S})}(z)$ for the queuing model with priority.

5. Numerical Example

Expected waiting time and expected number of customers in the queue for FM/FM/1 queue with two priority classes

Suppose that the arrival rates of first and second priority with the same service rate are fuzzy numbers represented by

$\tilde{A}_1 = [1, 2, 3, 4, 5]$, $\tilde{A}_2 = [6, 7, 8, 9, 10]$ and $\tilde{S} = [21, 22, 23, 24, 25]$ per hour.

The α-cut of the membership functions $\mu_{\tilde{A}_1}(\alpha), \mu_{\tilde{A}_2}(\alpha)$ and $M_{\tilde{S}}(\alpha)$ are [1+2α, 5-2α], [6+2α, 10-2α] and [21+2α, 25-2α] respectively. From equations (6) and (7), the parametric programming problems are formulated to derive the membership functions of

(i) $\tilde{L}_{q_1}$ = average queue length of first priority.

(ii) $\tilde{L}_{q_2}$ = average queue length of second priority

(iii) $\tilde{W}_{q_1}$ = average waiting time of units of first priority in the queue.

(iv) $\tilde{W}_{q_2}$ = average waiting time of units of second priority in the queue.

Objective Functions are

(i) The performance function of $\tilde{L}_{q_1}$ is

$$l_{Lq_1(\alpha)} = \min\left\{\frac{\left(\frac{r_1+r_2}{t}\right)\left(\frac{r_1}{t}\right)}{\left(1-\frac{r_1}{t}\right)}\right\}$$

ISSN: 2454-1907
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Such that

$1 + 2\alpha \le r_1 \le 5 - 2\alpha$

$6 + 2\alpha \le r_2 \le 10 - 2\alpha$

$21 + 2\alpha \le t \le 25 - 2\alpha$

Where $l_{Lq_1(\alpha)}$ is found when r_1 and r_2 approach their lower bounds and t approaches its upper bound.

The optimal solution is

$$l_{Lq_1(\alpha)} = \frac{7+18\alpha+8\alpha^2}{600-148\alpha+8\alpha^2} \text{ and}$$

$$u_{Lq_1(\alpha)} = \max\left\{\frac{\left(\frac{r_1+r_2}{t}\right)\left(\frac{r_1}{t}\right)}{\left(1-\frac{r_1}{t}\right)}\right\}$$

Such that $1 + 2\alpha \le r_1 \le 5 - 2\alpha$

$6+2\alpha \le r_2 \le 10-2\alpha$

$21 + 2\alpha \le t \le 25-2\alpha$

where $u_{Lq_1}(\alpha)$ is found when r_1 and r_2 approach their upper bounds and t approaches its lower bound.

$\therefore$ The optimal solution is

$$u_{Lq_1}(\alpha) = \frac{75-50\alpha+8\alpha^2}{336+116\alpha+8\alpha^2}$$

The membership function

$$\mu_{\tilde{L}q_1}(z) = \begin{cases} L(z) \text{ where } \left[l_{Lq_1(\alpha)}\right]_{\alpha=0} \le z \le \left[l_{Lq_1(\alpha)}\right]_{\alpha=1} \\ R(z) \text{ where} \left[u_{Lq_1(\alpha)}\right]_{\alpha=1} \le z \le \left[u_{Lq_1(\alpha)}\right]_{\alpha=0} \\ 0, \quad \text{otherwise} \end{cases}$$

Which is estimated as

$$\mu_{\tilde{L}q_1}(z) = \begin{cases} \dfrac{(148z+18)-\left(2704z^2+24752z+100\right)^{1/2}}{16(z-1)} & \text{where } 0.01 \le z \le 0.07 \\ \dfrac{-(116z+50)+\left(2704z^2+24752z+100\right)^{1/2}}{16(z-1)} & \text{where } 0.07 \le z \le 0.22 \\ 0, & \text{otherwise} \end{cases}$$

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(ii) The performance function of $\tilde{L}_{q_2}$ is

$$u_{Lq_2(\alpha)} = \min\left\{ \frac{\dfrac{r_2\left(r_1+r_2\right)}{t^2}}{\left(1-\dfrac{r_1+r_2}{t}\right)\left(1-\dfrac{r_1}{t}\right)} \right\}$$

And its optimal solution is

$$l_{Lq_2(\alpha)} = \frac{42+38\alpha+8\alpha^2}{432-216\alpha+24\alpha^2}$$

And $l_{Lq_2(\alpha)} = \max\left\{ \frac{\dfrac{r_2\left(r_1+r_2\right)}{t^2}}{\left(1-\dfrac{r_1+r_2}{t}\right)\left(1-\dfrac{r_1}{t}\right)} \right\}$

With its optimal solution

$$u_{Lq_2(\alpha)} = \frac{150-70\alpha+8\alpha^2}{96+120\alpha+24\alpha^2}$$

The membership function is

$$\mu_{\tilde{L}q_2}(z) = \begin{cases} \dfrac{\left(216z+38\right)-\left(5184z^2+34272z+100\right)^{1/2}}{16\left(3z-1\right)} & \text{where } 0.09 \le z \le 0.36 \\ \dfrac{-\left(120z+70\right)+\left(5184z^2+3427z+100\right)^{1/2}}{16\left(3z-1\right)} & \text{where } 0.36 \le z \le 1.56 \\ 0\,, & \text{otherwise} \end{cases}$$

(iii) The performance function of $\tilde{W}_{q_1}$ is

$$L_{wq_1(\alpha)} = \min\left\{ \frac{r_1+r_2}{t\left(t-r_1\right)} \right\}$$

Yields the optimal solution as

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$$L_{wq_1(\alpha)} = \min\left\{ \frac{7+4\alpha}{600-148\alpha+8\alpha^2} \right\}$$

And $u_{wq_2(\alpha)} = \max\left\{ \frac{r_1+r_2}{t(t-r_1)} \right\}$

Yields the solution as

$$u_{wq_2(\alpha)} = \frac{15-4\alpha}{336+116\alpha+8\alpha^2}$$

The membership function is

$$\mu_{\tilde{w}q_1}(z) = \begin{cases} \dfrac{(148z+4)-(2704z^2+1408z+16)^{1/2}}{16z} & \text{where } 0.01 \le z \le 0.023 \\ \dfrac{-(116z+4)-(2704z^2+1408z+16)^{1/2}}{16z} & \text{where } 0.023 \le z \le 0.044 \\ 0, & \text{otherwise} \end{cases}$$

(iv) The performance function of $\tilde{W}q_2$ is

$$l_{wq_2(\alpha)} = \min\left\{ \frac{r_1+r_2}{\left[t-(r_1+r_2)\right](t-r_1)} \right\}$$

Yields the optimal solution as

$$l_{wq_2(\alpha)} = \left\{ \frac{7+4\alpha}{432-216\alpha+24\alpha^2} \right\}$$

And $u_{wq_2(\alpha)} = \max\left\{ \frac{r_1+r_2}{t-(r_1+r_2)+(t-r_1)} \right\}$

With the solution

$$u_{wq_2(\alpha)} = \frac{15-4\alpha}{96+120\alpha+24\alpha^2}$$

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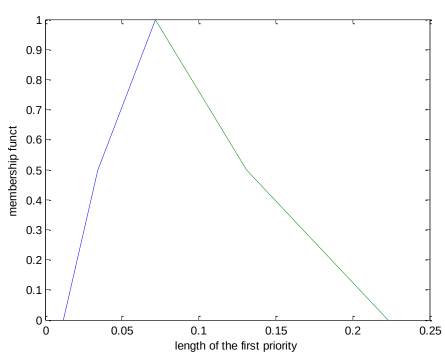
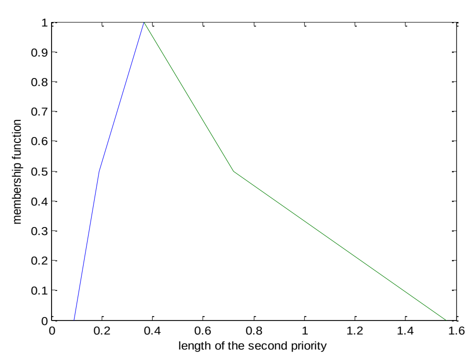
The membership function is

$$\mu_{\tilde{W}_{q_2}}(z)=\begin{cases}\dfrac{(216z+4)-\left(5184z^2+2400z+16\right)^{1/2}}{48z} & \text{where } 0.016\le z\le 0.04\\[2ex] \dfrac{-(120z+4)+\left(5184z+2400z+16\right)^{1/2}}{48z} & \text{where } 0.04\le z\le 0.15\\[2ex] 0, & \text{otherwise}\end{cases}$$

α-cuts of arrival rates, service rate, queue length and waiting time in queue of first and second priority

α	$\lambda_{x\alpha}$	$u_{x\alpha}$	$\lambda_{y\alpha}$	$u_{y\alpha}$	$\lambda_{Lq_1(\alpha)}$	$u_{Lq_1(\alpha)}$	$\lambda_{lq_2(\alpha)}$	$u_{Lq_2(\alpha)}$	$\lambda_{wq_1(\alpha)}$	$u_{wq_1(\alpha)}$	$\lambda_{wq_2(\alpha)}$	$u_{wq_2(\alpha)}$
0	1	5	21	25	0.0116	0.2232	0.09	1.5625	0.0116	0.0446	0.0616	0.1562
0.1	1.1	4.9	21.1	24.9	0.0151	0.2015	0.1117	1.3218	0.0126	0.0419	0.018	0.1348
0.2	1.2	4.8	21.2	24.8	0.0191	0.1816	0.1280	1.1269	0.0136	0.0394	0.0200	0.1173
0.3	1.3	4.7	21.3	24.7	0.0235	0.1634	0.1465	0.9669	0.0147	0.0371	0.0222	0.1028
0.4	1.4	4.6	21.4	24.6	0.0285	0.1466	0.1683	0.8338	0.0156	0.0349	0.02	0.0906
0.5	1.5	4.5	21.5	24.5	0.0340	0.1313	0.1909	0.7223	0.0170	0.0328	0.0272	0.0802
0.6	1.6	4.4	21.6	24.4	0.0402	0.1172	0.2175	0.6277	0.0182	0.0308	0.0302	0.0718
0.7	1.7	4.3	21.7	24.3	0.0470	0.1042	0.2478	0.5471	0.0195	0.0281	0.0334	0.0636
0.8	1.8	4.2	21.8	24.2	0.0544	0.0924	0.2823	0.4780	0.0209	0.0271	0.0371	0.0569
0.9	1.9	4.1	21.9	24.1	0.0627	0.0825	0.3216	0.4183	0.0223	0.0255	0.0412	0.0510
1	2	4	22	24	0.0717	0.0717	0.3667	0.3667	0.0239	0.0239	0.0458	0.0458





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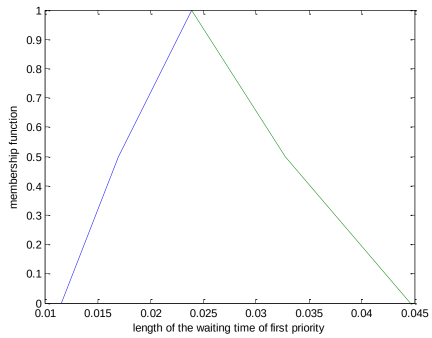

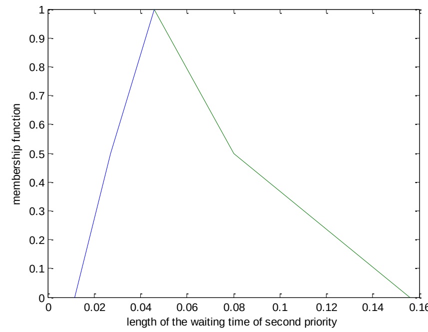

Conclusion

Fuzzy queuing models mark its richness for analysis of vague ideas. The fuzzy set theory has been applied to explore the queuing model of two priority classes using pentagon fuzzy numbers. Based on Zadeh's extension principle, system performance measures are structured. In forthcoming research, we shall deal with generalizing other methods for supporting decision-makers to clinch the direction for modification in priority queues.

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