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PENETRATIVE THERMO-GRAVITATIONAL AND SURFACE- TENSION DRIVEN CONVECTION IN A FERROFLUID LAYER THROUGH VOLUMETRIC INTERNAL HEATING WITH VARIABLE VISCOSITY

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ABSTRACT

This work pertaining to analytical and numerical studies on FTC in a FF layer with impact of coupled buoyancy-gravitational and surface-tension forces through the strength of internal heat source on the system subjected to the magnetic field dependent (MFD) viscosity effect. The lower boundary is considered to be rigid at either conducting or insulating to temperature perturbations, while upper boundary free open to the atmosphere is flat and subject to a Robin-type of thermal boundary condition. The Rayleigh-Ritz method with Chebyshev polynomials of the second kind as trial functions is employed to extract the critical stability parameters numerically. The onset of FTC is delayed with an increase in MFD (δ) parameter and Biot number (Bi) but opposite is the case with an increase in Rayleigh number (M_1), non-linearity of fluid magnetization (M_3) and strength of internal heat source (Ns). Their effects are complementary in the sense that the critical M_{ac} and R_{mc} decrease with an increase in R_t.

Keywords: Buoyancy-Gravitational, Surface-Tension Forces, Galerkin Technique, Ferrofluids, Volumetric Internal Heating, MFD Viscosity

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1. INTRODUCTION

Ferrofluids (FFs) are synthesized by suspending single domain ferromagnetic nanoparticles stabilized in various nonmagnetic carrier fluids, which exhibit both magnetic and fluid properties Rosensweig (1985), Shliomis (1974). These fluids are now termed as magnetic nanofluids and the study of such fluids has been a subject of intensive investigations over decades due to their potential applications in magnetically heat controlled thermosiphons for technological purposes Charles (2002), Blums (2002). Thermal convection in an FF layer in the occurrence of magnetic field, called ferro- thermal-convection (FTC), has been studied extensively both theoretically and experimentally over the years to understand the heat transfer systems and the details are sufficiently documented in the review article by Nkurikiyimfura et al. (2013).

Most FFs are either water based, or oil based. The viscosity of water is far more sensitive to temperature variations and oils are known to have viscosity decreasing exponentially with temperature rather than linearly. Realizing the importance, several investigators considered exponential variation in viscosity with temperature in analysing thermal convective instability in horizontal fluid layers, but the studies were limited to ordinary viscous fluids Kassoy and Zebib (1975), Blythe and Simpkins (1981), Patil and Vaidyanathan (1981), Patil and Vaidyanathan (1982). To our knowledge, no attention has been given to convective instability problems involving FFs, despite the importance of FFs in many heat transfer applications. For example, in a rotating shaft seal involving FFs the temperature may rise above 1000C at high shaft surface speeds. A similar situation may arise in the use of FFs in loudspeakers Lebon and Cloot (1986). The onset of FTC in a horizontal FF layer with temperature dependent viscosity in exponentially is examined Shivakumara et al. (2012).

In many natural phenomena, the study of penetrative FTC in a saturated porous layer Nanjundappa et al. (2011) with the internal heating source and applied Brinkman extended Darcy model in the momentum equation analyzed the internal heat generation effect on the onset of FTC in an FF saturated porous layer Nanjundappa et al. (2011), Nanjundappa et al. (2012). Savitha et al. (2021) investigated the penetrative FTC in an FF-saturated high porosity anisotropic porous layer via uniform internal heating. Thus, the purpose of the present chapter is to study a general problem of coupled thermo- gravitational and surface-tension FC in an FF layer with magnetic field dependent (MFD) viscosity. The study helps to understanding the control of FTC by MFD viscosity, which is constructive in various problems associated by heat transfer particularly in material-science processing. In the current study, the bottom surface is rigid with either constant temperature or uniform heat flux, while the upper is un-deformable free surface of surface tension forces. Besides, the Neumann-type of boundary condition is imposed on the upper surface. Several investigators have studied both types of instabilities in isolation or together in a horizontal FF layer.

2. PROBLEM FORMULATION

Consider a layer of horizontal Boussinesq FF of constant depth d with a uniformly volumetric heat source strength, Q, and in the occurrence of perpendicular magnetic field H_0. The surfaces are maintained the constant temperatures at $T_0 + \Delta T/2 \quad (z=0)$ and $T_0 - \Delta T/2 \quad (z=d)$. The gravity, $\vec{g} = -g\hat{k}$, acting downward direction, where $\hat{k}$ is the z-direction of unit vector. The stream of Bénard-Marangoni convection for thermocapillary force, σ (surface tension force), is given by

$$\sigma = \sigma_0 \left\{1 - \sigma_T \left(T - T_0\right)\right\} \quad \textbf{Equation 1}$$

where σT, σ_0 are positive constants.

The Maxwell's equations for magnetic field are implemented by

$$\nabla \times \vec{H} = 0 \text{ or } \vec{\mathrm{H}} = \nabla \varphi \quad \textbf{Equation 2}$$

$$\nabla \cdot \vec{B} = 0 \quad \textbf{Equation 3}$$

were

$$\vec{B} = \mu_0 \, (\vec{M} + \vec{H}) \quad \textbf{Equation 4}$$

With

$$\vec{M} = \frac{\vec{H}}{H} M(H, T) \quad \textbf{Equation 5}$$

and

$$M = M_0 - K(T - T_a) + \chi(H - H_0) \quad \textbf{Equation 6}$$

The equation of momentum with variable viscosity is

$$\rho_0 \left\{ \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \, \vec{q} \right\} = -\nabla \mathrm{p} + \rho \, \vec{g} + \mu_0 \, \vec{M} \cdot \nabla \vec{H} + \{\nabla \cdot \eta \underset{\sim}{D}\} \quad \textbf{Equation 7}$$

The heat equation with internal heating Q is

$$\left\{ \rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left[\frac{\partial \vec{M}}{\partial T} \right]_{V,H} \right\} \frac{DT}{Dt} + \mu_0 T \left[\frac{\partial \vec{M}}{\partial T} \right]_{V,H} \cdot \frac{D\vec{H}}{Dt} = k_t \nabla^2 T + Q \quad \textbf{Equation 8}$$

The conservation of mass equation is

$$\nabla \cdot \vec{q} = 0 \quad \textbf{Equation 9}$$

The state equation is

$$\rho = \rho_0 \left[1 - \alpha_t \, (T - T_0) \right] \quad \textbf{Equation 10}$$

Here $\vec{q} = (u, v, w)$ is the velocity, p is the pressure, t is the time, $\vec{B}$ is the magnetic induction and $\vec{H}$ is the intensity of magnetic field, $\vec{M}$ is the magnetization, μ_0 is the magnetic permeability of vacuum, $C_{V,H}$ is the specific heat capacity at constant volume and magnetic field per unit mass, and k_t is the thermal conductivity, $\chi = (\partial M / \partial H)_{H_0, T_0}$ is the magnetic susceptibility and $K = -(\partial M / \partial T)_{H_0, T_0}$ is the pyromagnetic co-efficient, α_t is the thermal expansion coefficient, ρ_0 is the density at $T = T_0$, $M_0 = M(H_0, T_0)$, $H = \left|\vec{H}\right|$, $M = \left|\vec{M}\right|$ and the last term of Equation 7 denotes as $\underset{\sim}{D} = [\nabla \vec{q} + (\nabla \vec{q})^T] / 2$ is the rate of strain tensor. The fluid is assumed to be incompressible having variable viscosity. Experimentally, it has been demonstrated that the magnetic viscosity has got exponential variation, with respect to magnetic field Rosenwieg et al. (1969) As a first approximation, for small field variation, linear variation of magnetic viscosity has been used in the form $\eta = \eta_0(1 + \vec{\delta} \cdot \vec{B})$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity and is considered to be isotropic Vaidyanathan et al. (2002), η_0 is taken as viscosity of the fluid when the applied magnetic field is absent.

The undisturbed quiescent state

$$\vec{q}_b = 0 \quad \textbf{Equation 11}$$

$$p_b(z) = p_0 - \rho_0 g z - \frac{\rho_0 \alpha_t g \beta z^2}{2} - \frac{\mu_0 M_0 \kappa \beta}{1+\chi} z - \frac{\mu_0 \kappa^2 \beta^2}{2(1+\chi)^2} z^2 \quad \textbf{Equation 12}$$

$$T_b(z) = -\frac{Q z^2}{2 k_1} + \frac{Q d z}{2 k_1} - \beta z + T_0 \quad \textbf{Equation 13}$$

$$\vec{H}_b(z) = \left[H_0 + \frac{K}{1+\chi} \{ T_b(z) - T_0 \} \right] \hat{k} \quad \textbf{Equation 14}$$

$$\vec{M}_b(z) = \left[M_0 - \frac{K}{1+\chi} \{ T_b(z) - T_0 \} \right] \hat{k} \quad \textbf{Equation 15}$$

Here we note that, $T_b(z)$, $\vec{H}_b(z)$, $\vec{M}_b(z)$ are distributed parabolically with the height of the FF layer due to the existence of volumetric heat source, Q. However, for $Q = 0$, the distributions of basic state are linear in *z*.

To study the stability of the quiescent state and perturb the relevant variables in the corresponding governing equations with framework of the linear theory

$$\vec{q} = \vec{q}\,', \; p = p_b(z) + p', \; T = T_b(z) + T', \; \vec{H} = \vec{H}_b(z) + \vec{H}', \; \vec{M} = \vec{M}_b(z) + \vec{M}' \quad \textbf{Equation 16}$$

Let the components of $\left\{ M_1', M_2', M_b(z) + M_3' \right\}$ $\left\{ H_1', H_2', H_b(z) + H_3' \right\}$

be perturbed the magnetization and magnetic field, respectively.

Using these in Equation 2,Equation 6, linearizing, we obtain

$$H_1 + M_1 = \left(1 + \frac{M_0}{H_0} \right) H_1 \quad \textbf{Equation 17}$$

$$H_2 + M_2 = \left(1 + \frac{M_0}{H_0}\right) H_2$$ **Equation 18**

$$H_3 + M_3 = -K\ T + \left(1 + \chi\right)\ H_3$$ **Equation 19**

From Equation 17,Equation 19 and it is considered that $K\beta\ d << \left(1+\chi\right) H_0$; $K\,Q\,d^2 << \left(1+\chi\right) H_0 2 k_1$.

Experimentally, Rosenwieg et al. (1969) has demonstrated the exponential variation in magneto-viscosity, $\eta = \eta e^{\delta\cdot\vec{B}}$, where δ is the variation of viscosity coefficient. Since the first approximation of small (linear) field variation in magneto-viscosity has been used. Substituting Equation 16 in Equation 7 and applying the basic state solutions, removing the pressure *p* by operating two times of curl on the resulting equations and linearizing together with $\vec{H} = \nabla\phi$, then gives

$$\left[\rho_0 \frac{\partial}{\partial t} - \eta_0 \left\{1 + \delta\,\mu_0 (M_0 + H_0)\right\} \nabla^2\right] \nabla^2 w = -\rho_0 \alpha_t g\ \nabla_1^2 T$$
$$-\mu_0 K \left(-\frac{Q\,z}{k_1} + \frac{Q\,d}{2\,k_1} - \beta\right) \frac{\partial}{\partial z}\left(\nabla_1^2 \varphi\right) + \frac{\mu_0 K^2}{1+\chi}\left(-\frac{Q\,z}{k_1} + \frac{Q\,d}{2\,k_1} - \beta\right)\left(\nabla_1^2 T\right)$$ **Equation 20**

As before, using Equation 16 ,Equation 8 and applying basic state solutions, and linearizing, we obtain

$$\frac{\partial T}{\partial t} - \mu_0\, T_0 K \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right) - k_1 \nabla^2 T = \left\{1 - \frac{\mu_0\, T_0 K^2}{1+\chi}\right\}\left(\frac{Q\,z}{k_1} - \frac{Q\,d}{2\,k_1} + \beta\right) w$$ **Equation 21**

Where $\rho_0 C_0 = \rho_0 C_{V,H} + \mu_0 K\ H_0$

Finally, Equation 2,Equation 3, after using Equation 16 together with Equation 17,Equation 19, yields (after neglecting primes)

$$\frac{\partial^2 \varphi}{\partial z^2} + \frac{1}{1+\chi}\left(1 + \frac{M_0}{H_0}\right) \nabla_1^2 \varphi - \frac{K}{1+\chi}\ \frac{\partial T}{\partial z} = 0$$ **Equation 22**

As the customary of convective instability analysis for each variable of $w,\ T,\ \phi$ is expanded in following form by assuming the normal mode hypothesis (separation of variables)

$$\left\{w,\quad T,\quad \phi\right\} = \left\{w,\quad T,\quad \phi\right\}(z,\ t)\ e^{i\,(l\,x + m\,y)}$$ **Equation 23**

Substituting into Equation 20,Equation 22, we get

$$\left[\rho_0 \frac{\partial}{\partial t} - \eta_0 \{1+\delta\, \mu_0 (M_0 + H_0)\} \left(\frac{\partial^2}{\partial z^2} - a^2\right)\right] \left(\frac{\partial^2}{\partial z^2} - a^2\right) w = a^2 \rho_0 \alpha_t g\, \theta + a^2 \mu_0 K \left(-\frac{Q z}{k_1} + \frac{Q d}{2 k_1} - \beta\right) \frac{\partial \phi}{\partial z} - a^2 \frac{\mu_0 K^2}{1+\chi} \left(-\frac{Q z}{k_1} + \frac{Q d}{2 k_1} - \beta\right) \theta$$ **Equation 24**

$$\frac{\partial \theta}{\partial t} - \frac{\mu_0 K T_0}{\rho_0 C_0} \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z}\right) = \kappa \left(\frac{\partial^2}{\partial z^2} - a^2\right)\theta + \left(1 - \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_0}\right)\left(\frac{Q z}{k_1} - \frac{Q d}{2 k_1} + \beta\right) w$$ **Equation 25**

$$(1+\chi)\frac{\partial^2 \varphi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right) a^2\, \varphi - K \frac{\partial \theta}{\partial z} = 0$$ **Equation 26**

Thus, Equation 24,Equation 26 are the governing linearized perturbation equations and they are non- dimensional zed using the following quantities:

$$z = z^* d,\;\; w = \frac{\nu}{d} w^*,\;\; a = \frac{1}{d} a^*,\;\; t = \frac{d^2}{\nu} t^*,\;\; \theta = \frac{\beta\, \nu\, d}{\kappa} \theta^*,$$

$$\varphi = \frac{K\, \beta\, \nu\, d^2}{(1+\chi)\, \kappa} \varphi^*,\quad \delta = \frac{1}{\mu_0 (M_0 + H_0)} \delta^*$$ **Equation 27**

After using Equation 25 in Equation 22, Equation 24, we obtain (ignoring the asterisks)

$$\{1+\delta\} \left(D^2 - a^2\right)^2 w - \frac{\partial}{\partial t}\left(D^2 - a^2\right) w = a^2 R_t\, \theta - a^2 R_m\, f(z)\left(D\phi - \theta\right)$$

$$\left(D^2 - a^2\right)\theta - \Pr \frac{\partial \theta}{\partial t} + \Pr M_2 \frac{\partial}{\partial t}(D\varphi) = f(z)(1 - M_2)\, w$$ **Equation 28**

$$\left(D^2 - a^2 M_3\right)\varphi - D\theta = 0$$ **Equation 29**

Where $f(z) = N_s (1 - 2z) - 1$. .

We set

$$\{w,\; \theta,\; \varphi\}(z, t) = \{W,\; \Theta,\; \Phi\}(z)\, exp[\omega t]$$ **Equation 30**

Here, ω denoted as the growth rate, which is complex frequency.

Substituting into Equation 26 ,Equation 28, we obtain

$$\left[(1+\delta)(D^2-a^2)-\omega\right]\left(D^2-a^2\right)W=a^2R_t\Theta-a^2R_m f(z)(D\Phi-\Theta)$$ **Equation 31**

$$\left(D^2-a^2-\Pr\omega\right)\Theta=[N_S(1-2z)-1](1-M_2)W$$ **Equation 32**

$$\left(D^2-a^2M_3\right)\Phi-D\Theta=0$$ **Equation 33**

The boundary conditions for these equations are

1. Lower boundary rigid-ferromagnetic at fixed temperature as

$$W(0)=DW(0)=\Theta(0)=\Phi(0)=0$$

2. Lower boundary rigid-ferromagnetic at fixed heat flux as

$$W(0)=DW(0)=D\Theta(0)=\Phi(0)=0$$ **Equation 34**

After linearizing the equations for balancing the surface tension gradient with shear stress at the free surfaces (Pearson 1958), we have

$$\tau_{xz}=\frac{\partial\sigma_s}{\partial x}=\eta_b(z)\frac{\partial u}{\partial z};\quad \tau_{yz}=\frac{\partial\sigma_s}{\partial x}=\eta_b(z)\frac{\partial v}{\partial z}$$ **Equation 35**

where σ_s is the surface tension and τ_{xz}, τ_{yz} are the shear stress Using Equation 37 yields

$$\frac{\partial^2\sigma_s}{\partial x^2}+\frac{\partial^2\sigma_s}{\partial y^2}=-\eta_0[1+\delta\,\mu_0(M_0+H_0)]\frac{\partial^2 w}{\partial z^2}$$ **Equation 36**

For most of the liquids as the temperature rises, the variation between the liquid and its vapor phase decreases. Thus, the suitable boundary conditions of surface tension at the free surfaces are

$$w=0;\eta_0[1+\delta\,\mu_0(M_0+H_0)]\frac{\partial^2 w}{\partial z^2}=\sigma_T\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$ **Equation 37**

Using Equation 39 and non-dimensionlizing the equations, we get

$$W = 0 \text{ and } (1+\delta)D^2W + Ma\,a^2\,\theta = 0$$ **Equation 38**

where, Ma denoted as the Marangoni number $\left(=\sigma_T \Delta T d / \mu\kappa\right)$.

Upper boundary free ferromagnetic at fixed heat flux is

$$W(1) = (1+\delta)D^2W(1) + Ma\ a^2\,\Theta(1) = D\Theta(1) + Bi\,\Theta(1) = D\Phi(1) = 0$$ **Equation 39**

were, Bi denoted as the Biot number $\left(=h\,d / k_t\right)$

3. METHOD OF SOLUTION

The GT is applied to obtain the problem of eigenvalue is to study the linear system of Equation 32 with Equation 35 and Equation 41. The unknown factors W, Θ and Φ can be expanded upon the complete set:

$$W_i = \sum_{i=1}^{n} A_i W_i(z), \quad \Theta_i = \sum_{i=1}^{n} C_i \Theta_i(z), \quad \Phi_i = \sum_{i=1}^{n} D_i \Phi_i(z)$$ **Equation 40**

Substitute in Equation 32 Equation 32, multiplying the resulting equations respectively by $W_i(z)$, $\Theta_i(z)$, $\Phi_i(z)$ and carrying out the integration by parts from z = 0 to z = 1 and using Equation 35 and Equation 41 we obtain

$$C_{ji}A_i + D_{ji}C_i + E_{ji}D_i = 0$$ **Equation 41**

$$G_{ji}A_i + H_{ji}C_i = 0$$ **Equation 42**

$$I_{ji}C_i + J_{ji}D_i = 0$$ **Equation 43**

From Equation 43,Equation 45 have a non-trivial solution if

$$\begin{vmatrix} C_{ji} & D_{ji} & E_{ji} \\ G_{ji} & H_{ji} & 0 \\ 0 & I_{ji} & J_{ji} \end{vmatrix} = 0.$$ **Equation 44**

Were

$$C_{ji} = (1+\delta)\left[< D^2W_j D^2W_i > + (2a^2+\omega) < DW_j DW_i > + a^2(a^2+\omega) < W_j W_i >\right]$$

$$D_{ji} = -a^2 R_t < W_j \Theta_i > + a^2 R_m < [N_s(1-2z)-1] W_j \Theta_i > + DW_j(1)\Theta_i(1) a^2 Ma$$

$$E_{ji} = -a^2 R_m < [N_s(1-2z)-1] W_j D\Phi_i >, G_{ji} = < [N_s(1-2z)-1] \Theta_j W_i >$$

$$H_{ji} = < D\Theta_j D\Theta_i + (a^2 + \omega \Pr) \Theta_j \Theta_i > + \Theta_i(1)\Theta_j(1)\ Bi$$

$$I_{ji} = < \Phi_j D\Theta_i >, \quad J_{ji} = < D\Phi_j D\Phi_i > + < \Phi_j \Phi_i > a^2 M_3$$

Where $< \cdots\cdots > = \int_0^1 (\cdots)\, dz.$

The eigenvalue is extracted from Equation 45. A trivial function W_i, Θ_i, Φ_i can be considered to satisfy the boundary conditions Equation 35, Equation 36 and Equation 41 by selecting the trial functions as

$$W_i = z^2(1-z) T^*_{i-1}, \quad \Phi_i = \xi_i = z^2\left(1 - \frac{2}{3}z\right) T^*_{i-1}$$

(i) For lower insulating case: $\Theta_i = z^{i-1} T^*_{i-1}$, **Equation 45**

(ii) For lower conducting case: $\Theta_i = z\left(1 - \frac{z}{2}\right) T^*_{i-1}$

Here T^*_i s denoted as the second kind Tchebyshev' polynomials, such that W_i, Θ_i, Φ_i satisfy Equation 35, Equation 36 and Equation 41 except, namely $(1+\delta)D^2W(1) + Ma\ a^2\,\Theta(1) = 0$ and $D\Theta(1) + Bi\,\Theta(1) = 0$ however the residual from this equations is incorporated as a residual from Eqs. Equation 32,Equation 34

4. NUMERICAL RESULTS AND DISCUSSION

It may be illustrated that Equation 41 with ω = 0 leads to the Marangoni number Ma corresponding wavenumber a with R_t, R_m, Ns, δ and M_3. The inner products concerned in the equations are assessed analytically in order to keep away from the errors in numerical integration. The reveals the computations that the convergence in resulting Ma_c crucially depends on δ. The presented results for $i = j = 8$ the order at which the convergence is attained, in general. The critical Marangoni number Ma_c is determined by corresponding critical wavenumber a_c. The results thus attained for different R_t, R_m, Ns, δ, M_3 and M_3 are existing graphically in Figure 2 and also in Table 1 and Table 2

To solve the eigenvalue problem from Equation 41 by employing the Galerkin-type of WRM. In order to confirm the numerical technique is applied, the values (Ma_c, a_c) and (R_{tc}, a_c) are very close to the existing values of Nield (1964) for $R_m = \delta = 0$ and Sparrow et al. (1964) for $R_m = Ma = \delta = 0$ under the limiting condition in Table 1 and Table 2, respectively. The comparisons of calculated present results agree well with results of previous numerical investigations are given in Table 1 and Table 2. It is evident that the results are in good agreement between the present and published previously. This validates the applicability and exactness of the method applied in solving the convective instability problem considered

Table 1 Comparison of (Ma_c, a_c) and (R_{tc}, a_c) for $R_m = \delta = 0$

	Nield [18]				Present analysis			
	Lower insulating		Lower conducting		Lower insulating		Lower Conducting	
Bi	*Mac*	*ac*	*Rc*	*ac*	*Mac*	*ac*	*Rc*	*ac*
0	79.607	1.993	669	2.086	79.6067	1.9929	668.998	2.0856
0.01	79.991	1.997	670.38	2.089	79.9913	1.9966	670.381	2.0888
0.1	83.427	2.028	682.36	2.117	83.4267	2.0281	682.36	2.1162
0.2	87.195	2.06	694.78	2.144	87.1951	2.0603	694.779	2.1437
0.5	98.256	2.142	727.42	2.212	98.2562	2.1423	727.422	2.2116
1	116.127	2.246	770.57	2.293	116.127	2.2462	770.57	2.2928
2	150.679	2.386	831.27	2.393	150.679	2.3864	831.27	2.3926
5	250.598	2.598	925.51	2.519	250.598	2.5978	925.51	2.519
10	413.44	2.743	989.49	2.589	413.44	2.7426	989.492	2.5889
20	736	2.852	1036.3	2.632	736	2.8524	1036.3	2.6323
50	1699.62	2.941	1072.19	2.661	1699.62	2.9406	1072.19	2.6615
100	3303.83	2.976	1085.9	2.672	3303.83	2.9755	1085.9	2.6718
1000	32170.1	3.01	1099.12	2.681	32170.1	3.0101	1099.12	2.6813
1010	32.073´10^{10}	3.014	1100.65	2.682	32.073´10^{10}	3.0141	1100.65	2.6823

Table 2 Comparison of (R_{tc}, a_c) for $Ma = R_m = \delta = 0$ (lower and upper insulating case)

Bi	Sparrow et al. [19]		Present study	
	R_c a_c		R_c a_c	
0	320	0	320	8.72918´10^{-17}
0.01	338.905	0.58	338.905	0.5831
0.03	353.176	0.76	353.158	0.7623
0.1	381.665	1.015	381.665	1.0151
0.3	428.29	1.3	428.29	1.2992
1	513.792	1.64	513.79	1.6438
3	619.666	1.92	619.666	1.9211
10	725.15	2.11	725.148	2.1055
30	780.24	2.18	780.2238	2.176
100	804.973	2.2	804.973	2.2029
¥	816.748	2.21	816.746	2.2147

Figure 1,Figure 6 illustrates the neutral stability curves corresponding for different Bi, δ, Ns, M_3, R_m and R_t as well as different bounding surfaces (surfaces of lower insulating and lower conducting). The neutral stability curves are concave growing for each of these surfaces and the curves of lower conducting case lie above lower insulating surfaces. The neutral stability curves shift growing with increasing Bi Figure 2, δ Figure 3 representing that their result is to increase the stability region. Besides, decrease the stability of the region by increasing Ns Figure 4, M_3 Figure 5 R_m Figure 6 and R_t Figure 7

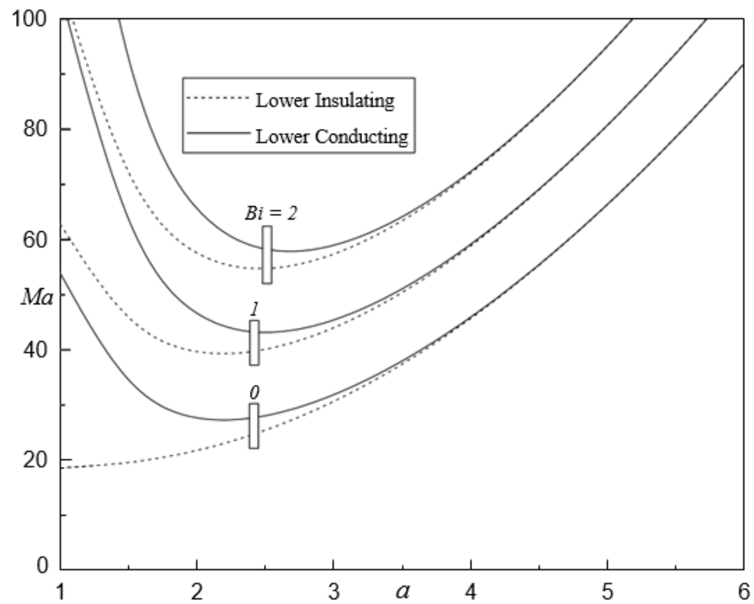

Figure 1 Ma against a for $R_1 = R_m = 50$, NS=5, $\delta = 0.5$ and $M_3 = 1$

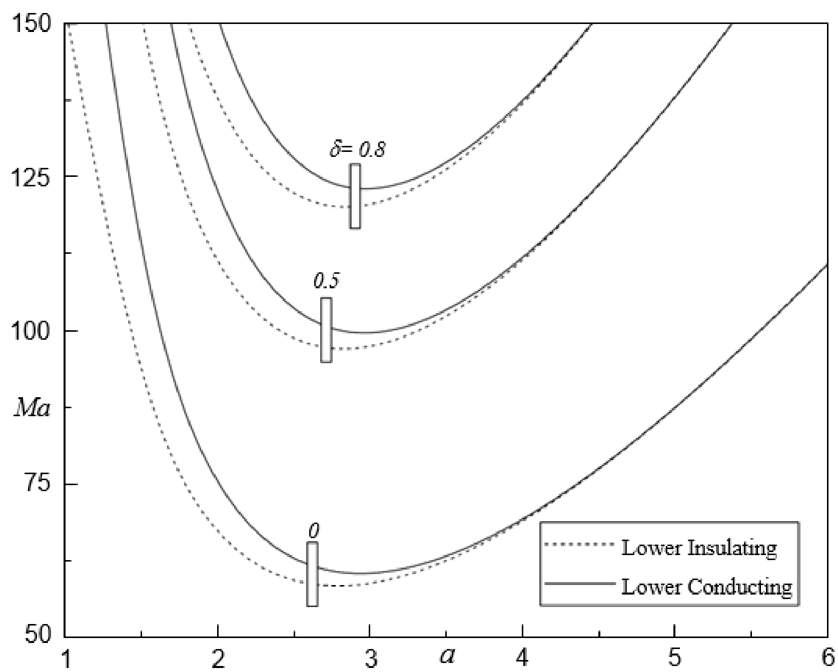

Figure 2 Ma against a for $R_t = R_m = 50$ $Ns = Bi = 5$ and $M_3 = 1$

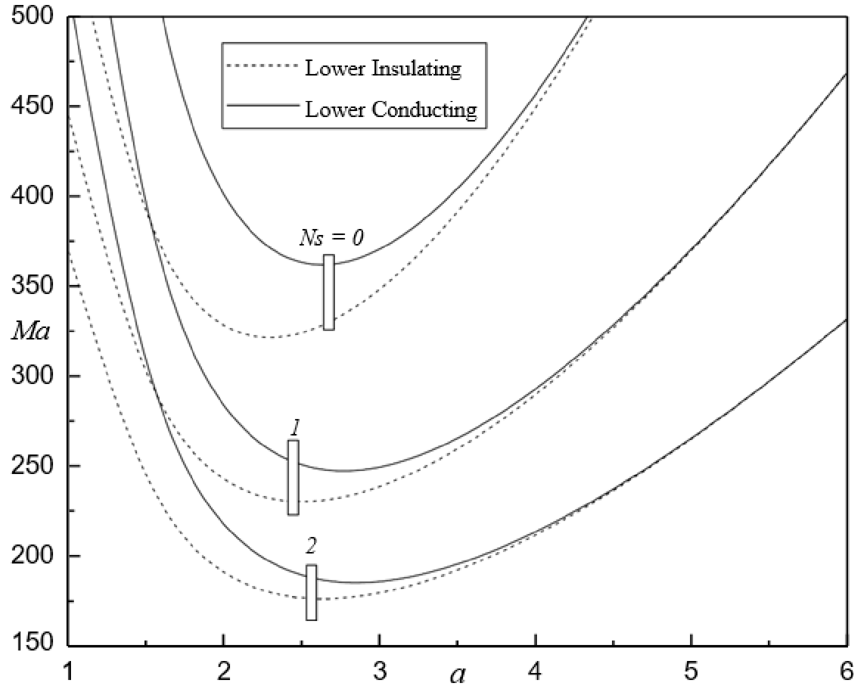

Figure 3 Ma against a for $R_t = R_m = 50$, $Bi = 5$, $\delta = 0.5$ and $M_3 = 1$

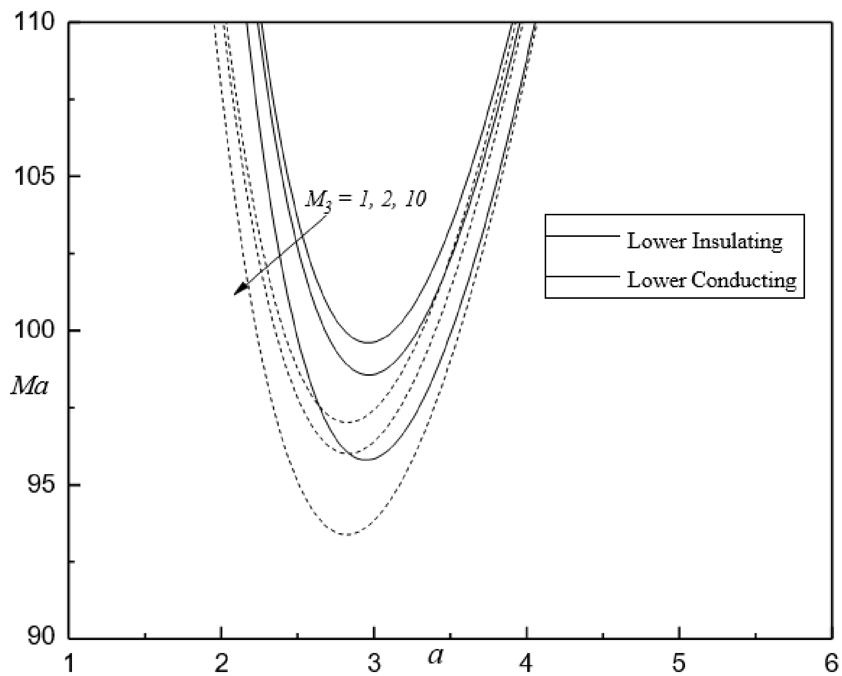

Figure 4 Ma against a for $R_t = R_m = 50$, $Bi = 5$, $\delta = 0.5$ and $Ns = 5$

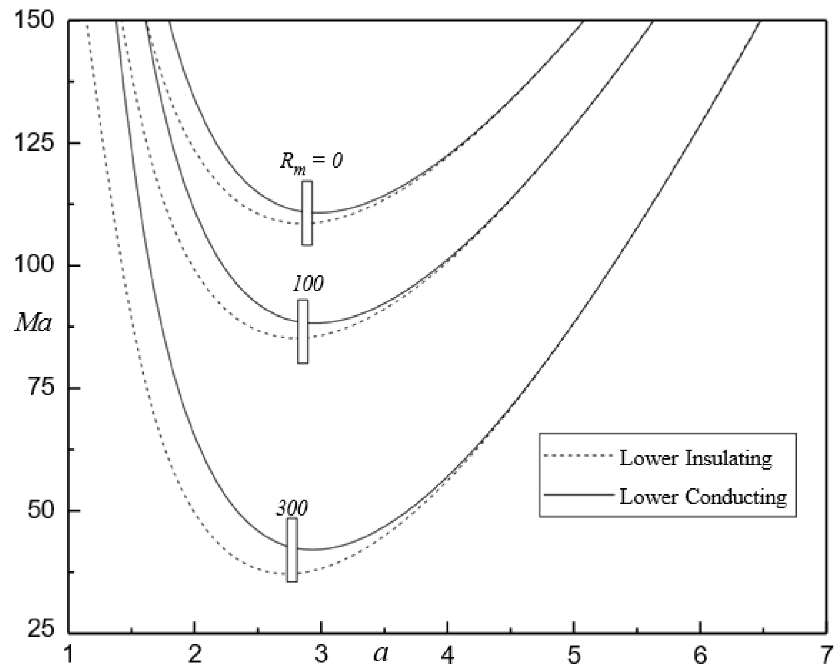

Figure 5 Ma against a for Rt 50, $Bi = Ns = 5, \delta = 0.5$ and $M_3 = 1$

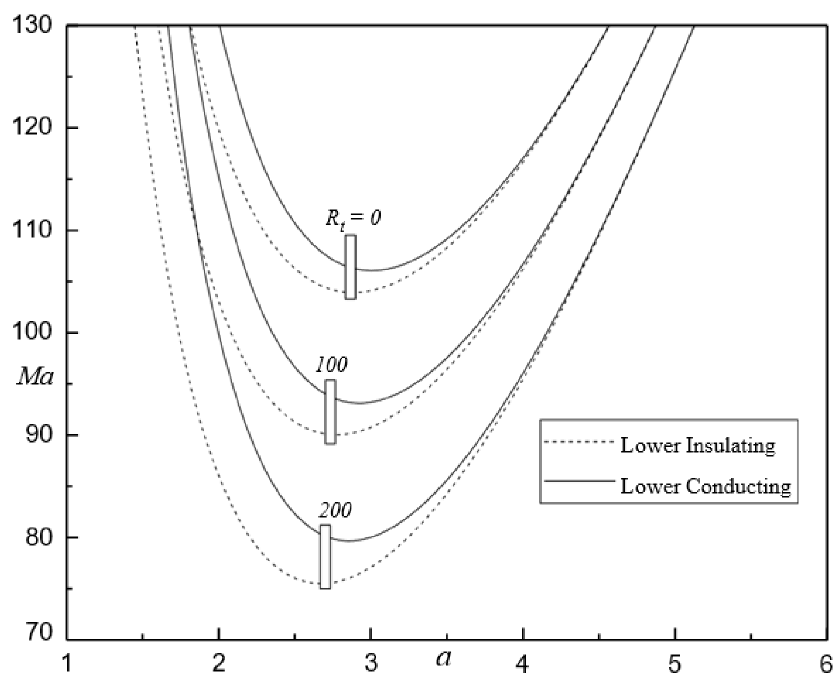

Figure 6 Ma against a for Rm $= 50, Bi = Ns = 5, \delta = 0.5$ and $M_3 = 1$

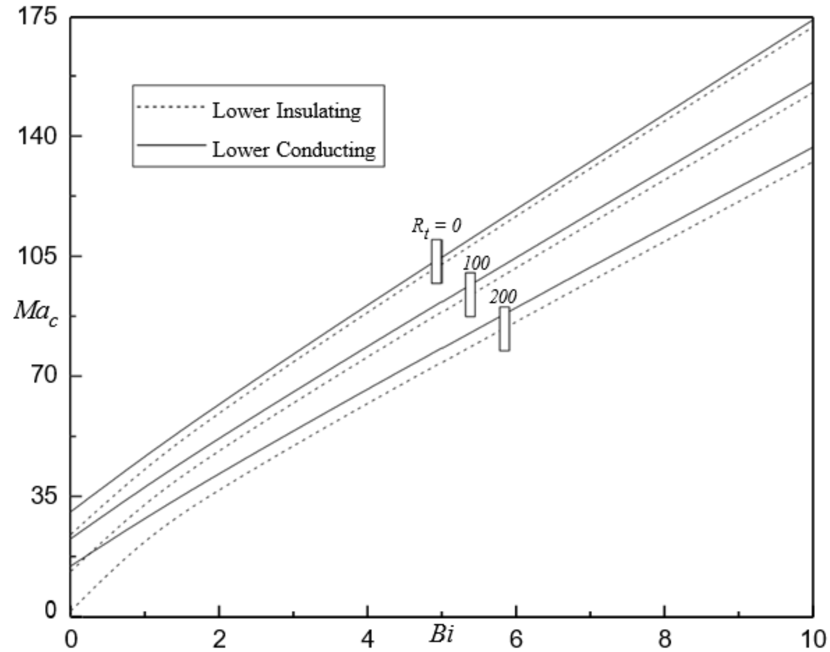

Figure 7 Ma_c against Bi for different R_t when $R_m = 50, Ns = 5, \delta = 0.5$ and $M_3 = 1$

In Figure 8,Figure 11 analogous to solid curves are corresponding to lower conducting and dotted curves corresponding to lower insulating. The plot of Ma_c against Bi for various R_t for $R_m = 50$, $Ns = 5$, $\delta = 0.5$ and $M_3 = 1$ Figure 7 It shows that the Ma_c value of lower conducting and lower insulating by increasing in Bi. Clearly, the results of lower insulating case are advancing the FTC compared to lower conducting. A further reveal that an increase in Bi is to delay the onset of FTC. This may be owing to the fact that with an increase in Bi, the boundary of free surface departs from good conductor of heat and hence there is an increase in Ma_c.

The effect of MFD viscosity parameter δ on the onset of FTC in a FF layer is presented in Figure 9 for fixed $R_m = 50$, $Ns = 5$, $Bi = 5$ and $M_3 = 1$. It is viewed that Ma_c increases with increasing δ indicating its effect is to stabilize the system. That is, the effect of δ increasing is to delay the FTC in the existence of magnetic field. To explore the effect of strength of dimensionless internal heat source N_s on the measure for the onset of FTC, the variation of Ma_c is displayed against N_s for $R_m = 50$, $\delta = 0.5$, $Bi = 5$ and $M_3 = 1$. in Figure 9. It is seen that R_{tc} decreases quite hastily first and then quite gradually monotonically with N_s representing the influence of increasing internal heating is to decrease Ma_c and thus destabilize the system. In particular, it is seen that the curves of Ma_c coalesce for various physical parameters as the strength of internal heating N_s is increased.

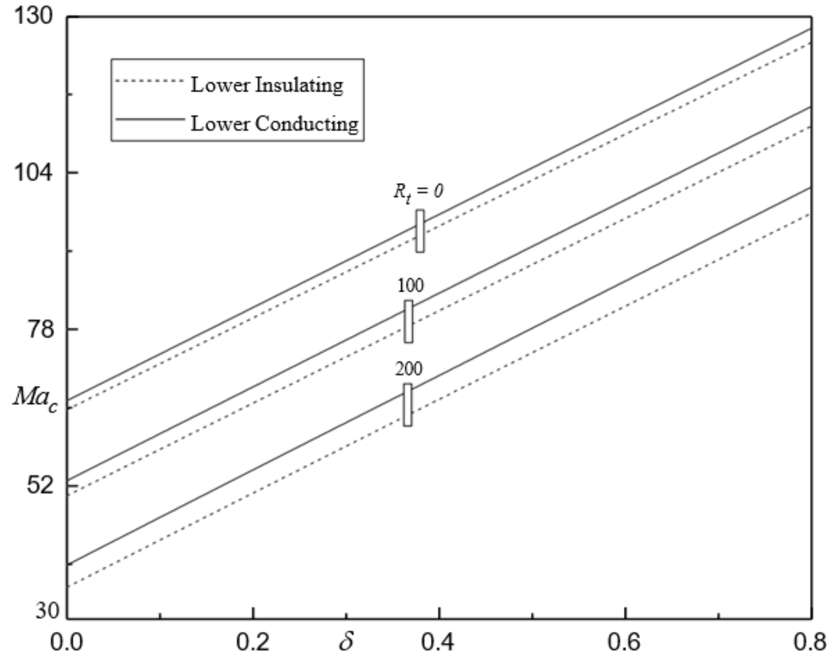

Figure 8 Ma_c against δ for different R_t when $R_m = 50$ $Ns = 5$ $Bi = 5$ and $M_3 = 1$

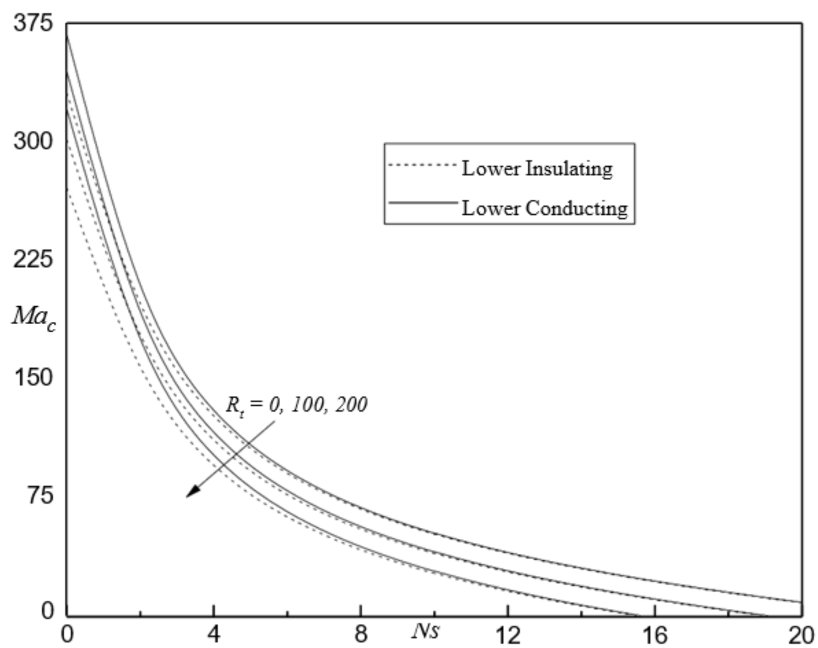

Figure 9 Ma_c against Ns for different R_t when $R_m = 50$, $\delta = 0.5$, $Bi = 5$ and $M_3 = 1$

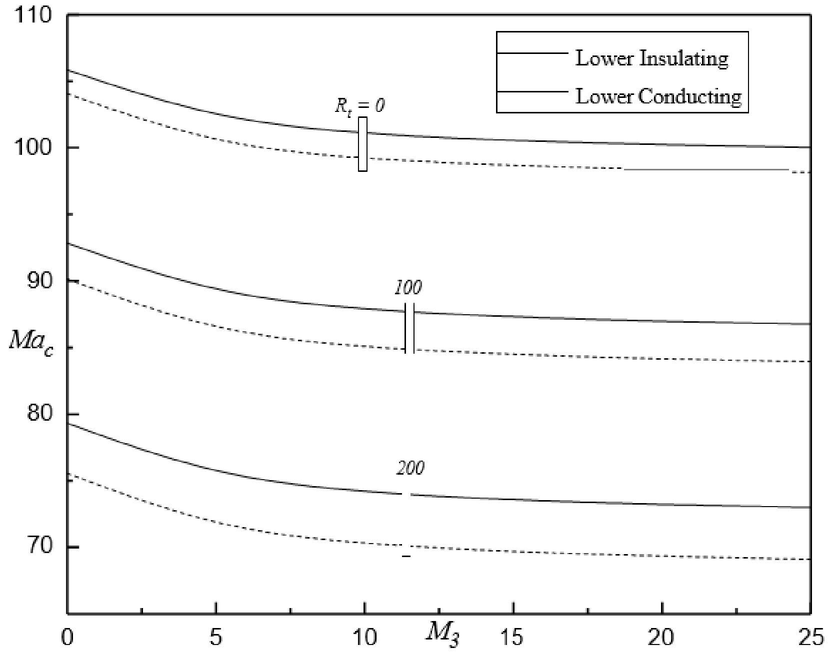

Figure 10 Ma_c against M_3 for different R_t when $R_m = 50$ $Ns = 5$ $Bi = 5$ and $\delta = 0.5$

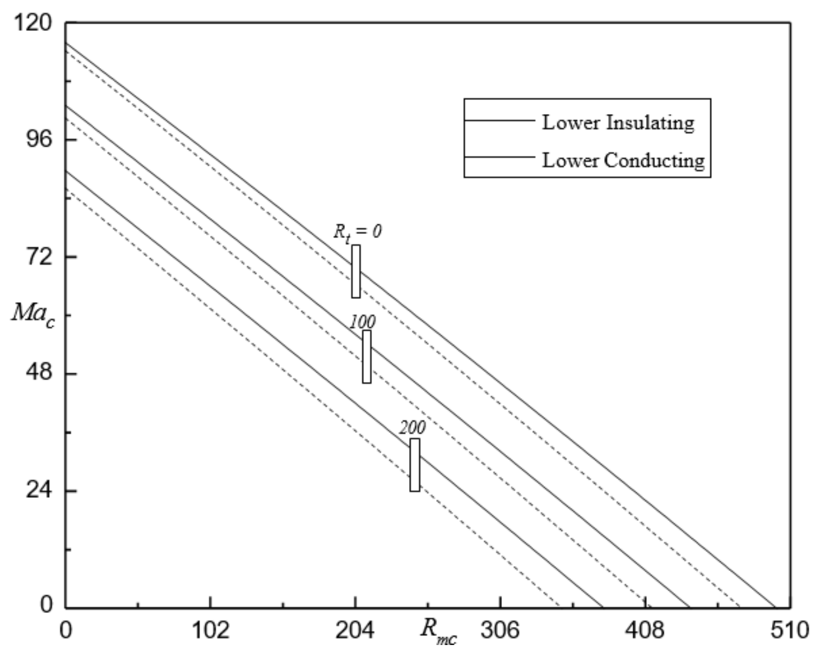

Figure 11 Ma_c against R_{mc} for different R_t when $M_3 = 1$ $Ns = 5$ $Bi = 5$ and $\delta = 0.5$

The effect of increase in nonlinearity of fluid magnetization (i.e. M_3) is shown in Figure 10 for different R_t when $R_m = 50$, $Ns = 5$, $Bi = 5$ and $\delta = 0.5$. From the figure, it is seen that an increase in M_3 is to decrease Ma_c and thus increase in the magnetization has destabilizing effects on the system but this effect is very insignificant.

The locus plot of Ma_c against for various R_t for $M_3 = 1$, $Ns = 5$, $Bi = 5$ and $\delta = 0.5$ Figure 11. It shows that they are bridging the space between lower insulating and conducting cases by increasing in R_t. For $R_t = 0$, the case corresponds to only the surface tension force are in effect. The amount of $R_t \neq 0$ is associated to the importance of buoyancy gravitational force. It is observed that increase in R_t leads to decrease Ma_c and R_{mc} signifying that the FF carries more heat efficiency than the ordinary viscous fluid case. This due to an increase the destabilizing the coupled magnetic and surface tension forces with increasing buoyancy gravitational force, R_t, thus the more easily for fluid flow in the system.

5. CONCLUSIONS

The linear stability theory is applied to study the effect of MFD viscosity on coupled buoyancy-gravitational and surface-tension forces on FTC in a FF layer through the strength of internal heat source on the system under the conditions of lower insulating/conducting case. The FF layer is heated from below and its top surface is subjected to a surface-tension force decreasing linearly with temperature. The problem of resulting eigenvalue is obtained numerically by utilizing the Galerkin WRT technique. It is shown that the effect of MFD viscosity is to enhance the onset of FTC and hence MFD viscosity plays a stabilizing role on the system. The increase in buoyancy-gravitational force, the forces of magnetic and surface-tension effect is to destabilize the system. Their effects are complementary in the sense that the critical Ma_c and R_{mc} decrease with an increase in R_t. The increase in Bi, δ and decrease in Ns, M_3 are having stabilizing effect on the system.

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