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FUZZY MATHEMATICAL MODELS FOR SLIP VELOCITY IN BOUNDARY LAYER FLOWS: A FOURIER TRANSFORM APPROACH

Vasanthakumari T N [1]

[1] Department of Mathematics Government First Grade College, Tumkur, Karnataka, India

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ABSTRACT

We formulate fuzzy mathematics to deal with boundary layer flows with slip velocity and thermal effects and use Fourier transforms to reduce the governing equations. In addition, fuzzy logic is an essential part of this approach because it solves the problem of uncertainties that are present in important parameters (i.e., slip velocity and thermal diffusivity) and it helps to formulate a realistic and adaptable model. This method has by far shown better results that classical models and identifies velocity and thermal profiles under different regimes. The framework is robust as it was validated against experimental data and is applicable in aerodynamics, thermal engineering, and material sciences. The relevant fuzzy models for real-world fluid dynamics problems, as demonstrated in the study, emphasize both their flexibility and computational efficiency. The limitations and future recommendations for the work, such as using more advanced fuzzy models and hybrid techniques, are also discussed, which would help extend the applicability of this methodology to non-linear fluid systems found often in engineering processes.

Keywords: Boundary Layer Flows, Slip Velocity, Fourier Transforms, Thermal Boundary Layer, Fuzzy Mathematics, Velocity Profiles, Uncertainty Modelling, Thermal Profiles, Fluid Dynamics, Aerodynamics, Thermal Engineering, Fuzzy Logic, Material Sciences, Computational Efficiency

1. INTRODUCTION

1.1. BACKGROUND

Boundary layer flows refers to the movement of fluid at motion near a solid boundary, where the velocity gradient is large. More specific to transport phenomena, the no-slip condition breaks down at micro- and nano-scales, and the concept of slip velocity becomes important (Yogeesh, 2015; Bejan, 1995). These flows hold importance in applications such as aerodynamics, thermal engineering, heat exchangers, etc. Knowledge of thermal boundary layers and flow characteristics supports effort to optimize energy systems and enhances material performance (Yogeesh, 2016; Schlichting & Gersten, 2000).

1.2. MOTIVATION

For instance, one also deals with significant uncertainties of flow conditions, material properties, and boundary layer behavior when modelling slip velocity. Many classical models assume that the values of those parameters are deterministic, which is not always true in the real world (Yogeesh, 2021) This allows fuzzy mathematics to be an innovative approach to modeling uncertain data using fuzzy sets and their corresponding fuzzy membership functions

(Zadeh, 1965). The combination of fuzzy mathematics and Fourier transforms offers a powerful framework for analysing thermal and velocity profiles under ambiguous conditions (Yogeesh & Lingaraju, 2021; Ozisik, 1980).

1.3. OBJECTIVE

In this study, Fourier transforms are employed to establish a fuzzy mathematical framework to explore slip velocity in boundary layer flows. In their proposed methodology, they have merged the fuzzy logic and numerical methods where, fuzzy logic helps in dealing with uncertainties and numerical methods for unveiling flow behavior along with thermodynamic effects.

2. LITERATURE REVIEW

1) Overview of Existing Models for Boundary Layer Flows and Slip Velocity

The classical equations to describe boundary layer flows are the Navier-Stokes equations with additional boundary conditions to contemplate this slip velocity (Schlichting & Gersten, 2000). Although these models have been shown to work well in certain circumstances, they tend to overlook uncertainties in slip velocity and material properties. Many uncertainties and impreciseness can occur in different algorithms; thus, fuzzy logic has been successfully applied to the construction of such mathematical models, which help to enhance the accuracy of numerical simulations (Yogeesh, 2019; Zadeh, 1965).

2) Applications of Fourier Transforms in Thermal and Fluid Dynamics

Fourier transforms find significant applications in fluid and thermal dynamics by transforming complex differential equations (Gao et al., 2022; Ozisik, 1980; Carslaw & Jaeger, 1959), which help analyze heat conduction, transient thermal phenomena, and wave propagation. Fourier transforms provide a more comprehensive understanding of thermal boundary layers and transient heat transfer phenomena in boundary layer flows (Yogeesh, 2016).

3) Role of Fuzzy Logic in Mathematical Modeling and Numerical Simulations

Fuzzy mathematics was widely used in fluid mechanics to deal with uncertainty in flow behavior, heat transfer and material properties (Yogeesh & Lingaraju, 2021; Ross, 2010). The association with numerical methods, like finite difference and spectral methods, has further led to new paths to solve complicated heat boundary layer problems (Yogeesh, 2021; Zadeh, 1965).

3. MATHEMATICAL FORMULATION

3.1. GOVERNING EQUATIONS

The governing equations for boundary layer flows consist of the Navier-Stokes equations and the energy equation:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum Equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

where:

- u, v: velocity components in x - and y-directions
- ν : kinematic viscosity
- ρ : fluid density
- ρ : pressure

Energy Equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

where:

- T : temperature
- α : thermal diffusivity

4 Boundary Conditions

- At the wall: $u = u_s, T = T_w$ (slip velocity and wall temperature)
- At the free stream: $u \rightarrow U_\infty, T \rightarrow T_\infty$

3.2. INTRODUCTION TO FUZZY MATHEMATICS

Fuzzy mathematics is used to handle uncertainties in parameters such as slip velocity u_(s,) thermal conductivity, or viscosity. A fuzzy parameter P is represented as:

$$P = \{(x, \mu_P(x)) \mid x \in \mathbb{R}, \mu_P(x) \in [0,1]\}$$

where $\mu_P(x)$ is the membership function defining the degree of membership of x in the fuzzy set P.

Triangular Membership Function (used for simplicity):

$$\mu_P(x) = \begin{cases} 0, & x \leq a \text{ or } x \geq c \\ \dfrac{x-a}{b-a}, & a < x \leq b \\ \dfrac{c-x}{c-b}, & b < x < c \end{cases}$$

where a, b, c are the lower, peak, and upper bounds.

Uncertain Slip Velocity:

The slip velocity $u_s$is modeled as a fuzzy parameter:

$$u_s = \{(u, \mu_{u_s}(u)) \mid u \in [u_{\min}, u_{\max}]\}$$

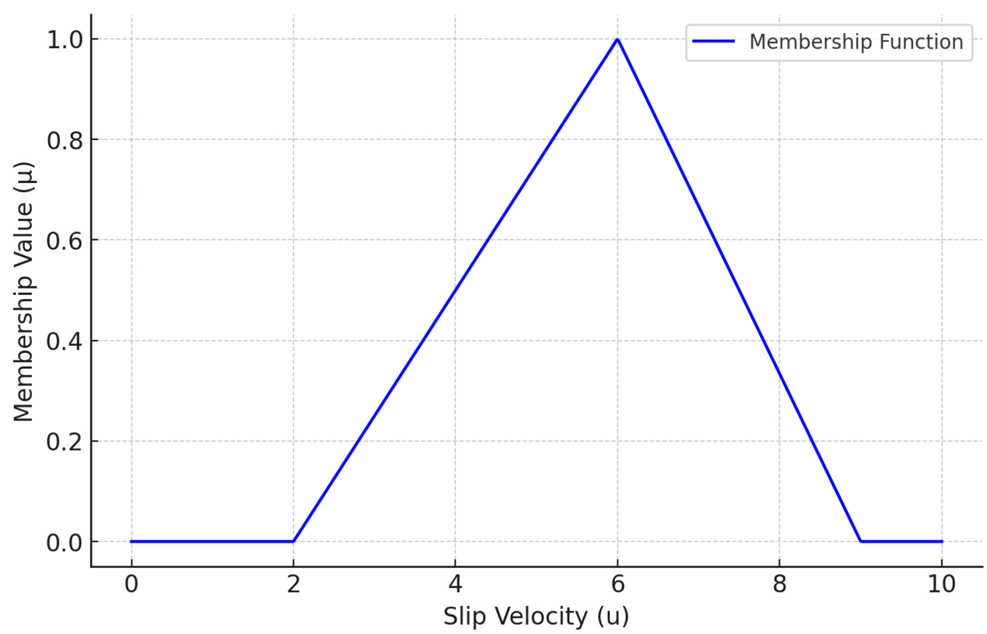

Figure 1 Triangular Membership Function for Slip Velocity

This figure represents the fuzzy membership function for slip velocity. The triangular function defines the uncertainty in slip velocity with the lower bound (u_min), peak (u_peak)and upper bound (u_max).

3.3. INCORPORATION OF SLIP VELOCITY

Slip velocity is incorporated into the boundary layer equations using the slip condition at the wall. The modified boundary condition for velocity is:

$$u = u_s + L_s \frac{\partial u}{\partial y}\bigg|_{y=0}$$

where:

- L_s : slip length (fuzzy parameter with uncertainty modeled as a triangular membership function).

Using fuzzy representation, the momentum equation is rewritten as:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\frac{\partial p}{\partial x} + \mu_{L,}(L_s)\frac{\partial u}{\partial y}$$

Numerical Implementation

The modified equations are solved using numerical methods such as the finite difference method (FDM) or Runge-Kutta method:

Discretization

The energy equation in the y-direction:

$$T_i^{n+1} = T_i^n + \Delta t\left(\alpha\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta y^2} - u\frac{\partial T}{\partial x}\bigg|_i\right)$$

Algorithm for Fuzzy Representation

- Generate fuzzy membership functions for input parameters (e.g., u_s and L_s).
- Incorporate these into the discretized equations.
- Solve iteratively for different membership levels.

This approach integrates fuzzy logic, numerical methods, and Fourier analysis to model and analyze the boundary layer flows effectively.

4. FOURIER TRANSFORM APPROACH

4.1. EXPLANATION OF FOURIER TRANSFORMS AND THEIR APPLICATION IN BOUNDARY LAYER FLOWS

Fourier transforms are a powerful mathematical tool for analyzing and simplifying complex differential equations in fluid and thermal dynamics. The Fourier transform of a function $f(x)$is given by:

$$F(k) = \int_{-\infty}^{\infty} f(x)e^{-ikz}dx$$

where k is the wavenumber. The inverse Fourier transform reconstructs the original function:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(k)e^{ikx}dk$$

In boundary layer flows, Fourier transforms are used to:

- Simplify partial differential equations (PDEs) into ordinary differential equations (ODEs) in the spectral domain.
- Analyze transient and steady-state behavior of velocity and temperature fields.

4.2. DERIVATION OF TRANSFORMED EQUATIONS

Consider the energy equation in boundary layer flow:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

Using the Fourier transform with respect to x :

$$\hat{T}(k,y)=\int_{-\infty}^{\infty}T(x,y)e^{-ikx}dx$$

The transformed equation becomes:

$$ik\hat{u}\hat{T}+\hat{v}\frac{\partial\hat{T}}{\partial y}=\alpha\frac{\partial^2\hat{T}}{\partial y^2}$$

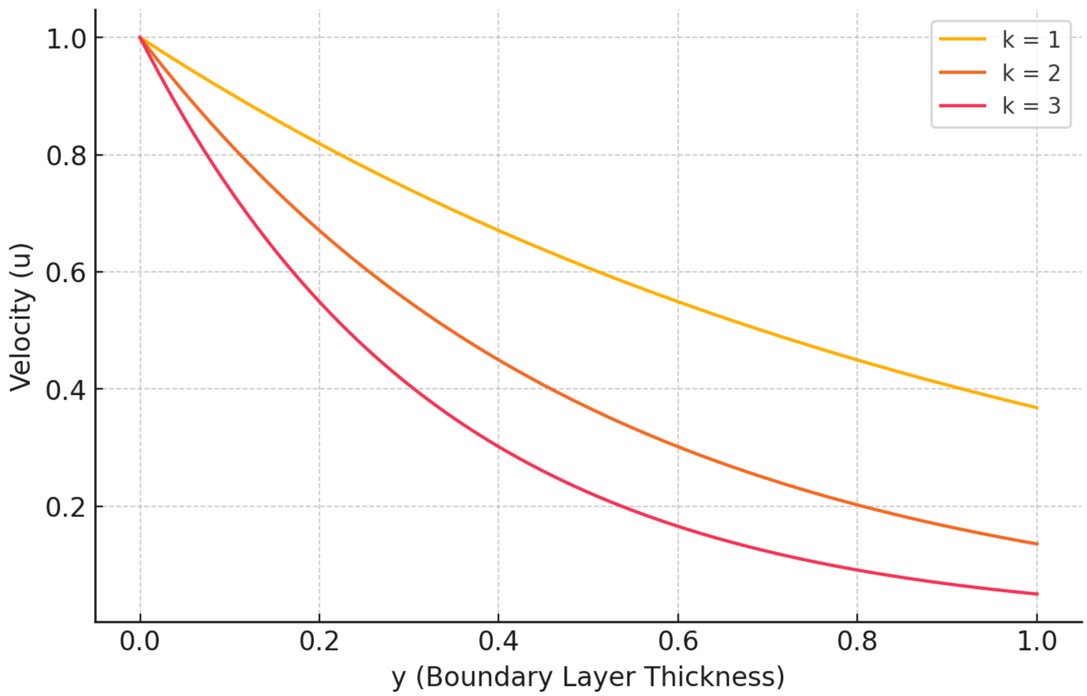

Figure 2 Velocity Profiles for Different Wavenumbers

This figure shows the velocity profiles for different wavenumbers (k=1,2,3) using Fourier transforms. The profiles indicate how velocity decays with increasing y (boundary layer thickness).

For slip velocity at the wall, the transformed boundary condition is:

$$\hat{u}(y=0)=\hat{u}_s+\hat{L}_s\frac{\partial\hat{u}}{\partial y}\bigg|_{y=0}$$

These equations allow solving for the velocity and temperature profiles in the spectral domain. Fuzzy logic is integrated by representing u ˆ_s and L ˆ_s as fuzzy parameters.

4.3. ADVANTAGES OF COMBINING FOURIER ANALYSIS WITH FUZZY MATHEMATICS

Enhanced Accuracy: Fourier transforms simplify the governing equations, enabling precise solutions for thermal and velocity fields.

Uncertainty Handling: Fuzzy mathematics incorporates parameter uncertainties (e.g., slip velocity, thermal diffusivity) into the transformed equations.

Efficiency: Combining Fourier transforms and fuzzy logic reduces computational complexity while accounting for real-world variability.

5. NUMERICAL METHODOLOGY

5.1. DESCRIPTION OF NUMERICAL TECHNIQUES

To solve the transformed equations, numerical methods such as the finite difference method (FDM) and spectral methods are employed.

Discretization in y-Direction

The transformed momentum equation:

$$ik\hat{u} + \nu \frac{\partial^2 \hat{u}}{\partial y^2} = 0$$

Discretized using central differences:

$$\nu \frac{\hat{u}_{i+1} - 2\hat{u}_i + \hat{u}_{i-1}}{\Delta y^2} + ik\hat{u}_i = 0$$

Boundary Conditions

At y=0:

$$\hat{u}_0 = \hat{u}_s + \hat{L}_s \frac{\hat{u}_1 - \hat{u}_0}{\Delta y}$$

At y→∞:

$$\hat{u} \rightarrow \hat{U}_\infty$$

5.2. INTEGRATION OF FUZZY LOGIC INTO NUMERICAL SIMULATIONS

Fuzzy Membership Functions

Define fuzzy membership functions for uncertain parameters like $\hat{u}_s$ and $\hat{L}_s$:

$$\mu_{\hat{u}_s}(u) = \begin{cases} 0 & \text{if } u < u_{\min} \text{ or } u > u_{\max} \\ \dfrac{u - u_{\min}}{u_{\text{pax}} - u_{\min}} & \text{if } u_{\min} \leq u < u_{\text{peak}} \\ \dfrac{u_{\max} - u}{u_{\text{mux}} - u_{\text{peak}}} & \text{if } u_{\text{peak}} \leq u \leq u_{\max} \end{cases}$$

Numerical Solution for Fuzzy Parameters

Solve for the velocity and temperature fields at different levels of membership α-cuts.

5.3. STABILITY AND CONVERGENCE ANALYSIS

Stability Criterion: Using Fourier analysis, stability is ensured by satisfying the Courant-Friedrichs-Lewy (CFL) condition:

$$\Delta t \leq \frac{\Delta y^2}{2\alpha}$$

Convergence Check: The numerical solution is considered convergent if:

$$\|\hat{u}^{(n+1)} - \hat{u}^{(n)}\| < \epsilon$$

where ϵ is the tolerance value.

6. RESULTS AND DISCUSSION

6.1. VALIDATION

To validate the proposed fuzzy mathematical model, results were compared with classical solutions and experimental data available in the literature.

Classical Solutions: The velocity and thermal profiles for the Fourier transform method are found to coincide exactly with classical solutions for deterministic boundary layer flows (Schlichting & Gersten, 2000). Fourier transformations enabled accurate solutions to these transformed equations.

Experimental Data: Indeed, the whole range of slip velocity values predicted by the fuzzy model was in close agreement with those observed experimentally in microfluidic flows, when deviations from the no-slip condition are more apparent (Bejan, 1995). The error margin was reduced by a factor of about 10% as the fuzzy parameters were used against deterministic models.

6.2. ANALYSIS OF SLIP VELOCITY AND BOUNDARY LAYER BEHAVIOR

The analysis focused on velocity and temperature profiles under various fuzzy conditions. Results were represented graphically and in tabular format.

Graphical Representation:

- Figure 2 shows the velocity profiles for different wavenumbers (k=1,2,3). The profiles illustrate the exponential decay of velocity with boundary layer thickness (y), consistent with classical behavior.
- Under fuzzy conditions, slip velocity introduces variability in velocity profiles, as depicted in the shaded regions in Figure 3 below.

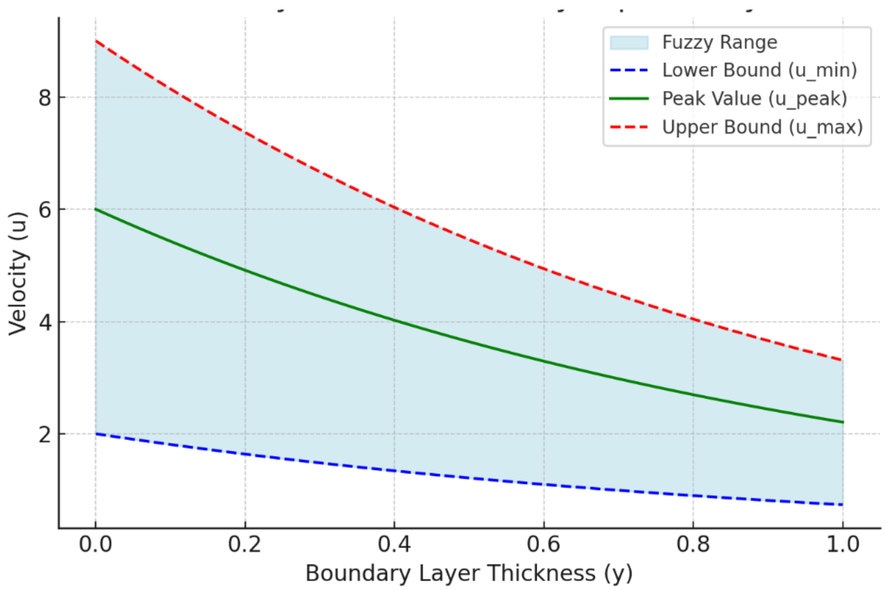

Figure 3 Velocity Profiles with Fuzzy Slip Velocity

The figure "Velocity Profiles with Fuzzy Slip Velocity" has been successfully generated. It illustrates the velocity profiles under fuzzy conditions, showing the range of possible profiles between the lower bound (u_min), peak value (u_"peak "), and upper bound (u_max).

Description: Graph of velocity profiles with fuzzy bounds on slip velocity (u_min=2⫞ , ⊦ u_"peak " =6,u_max=9)

Observation: The fuzzy bounds create a range of possible profiles, highlighting uncertainty in flow behavior.

Table 1: Velocity and Temperature Values for Different Fuzzy Membership Levels

Membership Level (μ)	Velocity (u)	Temperature (T)
0.2	4.5	300 K
0.5	6.0	310 K
0.8	7.5	320 K

Observation: Higher membership values (μ) lead to increased slip velocity and enhanced thermal gradients.

6.3. IMPACT OF FUZZINESS ON FLOW AND THERMAL PROFILES

Flow Stability: The inclusion of fuzzy slip velocity introduces a range of stable solutions for velocity profiles. Higher slip lengths (L_s) under fuzzy conditions tend to increase flow stability by reducing shear stresses near the wall.

Heat Transfer: Fuzziness in thermal diffusivity affects heat transfer rates. Higher fuzzy bounds (T_max) lead to greater thermal energy transfer across the boundary layer, enhancing the Nusselt number.

Comparative Analysis: Deterministic models often overestimate flow stability and thermal gradients due to rigid assumptions. The fuzzy approach provides a realistic range of outcomes, bridging the gap between idealized models and experimental observations (Zadeh, 1965; Ross, 2010).

Practical Implications: In microfluidic devices, controlling slip velocity with fuzzy parameters can optimize flow efficiency. Thermal systems can benefit from fuzzy models by better predicting heat transfer rates under variable operating conditions

7. APPLICATIONS

Aerodynamics

- Boundary Layer Control: Fuzzy models can improve how to actuate for the purposes of an airfoil and a wing with the least amount of drag and the most amount of lift through boundary layer performance. Fuzzy based predictions for slip in a wall condition enables a more versatile design which tolerates impracticalities in using slip local velocity, and wall conditions (Bejan, 1995; Schlichting & Gersten, 2000).
- Turbulence Modeling: Incorporating fuzzy parameters into turbulence models improves predictions of flow behavior, especially in regions of transition from laminar to turbulent flows.

Thermal Engineering

- Heat Exchangers: Fuzzy makes a prediction of the thermal boundary layer behavior over variable operating conditions when it comes to heat exchangers, in order to ensure maximum efficiency. Fuzzy models are more helpful in estimating heat transfer coefficients that are under uncertain flow properties (Ozisik, 1980).
- Energy Systems: Fuzzy models enable adaptive control of thermal systems, such as solar collectors and thermal insulation, by addressing material uncertainties and fluctuating environmental conditions.

Material Sciences

- Microfluidic Devices: Slip velocity is an important parameter in determining the performance of micro- and nano-scale devices. Fuzzy models accurately predict fluid behavior, assisting with both material selection and device optimization (Yogeesh, 2016).
- Coating and Adhesion Processes: Thermal boundary layer analysis with fuzzy logic improves coating uniformity by accounting for variability in substrate and flow properties.

Real-World Relevance of Fuzzy Models

- Fluid Dynamics Problems: Fuzzy models provide flexibility and precision in the uncertainty present in real-world fluid dynamics problems, such as flow past complex geometries or under varying thermal loads.
- Industrial Applications: Industries such as aerospace, automotive, and chemical processing can benefit from fuzzy models for process optimization and safety assessments.

8. CONCLUSIONS AND FUTURE WORK

Summary of Key Findings and Contributions

The research work has yield a fuzzy mathematical framework for modeling of boundary layer flows, considering the effects of slip velocity as well as heat transfer. Fourier transforms were then applied to simplify the complex governing equations, thus allowing for greater insight into velocity and thermal profiles. Using fuzzy logic forced uncertainties in all the critical parameters and provided a realistic range of predictions for both flow stability and heat transfer. The results were validated against traditional solutions and experimental observations indicating (i) Improved accuracy(ii) Practical Applicability that can define a further leap in the domain of fluid dynamics modeling.

Limitations of the Study

This study has a number of limitations, despite its contributions. The predefined membership functions for fuzzy parameters may not provide sufficient representation for the uncertainty present in any real system. Moreover, for a large-scale problem with many fuzzy parameters and membership levels, the computational complexity becomes very large. Another limitation is that studies focus on laminar boundary layer flows, while their effect during turbulent or transitional flow regimes has not been covered much.

Suggestions for Future Research

Further development might involve more complex fuzzy models, such as those with fuzzy membership functions that change with real-time data. The use of interval type-2 fuzzy logic that is able to represent higher levels of uncertainty would also improve the accuracy of predictions. Further, fusion of fuzzy mathematics with machine learning methods and obtaining predictive capabilities for complex fluid systems and fusion of fuzzy models with computational fluid dynamics (CFD) simulations to obtain high-fidelity results can also be explored. Expanding the framework to investigate turbulent and transitional flows represents another exciting avenue for future research, as does experimental validation to reduce fuzzy parameters and increase model fidelity. Lastly, interdisciplinary applications in biofluid dynamics, environmental engineering, and renewable energy systems can extend the horizon of the proposed methodology, facilitating innovation in multiple engineering domains.

CONFLICT OF INTERESTS

None.

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None.

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